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# Contra (i,j) (gsp)* - continuous Function in Bitopological Space

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***Abstract: In this paper we have introduced a new function of contra (i,j)$(gsp)^*$-continuous in bitopological spaces which is properly placed in between the class of closed sets and gsp-closed sets.***
***Key Words: contra (i,j) g-continuous, contra (i,j) gs-continuous,$\alpha$continuous, contra (i,j) gsp-continuous.***

## I. INTRODUCTION

A triple ( X, $\tau_i, \tau_j$ ) where X is a non-empty set and $\tau_i$ and $\tau_j$ are topologies on X is called a bitopological space." Kelly [20] introduced study of such spaces.In 1985, Fukutake [19 ] introduced the concepts of g-closed sets[10 ] in bitopological spaces. In the year 1994 , Maki.et.al [12] defined $\alpha g$ -closed sets in topological space.S.P. Arya and N. Tour [3] defined $gs$ -closed sets in 1990. Dontchev [8], Gnanambal [9] and Palaniappan and Rao [17] introduced gsp-closed sets. J.Dontchev [8] introduced gsp-closed sets in 1995. Levine [10] Devi.et.al is. In this paper the new function contra (i,j)$(gsp)^*$-continuousfunction is introduced. The concepts contra (i,j) g-continuous,contra (i,j) gs-continuous,contra (i,j) $\alpha$g- continuous,contra (i,j) gsp-continuous are defined few of their properties are studied.

## II. PRELIMINARIES

### *A. Definition 2.1*

"A subset A of topological space ( X, $\tau_i, \tau_j$ ) is called

1) a pre-open set[14] if $A \subseteq \text{int}(cl(A))$ and a pre-closed set if cl(int(A))$\subseteq A$
2) a semi-open set [11] if $A \subseteq cl(\text{int}(A))$ and a semi-closed set if
3) a semi-pre open set[1] if $A \subseteq cl(\text{int}(cl(A))$ and a semi-pre closed set[1] if
4) an $\alpha$-open set [15]if $A \subseteq \text{int}(cl(\text{int}(A)))$ and an $\alpha$-closed set[15] if
5) $regular\text{–}open$ set[14] if int(cl(A))=A and an $regular\text{–}closed$ set[14] if A=int(cl(A))"

### *B. Definition 2.2*

"A subset A of topological space ( X, $\tau_i, \tau_j$ ) is called

1) a generalized closed set (briefly (i,j) g-closed) [10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in
2) generalized semi-closed set(briefly) (i,j) gs-closed [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in $\tau_i$.
3) an $\alpha$- generalized closed set (briefly (i,j)$\alpha$g-closed) [12] if $\alpha\, cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in $\tau_i$.
4) a generalized semi pre-closed set (briefly (i,j) gsp-closed) [8] if sp$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in $\tau_i$.

### *C. Definition 2.3*

" A function $f$: ( X, $\tau_i, \tau_j$ ) $\rightarrow$ ( Y,$\sigma_i, \sigma_j$) is called

1) Contra (i,j) g-continuous [4] if $f^{-1}$(V) is a g-closed set of ( X, $\tau_i, \tau_j$ ) for every closed set V of ( Y,$\sigma_i, \sigma_j$).
2) Contra (i,j) $\alpha$g-continuous[9] if $f^{-1}$(V) is a $\alpha$g-closed set of ( X, $\tau_i, \tau_j$ ) for every closed set V of ( Y,$\sigma_i, \sigma_j$).
3) Contra (i,j) gs-continuous [7] if $f^{-1}$(V) is a gs-closed set of ( X, $\tau_i, \tau_j$ ) for every closed set V of ( Y,$\sigma_i, \sigma_j$).
4) contra (i,j) gsp-continuous [8] if $f^{-1}$(V) is a gsp-closed set of ( X, $\tau_i, \tau_j$ ) for every closed set V of ( Y,$\sigma_i, \sigma_j$).

### *D. Definition 3*

A function $f:(X,\tau_i,\tau_j)\rightarrow(Y,\sigma_{i,}\sigma_j)$ is called contra (i,j) (gsp)*-continuous if $f^{-1}$(v) is (i,j) (gsp)*-closed in $(X,\tau_i,\tau_j)$ for each open set v of $(Y,\sigma_{i,}\sigma_j)$ .

*E. Theorem 3.1*

Every contra continuous function is contra (i,j) (gsp)*-continuous

Let $f:(X,\tau_i,\tau_j)\rightarrow(Y,\sigma_{i,}\sigma_j)$ be a contra continuous map

Let v be any open set in $(Y,\sigma_{i,}\sigma_j)$.

Then the inverse image of $f^{-1}$(v) is (i,j) (gsp)*- closed in $(X,\tau_i,\tau_j)$.

Since every closed set is (i,j) (gsp)*- closed.

$f^{-1}$(v) is (i,j) (gsp)*- closed in $(X,\tau_i,\tau_j)$.

