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A Note on an Upper Bound for B_q(n, d)

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Abstract: In this correspondence, we have to obtain an upper bound for the value of B_q(n,d), we have related to the bounds on the number of code words in a linear code C of length n. In particular we have given the exact inequality for B_q(n, d).
Keywords: Minimum distance, upper bound, minimum Hamming distance, lower bound.

I. INTRODUCTION

Let f_q be a field having q elements, where $q=P^m$ (P a prime and $m\geq 1$). A linear code C of dimension k is a subspace of the vector space $f_q{}^n$ over f_q .C contains n elements which are n-tuples $(x_1, x_2, \ldots\ldots x_n); x_i \epsilon f_q$, i=1 to n) and elements. The elements of C are called code words of length n. The distance between two code words is define as follows.

First, the Hamming weight of a vector $\bar{u} = u_1, u_2, .. u_n$ is the number of non-zero $u_1$in $\bar{u}$ written $wt(\bar{u})$.

Secondly, the Hamming distance between two vectors $\bar{u} = u_1, u_2, .. u_n$ and

$\bar{v} = v_1, v_2, .. v_n$ is the number of places where co-ordinates of $\bar{u}$ and $\bar{v}$ differ and it is denoted by $d(\bar{u}, \bar{v})$.

Evidently, $d(\bar{u}, \bar{v}) = wt(\bar{u} - \bar{v})$ as $f_q{}^n$ is an abelian group with identity

$\bar{0} = 0,0,0, \ldots.0$, $\bar{u} - \bar{v} \epsilon$ $f_q{}^n$ and $wt(\bar{u} - \bar{v})$ is also well defined.

II. PRELIMINARY RESULTS

Where not given, Proofs or references for the results of this section may be found in section 2 of [7]

The Hamming weight of a vector $\bar{u}$ denoted by $wt(\bar{u})$ is the number of non- zero entries in $\bar{u}$.For a linear code, the minimum distance is equal to the smallest of the weights of the non- zero code words. If C is an (n-k) code, we let A_i and B_i denoted the number of code words of weight i in C.

2.1 Definition The minimum distance of the code is the minimum Hamming distance between its code words. That is,

$$d = \min d(\bar{u}, \bar{v})$$

$$= \min wt(\bar{u} - \bar{v}), \bar{u}, \bar{v} \epsilon C, \bar{u} \neq \bar{v}$$

$$(or) = \min wt(\bar{u}), \bar{u} \epsilon C, \bar{u} \neq 0.$$

It is known that the minimum distance of a linear code is the minimum weight of any non-zero code word.

2.2 Definition A linear code of length n, dimension k, and minimum distance d is known as an

[n, k, d] code.

Bounds on the number of code words in a linear code C of length n, and minimum distance d having studied by various authors. See carry Huffman and Vera pless [1].

Theorem 2.3 (The Mac William's Identities)

Let C be an [n, k] code over GF (q). Then the A_i's and B_i's satisfy

$$\sum_{j=0}^{n-t} \binom{n-j}{t} A_j - q^{k-t} \sum_{j=0}^{t} \binom{n-j}{n-t} B_j \text{ for } t=0, 1 \ldots n$$

Lemma 2.4 For an (n, k, d) code over GF (q), Bi =0 for each value of i (where $1 \leq i \leq k$) such that there does not exist an (n-i, k-i+1, d) code.

Lemma 2.5 Suppose $\bar{u}$ and $\bar{v}$ are linearly independent vector in V (n, q) then

$$wt(\bar{u}) + wt(\bar{v}) + \sum_{\lambda \in GF(q)\setminus\{0\}} wt(\bar{u} + \lambda\,\bar{v}) = q(n - z)$$

Where Z denotes the number of co-ordinates places in which both $\bar{u}$ and $\bar{v}$ have zero entries.

III. AN INEQUALITY FOR B_q(n, d)

It is known that B_q(n, d) is a non-negative integer power of q. For an [n, k, d] code $B_q(n, d) = q^k$

If d>1 then $B_q(n, d) \leq B_q(n-1, d-1)$, for q=2 $B_2(n, d) = B_2(n-1, d-1)$. Also $B_q(n, n) = q$.

Theorem 3.1 For $d \geq 1, if\ n \geq 2d-1$, then $B_q(n, d) \leq q^{d-1}(q-1)^{n-d+1}$ .... (1.1)

Proof In [1] it is shown that $B_q(n, d) \leq q\, B_q(n-1, d)$ .... (1.2)

Changing d to d-1 in (1.2) we obtain

$$B_q(n, d-1) \leq B_q(n-1, d-1) \qquad \text{................ (1.3)}$$

As a code word of length n and minimum distance atleast d is counted in a code word of minimum distance atleast d-1

$$B_q(n,d) \leq B_q(n, d-1) \qquad \text{.................. (1.4)}$$

Form (1.3) and (1.4) we deduce that

$$B_q(n,d) \leq q\, B_q(n-1, d-1) \qquad \text{.................. (1.5)}$$

Suppose $n - d = m,\ m \geq 0$; Successive application of (1.5) $d-1$ times.

$$B_q(n,d) \leq q^{d-1} B_q(m+1, d) \qquad \text{.................. (1.6)}$$

As $m + 1 = n - (d-1)$. We arrive at

$$B_q(n,d) \leq q^{d-1} B_q(n - (d-1), d) \qquad \text{.................... (1.7)}$$

When $n - d + 1 \geq d$ (1.7) holds for $n \geq 2d - 1$

But $B_q(m+1, d) \leq (q-1)^{m+1}$

Then form (1.7) $B_q(n,d) \leq q^{d-1}(q-1)^{n-d+1}$

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