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Heat Transfer for Fluid Flow over an Exponentially Stretching Porous Sheet with Surface Heat Flux in Porous Medium by means of Homotopy Analysis Method

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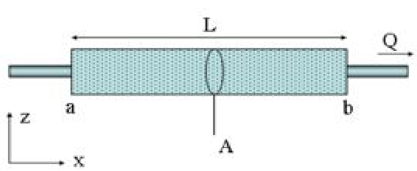
Abstract: In this study, the analytical solution of a viscous and incompressible fluid towards an exponentially stretching porous sheet with surface heat flux in porous medium, for the boundary layer and heat transfer flow, is presented. Similarity transformations are used to convert the partial differential equations corresponding to the momentum and heat equations into highly non-linear ordinary differential equations. Analytical solutions of these equations are obtained by homotopy analysis method (HAM). It is found that the skin-friction coefficient increases with increasing the permeability parameter as well as with the suction parameter. Momentum and thermal boundary layer thickness decrease with increasing exponential parameter.

I. INTRODUCTION

The steady two-dimensional laminar flow of a viscous incompressible electrically conducting fluid over an exponentially stretching sheet in the presence of a uniform transverse magnetic field with viscous dissipation and radiative heat flux. A porous medium or a porous material is a material containing pores (voids). The skeletal portion of the material is often called the "matrix" or "frame". The pores are typically filled with a fluid (liquid or gas). The skeletal material is usually a solid, but structures like foams are often also usefully analyzed using concept of porous media. Heat transfer over a stretching surface with uniform or variable heat flux in micropolar fluids is investigated in this Letter. The boundary layer equations are transformed into ordinary differential equations, and then they are solved numerically by a finite-difference method. The Stretching surface is a quiescent or moving fluid is important in number of industrial manufacturing processes that includes both metal and polymer sheets. An interesting fluid mechanical application is found in polymer extraction processes, where the object on passing between two closely placed vertical solid blocks is stretched in a region of fluid saturated porous medium. The stretching imparts a unidirectional orientation to the extradite, thereby improving its mechanical properties. The liquid is meant to cool the stretching sheet whose property depends greatly on the rate which it is cooled and stretched in porous medium. is an equation that describes the flow of a fluid through a porous medium. The law was formulated by Henry Darcy based on the results of experiments on the flow of water through beds of sand, forming the basis of hydrogeology, a branch of earth sciences.



The homotopy analysis method (HAM) is a semi-analytical technique to solve nonlinear ordinary/partial differential equations. The homotopy analysis method employs the concept of the homotopy from topology to generate a convergent series solution for nonlinear systems.

A. Formulation Of The Problem

Consider the flow of an incompressible viscous fluid past a flat sheet coinciding with the plane y = 0 in a porous medium with a non-uniform permeability k. In this analysis of the flow in the porous medium, the differential equation governing the fluid motion is based on Darcy's law. Consider Cartesian coordinates (x; y; z). It is assumed that the sheet is associated to a variable heat flux (VHF) $q_w(x)$. The fluid flow is limited to $y > 0$. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed. The effect of these two equal and opposite forces cause a symmetric boundary at the center (the origin as shown in Fig) of the porous medium. For two-dimensional flow, the velocity field is considered as

$$V = [u(x,y), v(x,y), 0]$$

where u and v are the velocity components in x and y directions respectively.

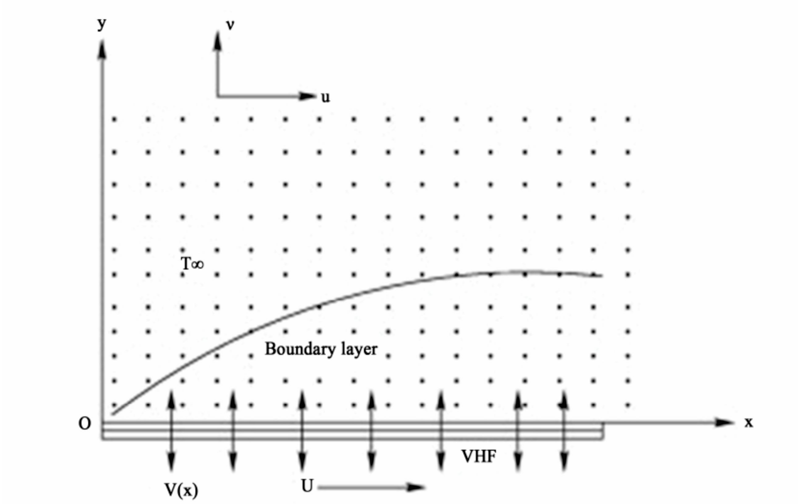


Figure : Geometrical representation of the problem.