Therefore $f$ is contra (i,j) (gsp)*-continuous.

Converse is not true.

*F. Theorem 3.2*

Every contra (i,j) (gsp)*-continuous map is contra (i,j) g-continuous.But the converse is not true.

*1) Proof:* Let $f:(X,\tau_i,\tau_j)\rightarrow(Y,\sigma_{i,}\sigma_j)$ be a contra (i,j) (gsp)* continuous map.

Let v be any open set in $(Y,\sigma_{i,}\sigma_j)$.

Then the inverse image of $f^{-1}$(v) is (i,j) (gsp)*- closed in $(X,\tau_i,\tau_j)$.

Since every (i,j) (gsp)*- closed set is (i,j) g-closed.

$f^{-1}$(v) is (i,j) g-closed in $(X,\tau_i,\tau_j)$.

Therefore $f$ is contra (i,j) g-continuous.

*G .Example 3.3*

Let X= {a, b, c}= Y, $\tau_i$ ={$\varphi$,X,{c},{a, c}}, $\tau_j$ ={$\varphi$,X,{b},{a, b}}, $\sigma_i=\varphi$,Y,{a,c},

$\sigma_j=\varphi$,Y,{b,c}.

Let $f:(X,\tau_i,\tau_j)\rightarrow(Y,\sigma_{i,}\sigma_j)$ be the identity map.

Let us prove that $f$ is contra ( i,j) g- continuous.But not contra (i,j) (gsp)*-continuous.

We have proved that the (i,j) g-closed sets are all the subsets of X.

And the (i,j) (gsp)*- closed sets are $\varphi$, X, $\{c\}$,$\{a,b\}$,{a,c}.

$f^{-1}${a} ={a} is (i,j) g-closed in $(X,\tau_i,\tau_j)$.

But it is not (i,j) (gsp)*- closed in $(X,\tau_i,\tau_j)$.

Hence $f$ is contra ( i,j) g- continuous but not (i,j) (gsp)*-continuous.

*G. Theorem 3.4*

Every contra (i,j) (gsp)*-continuous map is contra (i,j) gs-continuous

*1) Proof:* Let $f:(X,\tau_i,\tau_j)\rightarrow(Y,\sigma_{i,}\sigma_j)$ be a contra (i,j) (gsp)* continuous map.

Let v be any open set in $(Y,\sigma_{i,}\sigma_j)$.

Then the inverse image of $f^{-1}$(v) is (i,j) (gsp)*- closed in $(X,\tau_i,\tau_j)$.

Since every (i,j) (gsp)*- closed set is (i,j) gs-closed.

$f^{-1}$(v) is (i,j) gs-closed in $(X,\tau_i,\tau_j)$.

Therefore $f$ is contra (i,j) gs-continuous.

*H. Example 3.5*

Let X= {a, b, c}= Y, $\tau_i$ ={$\varphi$,X,{c},{a, c}}, $\tau_j$ ={$\varphi$,X,{b},{a, b}}, $\sigma_i=\varphi$,Y,{a,c},

$\sigma_j=\varphi$,Y,{b,c}.

Let $f:(X,\tau_i,\tau_j)\rightarrow(Y,\sigma_{i,}\sigma_j)$ be the identity map.

Let us prove that $f$ is contra ( i,j) gs- continuous.But not contra (i,j) (gsp)*-continuous.

We have proved that the (i,j) gs-closed sets are all the subsets of X.

And the (i,j) (gsp)*- closed sets are $\varphi$, X, $\{c\}$,$\{a,b\}$,{a,c}.

$f^{-1}\{a\}$ ={a} is (i,j) gs-closed in ( X, $\tau_i, \tau_j$ ).

But it is not (i,j) (gsp)*- closed in ( X, $\tau_i, \tau_j$ ).

Hence $f$ is contra ( i,j) gs- continuous but not (i,j) (gsp)*-continuous.

*I. Theorem 3.6*

Every contra (i,j) $(gsp)^*$-continuous map is contra (i,j) $\alpha$g-continuous.

*1) Proof:* Let $f$ : ( X, $\tau_i, \tau_j$ ) → ( Y, $\sigma_i, \sigma_j$ ) be a contra (i,j) $(gsp)^*$ continuous map.

Let v be any open set in ( Y, $\sigma_i, \sigma_j$ ).

Then the inverse image of $f^{-1}$(v) is (i,j) (gsp)*- closed in ( X, $\tau_i, \tau_j$ ).

Since every (i,j) (gsp)*- closed set is (i,j) $\alpha$g-closed.

$f^{-1}$(v) is (i,j) $\alpha$g-closed in ( X, $\tau_i, \tau_j$ ).

Therefore $f$ is contra (i,j) $\alpha$g-continuous.

