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Contra (gsp)*-Continuous Function in Topological Spaces

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Abstract: In this paper we have introduced a new function of contra (gsp)*-continuous in topological spaces which is properly placed in between the class of closed sets and gsp-closed sets.

Key Words: contra g-continuous, contra gs-continuous, α g-continuous, contra rg-continuous.

I. INTRODUCTION

Levine [10] introduced the class of g -closed sets in 1970. Maki.et.al [12] defined αg -closed sets in 1994. Arya and Tour [3] defined gs -closed sets in 1990. Dontchev [8], Gnanambal [9] Palaniappan and Rao[17] introduced gsp-closed set, rg -closed sets respectively.Veerakumar [18] introduced g^* -closed sets in 1991.J.Dontchev [8] introduced gsp-closed sets in 1995..The purpose of this paper is to introduce the concepts contra (gsp)*-continuous function.

II. PRELIMINARIES

A. *Definition 2.1:* A subset A of topological space (X, τ) is called

- 1) a pre-open set [14] if $A \subseteq \text{int}(cl(A))$ and a pre-closed set if $cl(\text{int}(A)) \subseteq A$
- 2) a semi-open set [11] if $A \subseteq cl(\text{int}(A))$ and a semi-closed set if $\text{int}(cl(A)) \subseteq A$
- 3) a semi-preopen set [1] if $A \subseteq cl(\text{int}(cl(A)))$ and a semi-preclosed set [1] if
- 4) an α -open set [15] if $A \subseteq \text{int}(cl(\text{int}(A)))$ and an α -closed set [15] if $cl(\text{int}(cl(A))) \subseteq A$
- 5) a *regular-open* set [14] if $\text{int}cl(A)=A$ and an *regular-closed* set [14] if $A = \text{int}cl(A)$

B. *Definition 2.2:* A subset A of topological space (X, τ) is called

- 1) a generalized closed set (briefly g -closed) [10] if $cl(A) \subseteq U$ whenever $A \subseteq U$
- 2) generalized semi-closed set (briefly gs -closed [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- 3) an α -generalized closed
- 4) set (briefly αg -closed) [12] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ)
- 5) a regular generalized closed set (briefly rg -closed) [17] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ)

C. *Definition 2.3:* A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called

- 1) Contra g -continuous [4] if $f^{-1}(v)$ is a g -closed set of (x, τ) for every open set v of (y, σ)
- 2) Contra αg -continuous [9] if $f^{-1}(v)$ is a αg -closed set of (x, τ) for every open set v of (y, σ)
- 3) Contra gs -continuous [7] if $f^{-1}(v)$ is a gs -closed set of (x, τ) for every open set v of (y, σ)
- 4) Contra rg -continuous [17] if $f^{-1}(v)$ is a rg -closed set of (x, τ) for every open set v of (y, σ)

D. *Definition 2.4 :* A function $f: (X, \tau) \rightarrow (Y, \sigma)$ from a topological space X into a topological space Y is said to be contra – continuous if $f^1(V)$ is closed in X for each open set V of Y .

E. Definition 2.5: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called contra(gsp)*-continuous if $f^{-1}(V)$ is (gsp)*-closed in (X, τ) for each open set V of (Y, σ) .

1) *Theorem:* Let a function $f: (X, \tau) \rightarrow (Y, \sigma)$ be a map, where $(X, \tau), (Y, \sigma)$ are spaces then f is contra(gsp)*-continuous iff the inverse image of every closed subset of (Y, σ) is gsp-open in (X, τ)

2) *Proof*

Let F be a closed subset in (Y, σ) .

Then $Y-F$ is open in (Y, σ)

Since f is contra(gsp)*-continuous, $f^{-1}(Y-F)$ is (gsp)*-closed.

But $f^{-1}(Y-F) = X - f^{-1}(F)$

Thus $f^{-1}(F)$ is gsp open in (X, τ)

Conversely,

Let G be an open subset in (Y, σ)

Then $Y-G$ is closed in (Y, σ)

Since the inverse image of every closed subset in (Y, σ) is gsp-open in (X, τ)

$f^{-1}(Y-G)$ is gsp open in (X, τ)

But $f^{-1}(Y-G) = X - f^{-1}(G)$

Thus $f^{-1}(G)$ is (gsp)*-closed.

Therefore f is contra(gsp)*-continuous.

F. Theorem: Every contra-continuous function is contra(gsp)*-continuous.

1) *Proof:* Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be contra-continuous.

Let V be any open set in Y .

Then the inverse image $f^{-1}(V)$ is closed in X .

Since every closed set is (gsp)*-closed.

$f^{-1}(V)$ is (gsp)*-closed in X .

Therefore f is contra(gsp)*-continuous.

G. Theorem: Every contra(gsp)*-continuous map is contra g-continuous.

But the converse is not true.

1) *Proof*

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be contra(gsp)*-continuous.

Let V be any open set in Y .

Then the inverse image $f^{-1}(V)$ is (gsp)*-closed in X .

Since every gsp*-closed set is g-closed. $f^{-1}(V)$ is g-closed in X . hence f is contra g-continuous.

2) *Example*

Let $X=Y= \{a, b, c\}$

$\tau= \{\emptyset, X, \{a\}, \{a,b\}\}$

$\sigma= \{\emptyset, Y, \{b\}\}$ Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Let us prove that f is contra g-continuous.

But not contra(gsp)*-continuous.

We have proved that the g-closed sets are

$\emptyset, X, \{c\}, \{a,c\}, \{b,c\}$

And the gsp*-closed sets are

$\emptyset, X, \{c\}, \{b,c\}$

$f^{-1}(\{a,c\}) = \{a,c\}$ is g-closed in (X, τ)

Thus the inverse of every closed set of (Y, σ) is g-closed in (X, τ) but not (gsp)*-closed in (X, τ) .

Hence f is contra g-continuous but not contra(gsp)*-continuous.

H. Theorem: Every contra(gsp)*-continuous map is contra α g-continuous.

1) *Proof*

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be contra(gsp)*-continuous.

To prove that, f is contra αg -continuous.

V be any open set in Y . Then the inverse image $f^{-1}(V)$ is $(gsp)^*$ -closed in X . Since every $(gsp)^*$ -closed set is αg -closed. $f^{-1}(V)$ is αg -closed in X .

Therefore f is contra αg -continuous.

2) *Example*

Let $X=Y= \{a, b, c\}$

$\tau= \{\varphi, X, \{a\}, \{a,b\}\}$

$\sigma =\{ \phi, Y, \{b\}\}$

Let $f: (X,\tau) \rightarrow (Y,\sigma)$ be the identity map.

Let us prove that f is contra αg -continuous

But not contra $(gsp)^*$ -continuous We have proved that the αg -closed sets are $\varphi, X, \{b\}, \{c\}, \{a,c\}, \{b,c\}, \{a,c\}, \{b,c\}$

$f^{-1}(\{a,c\})=\{a,c\}$ is αg -closed in (X,τ) but not $(gsp)^*$ -closed in (X,τ) . Hence f is contra αg -continuous but not contra $(gsp)^*$ -continuous.

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