The governing equations of continuity, momentum and energy are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\nu}{k}u, \qquad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2}, \qquad (3)$$

Thus $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity, ρ is the fluid density (assumed constant), μ is the coefficient of fluid viscosity, c_p is specific heat at constant pressure, κ is the thermal conductivity of the fluid. The appropriate boundary conditions for the problem are given by

$$at \quad y = 0, u = U, v = -V(x), \ \frac{\partial T}{\partial y} = -\frac{q_w(x)}{\kappa}, as \quad y \to \infty, u \to 0, T \to 0.$$

Here $U = U_0 e^{\frac{Nx}{L}}$ is the stretching velocity, $q_w(x) = q_{w0} T_0 \sqrt{\frac{U_0}{2\nu L}} e^{\frac{Nx}{L}}$ is the variable surface heat flux, U_0, T_0, q_{w0} are the reference velocity, temperature and heat flux respectively, $V(x) = V_0 e^{\frac{Nx}{2L}}$, a special type of velocity at the wall. Where V_0 is a constant, $V(x) > 0$ is the velocity of suction and $V(x) < 0$ is the velocity of blowing, $k = k_0 e^{-\frac{Nx}{L}}$ is the non-uniform permeability of the medium, k_0 is a constant which gives the initial permeability, N is the exponential parameter. However, for the sake of comparison, we shall consider the case of prescribed surface temperature (PST), $T = T_w$ at $y = 0$.

B. *Solution*

Introducing the similarity variables as

$$\eta = \sqrt{\frac{U_0}{2\nu L}} e^{\frac{Nx}{2L}} y, \qquad (4a)$$

$$u = U_0 e^{\frac{Nx}{L}} f'(\eta), \qquad (4b)$$

$$v = -N\sqrt{\frac{\nu U_0}{2L}} e^{\frac{Nx}{2L}} \{f(\eta) + \eta f'(\eta)\}, \qquad (4c)$$

$$T = \frac{q_{w0}}{\kappa} T_0 e^{\frac{Nx}{2L}} \theta(\eta), \qquad (4d)$$

By substituting the above equations of (4) in (2) and (3) we get the required solution

$$\boldsymbol{f''' - 2Nf'^2 + Nff'' - k_1 f'} \qquad (5)$$

$$\boldsymbol{\theta''(\eta) - Pr(Nf'\theta - N\theta' f) = 0} \qquad (6)$$

And the boundary conditions take the following form:

$at \quad \eta = 0, f' = 1, f = S, \theta' = -1, \quad and\ as \quad \eta \to \infty, f' \to 0, \theta \to 0$ Where the prime denotes differentiation with respect to η, $S = \frac{V_0}{\sqrt{\frac{\nu U_0}{2L}}} > 0\ (or < 0)$ is the suction parameter, $k_1 = \frac{2\nu L}{k_0 U_0}$ is the permeability parameter, $Pr = \frac{\mu c_p}{\kappa}$ is thePrandtl number.

C. The Solution Of The Problem By Means Of Ham

1) Zeroth-Order Deformation Problem

For the problem, the zeroth-order deformation is

$$(1-p)\mathcal{L}_1[\varphi(t,p) - f_0(t)] = ph_1 A_1[\varphi(t,p)]$$

$$\varphi(0,p) = S, \left.\frac{\partial\varphi(\eta,p)}{\partial\eta}\right|_{\eta=0} = 1$$

$$\left.\frac{\partial\varphi(\eta,p)}{\partial\eta}\right|_{\eta=\infty} = 0$$

$$(1-p)\mathcal{L}_2[\Psi(t,p) - \theta_0(t)] = ph_2 A_2[\varphi(t,p), \Psi(t,p)]$$

$$\Psi(\infty,p) = 0, \left.\frac{\partial\Psi(\eta,p)}{\partial\eta}\right|_{\eta=0} = 1$$