*J. Example 3.7*

Let X= {a, b, c}= Y, $\tau_i$ ={$\varphi$,X,{c},{a, c}}, $\tau_j$ ={$\varphi$,X,{b},{a, b}}, $\sigma_i$ = $\varphi$,Y,{a,c},

$\sigma_j$ = $\varphi$,Y,{a,c}.

Let $f$ : ( X, $\tau_i, \tau_j$ ) → ( Y, $\sigma_i, \sigma_j$ ) be the identity map.

Let us prove that $f$ is contra ( i,j) $\alpha$g- continuous.But not contra (i,j) (gsp)*-continuous.

We have proved that the (i,j) $\alpha$g-closed sets are all the subsets of X.

And the (i,j) (gsp)*- closed sets are $\varphi$, X, $\{c\}$,$\{a, b\}$,{a,c}.

$f^{-1}\{b\}$ ={b} is (i,j) $\alpha$g-closed in ( X, $\tau_i, \tau_j$ ).

But it is not (i,j) (gsp)*- closed in ( X, $\tau_i, \tau_j$ ).

Hence $f$ is contra ( i,j) $\alpha$g- continuous but not (i,j) (gsp)*-continuous.

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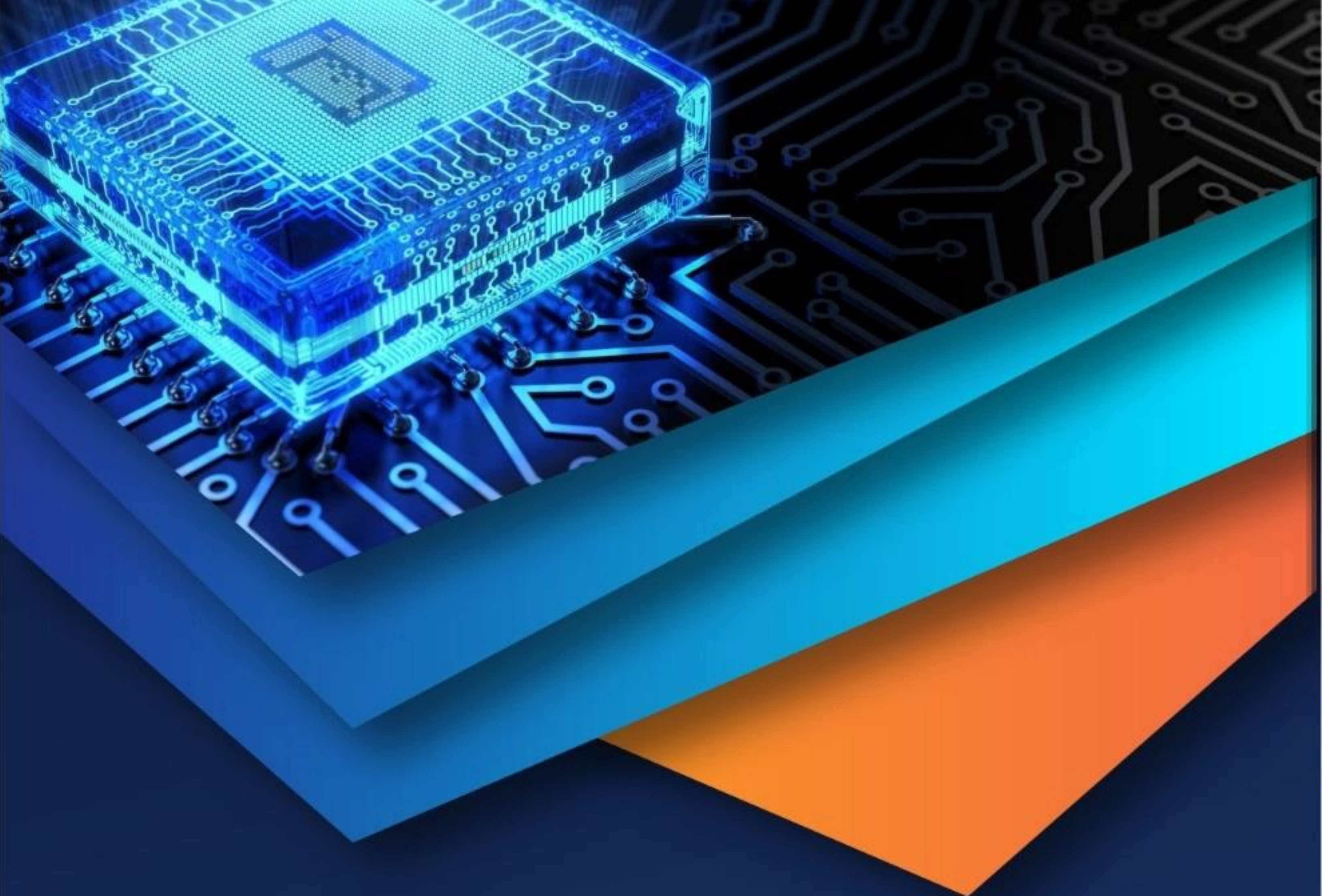


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