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A Note on an Upper Bound for $B_q(n, d)$

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Abstract: In this correspondence, we have to obtain an upper bound for the value of $B_q(n, d)$, we have related to the bounds on the number of code words in a linear code C of length n. In particular we have given the exact inequality for $B_q(n, d)$.

Keywords: Minimum distance, upper bound, minimum Hamming distance, lower bound.

I. INTRODUCTION

Let f_q be a field having q elements, where $q = P^m$ (P a prime and $m \geq 1$). A linear code C of dimension k is a subspace of the vector space f_q^n over f_q . C contains n elements which are n-tuples $(x_1, x_2, \dots, x_n); x_i \in f_q, i=1$ to n) and elements. The elements of C are called code words of length n. The distance between two code words is define as follows.

First, the Hamming weight of a vector $\bar{u} = u_1, u_2, \dots, u_n$ is the number of non-zero u_i in \bar{u} written $wt(\bar{u})$.

Secondly, the Hamming distance between two vectors $\bar{u} = u_1, u_2, \dots, u_n$ and

$\bar{v} = v_1, v_2, \dots, v_n$ is the number of places where co-ordinates of \bar{u} and \bar{v} differ and it is denoted by $d(\bar{u}, \bar{v})$.

Evidently, $d(\bar{u}, \bar{v}) = wt(\bar{u} - \bar{v})$ as f_q^n is an abelian group with identity

$\bar{0} = 0, 0, 0, \dots, 0, \bar{u} - \bar{v} \in f_q^n$ and $wt(\bar{u} - \bar{v})$ is also well defined.

II. PRELIMINARY RESULTS

Where not given, Proofs or references for the results of this section may be found in section 2 of [7]

The Hamming weight of a vector \bar{u} denoted by $wt(\bar{u})$ is the number of non-zero entries in \bar{u} . For a linear code, the minimum distance is equal to the smallest of the weights of the non-zero code words. If C is an (n-k) code, we let A_i and B_i denoted the number of code words of weight i in C.

2.1 Definition The minimum distance of the code is the minimum Hamming distance between its code words. That is,

$$d = \min d(\bar{u}, \bar{v})$$

$$= \min wt(\bar{u} - \bar{v}), \bar{u}, \bar{v} \in C, \bar{u} \neq \bar{v}$$

$$(or) = \min wt(\bar{u}), \bar{u} \in C, \bar{u} \neq \bar{0}.$$

It is known that the minimum distance of a linear code is the minimum weight of any non-zero code word.

2.2 Definition A linear code of length n, dimension k, and minimum distance d is known as an

[n, k, d] code.

Bounds on the number of code words in a linear code C of length n, and minimum distance d having studied by various authors. See carry Huffman and Vera pless [1].

Theorem 2.3 (The Mac William's Identities)

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Let C be an $[n, k]$ code over GF (q). Then the A_i 's and B_i 's satisfy

$$\sum_{j=0}^{n-t} \binom{n-j}{t} A_j - q^{k-t} \sum_{j=0}^t \binom{n-j}{n-t} B_j \text{ for } t=0, 1 \dots n$$

Lemma 2.4 For an (n, k, d) code over GF (q), $B_i = 0$ for each value of i (where $1 \leq i \leq k$) such that there does not exist an $(n-i, k-i+1, d)$ code.

Lemma 2.5 Suppose \vec{u} and \vec{v} are linearly independent vector in $V(n, q)$ then

$$wt(\vec{u}) + wt(\vec{v}) + \sum_{\lambda \in GF(q) \setminus \{0\}} wt(\vec{u} + \lambda \vec{v}) = q(n - z)$$

Where Z denotes the number of co-ordinates places in which both \vec{u} and \vec{v} have zero entries.

III. AN INEQUALITY FOR $B_q(n, d)$

It is known that $B_q(n, d)$ is a non-negative integer power of q. For an $[n, k, d]$ code $B_q(n, d) = q^k$

If $d > 1$ then $B_q(n, d) \leq B_q(n-1, d-1)$, for $q=2$ $B_2(n, d) = B_2(n-1, d-1)$. Also $B_q(n, n) = q$.

Theorem 3.1 For $d \geq 1$, if $n \geq 2d - 1$, then $B_q(n, d) \leq q^{d-1}(q - 1)^{n-d+1}$ (1.1)

Proof In [1] it is shown that $B_q(n, d) \leq q B_q(n - 1, d)$ (1.2)

Changing d to d-1 in (1.2) we obtain

$$B_q(n, d - 1) \leq B_q(n - 1, d - 1) \text{ (1.3)}$$

As a code word of length n and minimum distance atleast d is counted in a code word of minimum distance atleast d-1

$$B_q(n, d) \leq B_q(n, d - 1) \text{ (1.4)}$$

Form (1.3) and (1.4) we deduce that

$$B_q(n, d) \leq q B_q(n - 1, d - 1) \text{ (1.5)}$$

Suppose $n - d = m$, $m \geq 0$; Successive application of (1.5) $d - 1$ times.

$$B_q(n, d) \leq q^{d-1} B_q(m + 1, d) \text{ (1.6)}$$

As $m + 1 = n - (d - 1)$. We arrive at

$$B_q(n, d) \leq q^{d-1} B_q(n - (d - 1), d) \text{ (1.7)}$$

When $n - d + 1 \geq d$ (1.7) holds for $n \geq 2d - 1$

But $B_q(m + 1, d) \leq (q - 1)^{m+1}$

Then form (1.7) $B_q(n, d) \leq q^{d-1}(q - 1)^{n-d+1}$

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