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Strongly (ĝ)* Closed Sets In Topological Space

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Abstract: In this paper, we study the concept of strongly (ĝ)* - closed sets and strongly (ĝ)*-continuous functions and check how they deal with the topological spaces and their sub-sets. We also read out how the strongly (ĝ)*-closed sets and maps inter-relates with other sets with a change or transformation in their properties.
Key words: cl(A), int(A), strongly (ĝ)*-closed set, g-closed set, g*-closed set, (ĝ)*-closed set, strongly (ĝ)*-continuous map, g-continuous map, g*-continuous map, (ĝ)*-continuous map

I. INTRODUCTION

Levine [4] introduced the class of g -closed sets in 1970. Veerakumar [5] introduced ĝ -closed sets in 1991. A. Gayathri [8] introduced the class of (ĝ)* sets in 2014. The intention of this paper is to give the basic properties of strongly (ĝ)*-closed set and strongly (ĝ)*-continuous map and how they work in relation with other sets and maps.

II. PRELIMINARIES

We see the non-empty topological space (X, τ), a subset A of X and an open set U of X. We also see the terms of closure of A i.e. Cl(A) and interior of A i.e. int(A).

A. Definition 2.1: Let A be a subset of a topological space (X, τ). The interior of A is defined as the union of all open sets contained in A. It is denoted by int(A).

B. Definition 2.2: Let A be a subset of a topological space (X, τ). The closure of A is defined as the intersection of all closed sets containing A. It is denoted by cl(A).

C. Definition 2.3: A subset A of the topological space (X, τ) is called

1) a pre-open set [7] if A $\subseteq$ int(cl(A))

2) a pre-closed set [7] if cl(int(A)) $\subseteq$ A

3) a semi-open set [4] if A $\subseteq$ cl(int(A))

4) a semi-closed set [4] if int(cl(A)) $\subseteq$ A

5) a semi-pre open set [1] if A $\subseteq$ cl(int(cl(A)))

6) a semi-pre closed set [1] if int(cl(int(A))) $\subseteq$ A

D. Definition 2.4: A subset A of a topological space (X, τ) is called

1) g-closed or generalized closed set [3] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).

2) g*-closed set [5] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in (X, τ).

3) ĝ -closed set [6] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in (X, τ).

4) (ĝ)*-closed set [8] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is ĝ -open in (X, τ).

E. Definition 2.5: A map f : (X, τ) → (Y, σ) is called

1) g-continuous [2] if f^{-1}(V) is a g-closed set of (X, τ) for every closed set V of (Y, σ).

2) g*-continuous [5] if f^{-1}(V) is a g*-closed set of (X, τ) for every closed set V of (Y, σ).

3) (ĝ)*-continuous [8] if f^{-1}(V) is a (ĝ)*-closed set of (X, τ) for every closed set V of (Y, σ).

III. BASIC PROPERTIES OF STRONGLY (Ĝ)*-CLOSED SET

A. Definition 3.1

A subset 'A' of a topological space (X, τ) is said to be a strongly (ĝ)*-closed set, if cl(int(A)) $\subseteq$ U, whenever A $\subseteq$ U and U is ĝ -open in X.

Theorem 3.2:
Every closed set is strongly ($\hat{g}$)* -closed.
Proof Let (X, τ) be a topological space.
And A $\subseteq$ (X, τ) is a closed set.
i.e. Cl(A) = A.
To prove: A is strongly ($\hat{g}$)*-closed.
Let A $\subseteq$ U and U be $\hat{g}$ open.
Then cl(A) $\subseteq$ U.
Also, cl(int(A)) $\subseteq$ cl(A)
We get, cl(int(A)) $\subseteq$ cl(A) $\subseteq$ U
i.e. cl(int(A)) $\subseteq$ U, whenever A $\subseteq$ U and U is $\hat{g}$ open.
$\therefore$ A is strongly ($\hat{g}$)*-closed.

B. Theorem 3.3

Every g-closed set is strongly ($\hat{g}$)*-closed.
1) Proof: Let A be a g-closed set.
By the definition 2.4.1,
Cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is open in (X, τ).
To prove: A is strongly ($\hat{g}$)*-closed.
Let A $\subseteq$ U and U is $\hat{g}$ open.
We've, cl(A) $\subseteq$ U
Also, cl(int(A)) $\subseteq$ cl(A)
Then, cl(int(A)) $\subseteq$ cl(A) $\subseteq$ U
i.e. cl(int(A)) $\subseteq$ U, whenever A $\subseteq$ U and U is $\hat{g}$ open in (X, τ).
$\therefore$ A is strongly ($\hat{g}$)*-closed.
The converse of the above theorem need not be true as shown in the following example.

C. Example 3.4

Let X = {a, b, c} and τ = {ϕ, X, {a}, {a, c}}
Closed sets are X, ϕ, {b, c}, {b}.
Semi-open sets are {a, c}, {a, b}, {a}, ϕ, X.
$\hat{g}$ -open sets are {a, c}, {a}, ϕ, X.
Strongly ($\hat{g}$)*-closed sets are {b}, {c}, {a, b}, {b, c}, ϕ, X.
g- closed sets are {b}, {a, b}, {b, c}, ϕ, X.
$\therefore$ A = {c} is a strongly ($\hat{g}$)*-closed set but not g-closed.
Hence, every strongly ($\hat{g}$)*-closed set need not be g-closed.