Thus $A_1[\varphi(t,p)] = \frac{\partial^3\varphi(\eta,p)}{\partial\eta^3} + N\varphi(\eta,p)\frac{\partial^2\varphi(\eta,p)}{\partial\eta^2} - 2N\left(\frac{\partial\varphi(\eta,p)}{\partial\eta}\right)^2 - K_1\frac{\partial\varphi(\eta,p)}{\partial\eta}$

and $A_2[\varphi(t,p),\Psi(t,p)] = \frac{\partial\Psi^2(\eta,p)}{\partial\eta^2} + Pr\left(N\varphi(t,p)\frac{\partial\Psi(\eta,p)}{\partial\eta} - N\frac{\partial\varphi(\eta,p)}{\partial\eta}\Psi(t,p)\right)$

Where (p[0,1]) $h_i \neq (i = 1,2)$ are the respective embedding and auxiliary parameter such that $\phi(\eta,0) = f_0(\eta), \phi(\eta,1) = f(\eta)\ \ and\ \ \Psi(\eta,0) = \theta_0(\eta), \Psi(\eta,1) = \theta(\eta)$. Obviously when p varies from 0 to 1, $\phi(\eta,p)$ changes from the initial guess $f_0(\eta)$ to exact solution $f(\eta)$ and $\Psi(\eta,p)$ from $\theta_0(\eta)$ to $\theta(\eta)$.

By Taylor's series, we have,

$$\phi(\tau,p) = f_0(\tau) + \sum_{m=1}^{\infty} f_m(\tau)p^m$$

$$\Psi(\tau,p) = \theta_0(\tau) + \sum_{m=1}^{\infty} \theta_m(\tau)p^m$$

$$f_m(\tau) = \frac{1}{m!}\left.\frac{\partial^m\phi(\tau,p)}{\partial p^m}\right|_{p=0,} \qquad (7)$$

$$\theta_m(\tau) = \frac{1}{m!}\left.\frac{\partial^m\Psi(\tau,p)}{\partial p^m}\right|_{p=0,} \qquad (8)$$

D. Higher-Order Deformation Problem

The mth-order deformation problems are

$$\mathcal{L}_1[f_m(\tau) - \chi_m f_{m-1}(\tau)] = h_1 R_{1m}(f_{m-1}) \qquad (9)$$

$$f'_m(0) = 1,\ f'_m(\infty) = 0,\ f_m(0) = S \qquad (10)$$

$$\mathcal{L}_2[\theta_m(\tau) - \chi_m\theta_{m-1}(\tau)] = h_2 R_{2m}(f_{m-1},\theta_{m-1}) \qquad (11)$$

$$\theta'_m(0) = -1, \quad \theta_m(\infty) = 0 \qquad (12)$$

Where

$$R_{1m}(f_{m-1}) = f'''_{m-1} + \sum_{n=0}^{m-1}[Nf_n(\eta)f''_{m-1-n}(\eta) - 2Nf'_n(\eta)f'_{m-1-n}(\eta) - K_1 f'_{m-1}(\eta)]$$

$$R_{2m}(f_{m-1},\theta_{m-1}) = \theta''_{m-1}(\eta) + Pr\sum_{n=0}^{m-1}[Nf_n(\eta)\theta'_{m-1-n}(\eta) - Nf'_n(\eta)\theta_{m-1}(\eta)]$$

And $\chi_m = \begin{cases} 0, & m \leq 1, \\ 1, & m > 1 \end{cases}$

The general solution of the Equations (9)-(12) is

$$f_m(\eta) = f_m^*(\eta) + C_1^m + C_2^m\eta + C_3^m e^{-\eta}$$

$$\theta_m(\eta) = \theta_m^*(\eta) + C_4^m + C_5^m e^{-\eta}$$

In which $f_m^*(\eta)$ and $\theta_m^*(\eta)$ represent the special solution of Equations (9) and (11) and the integral constants $C_i(i = 1,\ldots,5)$ can be computed by employing the boundary conditions (10) and (12) as:

$$\boldsymbol{C_1^m + C_3^m = f_m^*(0)}, \qquad \boldsymbol{C_2^m - C_3^m = f_m^*(0)}, \qquad \boldsymbol{C_2^m = f_m^*}(\infty)$$

$$\boldsymbol{C_4^m = \theta_m^*}(\infty), \qquad \boldsymbol{C_5^m = \theta_m^*(0)}$$

II. CONCLUSION

In this paper, the analytical solutions for steady boundary layer flow and heat transfer over an exponentially stretching surface in porous medium with variable surface heat flux. The effect of suction parameter on a viscous incompressible fluid is to suppress the velocity field which in turn causes the enhancement of the skin-friction coefficient. Skin-friction coefficient is higher for suction than that of blowing. Momentum and thermal boundary layer thickness decrease with increasing N. The surface shear stress increases as the permeability parameter increases. With increasing Pr, temperature decreases and an increase in Prandtl number reduces the thermal boundary layer thickness. The homotopy analysis method is used to obtain the analytical solutions of a non linear Ordinary differential equations related to the boundary layer flow and heat transfer flow of a viscous and incompressible fluid towards an exponentially stretching porous sheet with surface heat flux in porous medium. The effect of the emerging parameters is discussed and the results are presented graphically.

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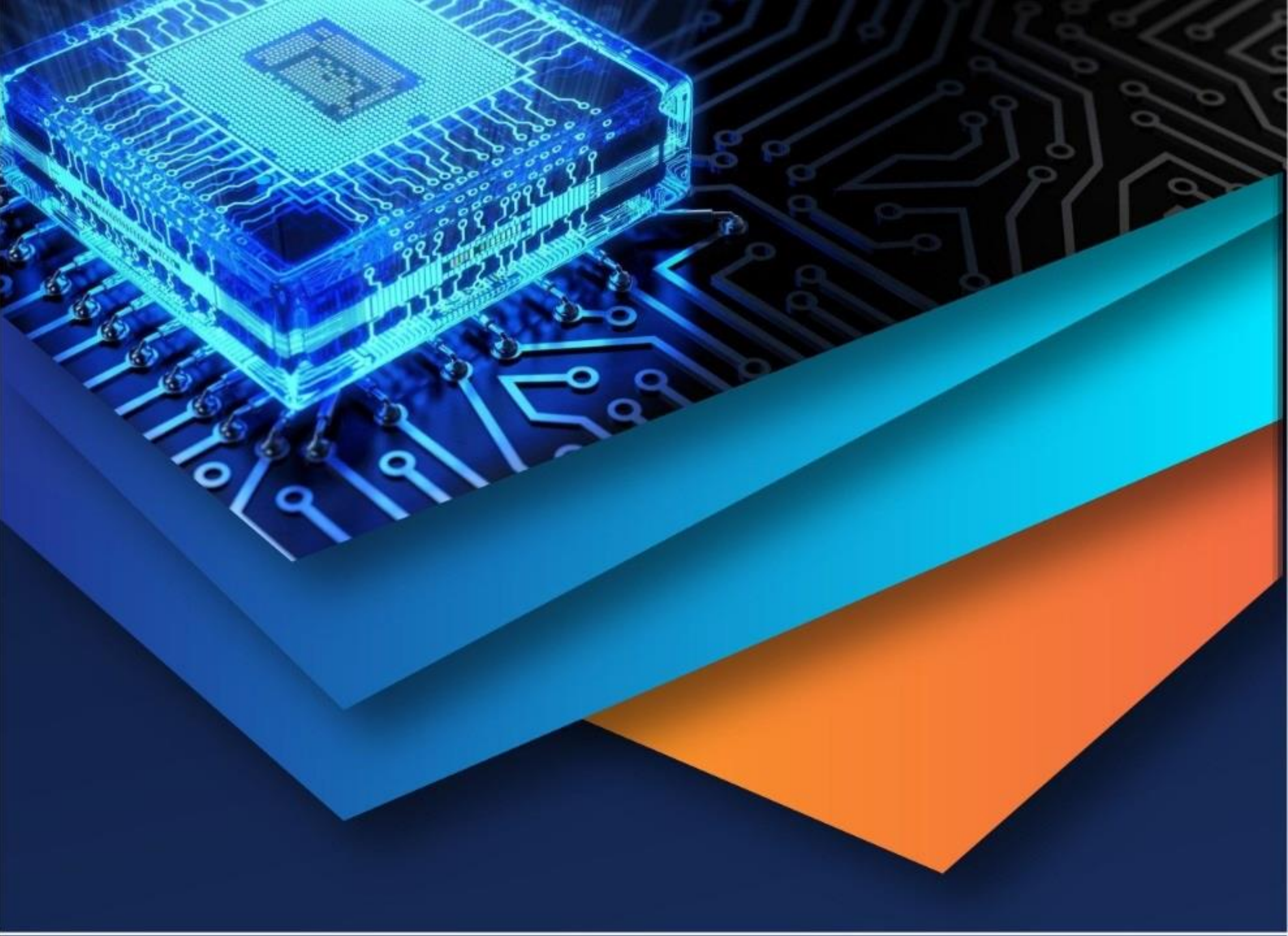

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