D. Theorem 3.5

Every g*-closed set is strongly ($\hat{g}$)*-closed.
1) Proof: Let A be a g*-closed set.
By the definition 2.4.2,
Cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is g-open in (X,τ).
To prove: A is strongly ($\hat{g}$)*-closed.
Let A $\subseteq$ U and U is $\hat{g}$ -open.
We've, cl(A) $\subseteq$ U
Also, cl(int(A)) $\subseteq$ cl(A)
Then, cl(int(A)) $\subseteq$ cl(A) $\subseteq$ U
i.e. cl(int(A)) $\subseteq$ U, whenever A $\subseteq$ U and U is $\hat{g}$ -open in (X, τ).

$\therefore$ A is strongly ($\hat{g}$)*-closed.
The converse of the above theorem need not be true as shown in the following example.

E. Example 3.6

Let X = {a, b, c} and τ = {ϕ, X, {a}, {a, c}}
Closed sets are X, ϕ, {b, c}, {b}
g-open sets are {a, c}, {c}, {a}, ϕ, X.
g*-closed sets are {b}, {a, b}, {b, c}, ϕ, X.
Strongly ($\hat{g}$)*-closed sets are {b}, {c}, {a, b}, {b, c}, ϕ, X.
$\therefore$ A = {c} is a strongly ($\hat{g}$)*-closed set but not g*-closed.
Hence, every strongly ($\hat{g}$)*-closed set need not be g*-closed.

F. Theorem 3.7

Every ($\hat{g}$)*-closed set is strongly ($\hat{g}$)*-closed.
*1) Proof:*Let A be a ($\hat{g}$)*-closed set.
By the definition 2.4.4,
Cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is $\hat{g}$ -open in (X, τ).
To prove: A is strongly ($\hat{g}$)*-closed.
Let A $\subseteq$ U and U is $\hat{g}$-open.
We've, cl(A) $\subseteq$ U
Also, cl(int(A)) $\subseteq$ cl(A)
Then, cl(int(A)) $\subseteq$ cl(A) $\subseteq$ U
i.e. cl(int(A)) $\subseteq$ U, whenever A $\subseteq$ U and U is open in (X, τ).
$\therefore$ A is strongly ($\hat{g}$)*-closed.
The converse of the above theorem need not be true as shown in the following example.

G. Example 3.8:

Let X = {a, b, c} and τ = {ϕ, X, {a}, {a, c}}
Closed sets are X, ϕ, {b, c}, {b}
g-open sets are {a, c}, {c}, {a}, ϕ, X.
($\hat{g}$)*-closed sets are {b}, {a, b}, {b, c}, ϕ, X.
Strongly ($\hat{g}$)*-closed sets are {b}, {c}, {a, b}, {b, c}, ϕ, X.
$\therefore$ A = {c} is a strongly ($\hat{g}$)*-closed set but not ($\hat{g}$)*-closed.
Hence, every strongly ($\hat{g}$)*-closed set need not be ($\hat{g}$)*-closed.

IV. BASIC PROPERTIES OF STRONGLY ($\hat{G}$)* -CLOSED CONTINUOUS MAPS

A. Definition 4.1

A map f : (X, τ) $\rightarrow$ (Y, σ) is called a strongly ($\hat{g}$)*- continuous map if f^{-1}(V) is a strongly ($\hat{g}$)*-closed set in (X, τ) for every closed set V of (Y, σ).

B. Theorem 4.2

Every continuous map is strongly ($\hat{g}$)*-continuous.
1) Proof: Let f : (X, τ) $\rightarrow$ (Y, σ) be a continuous map.
To prove: f is strongly ($\hat{g}$)*-continuous.
Let V be a closed set in (Y, σ).
Since, f is continuous; there exists a closed set f^{-1}(V) in (X, τ).
By theorem 3.2,
"Every closed set is a strongly ($\hat{g}$)* -closed."
Hence, f^{-1}(V) is a strongly ($\hat{g}$)* -closed set in (X, τ).

$\therefore f$ is strongly ($\hat{g}$)*-continuous.
The converse of the above theorem need not be true as shown in the following example.

C. Example 4.3:
Let X = Y = {a, b, c}
And τ = {ϕ, X, {a, b}}
Closed sets in (X, τ) are X, ϕ, {c}.
And σ = {ϕ, Y, {b, c}}
Closed sets in (Y, σ) are Y, ϕ, {a}.
Let f : (X, τ) → (Y, σ) be an identity map.
Semi-open sets are {a, b}, ϕ, X.
$\hat{g}$ -open sets are {a, b}, {a}, {b}, ϕ, X.
Strongly ($\hat{g}$)*-closed sets are {a}, {b}, {c}, {b, c}, {a, c}, ϕ, X.
$\therefore f^{-1}\{a\} = \{a\}$ is a strongly ($\hat{g}$)*-closed set in (X, τ) but not a closed set in (X, τ).
Thus, the converse of the above theorem is not true.
Hence, every strongly ($\hat{g}$)*- continuous map need not be continuous.

D. Theorem 4.4
Every g-continuous map is strongly ($\hat{g}$)*-continuous.
1) Proof: Let f : (X, τ) → (Y, σ) be a continuous map.
By the definition 2.5.1,
f^{-1}(V) is a g-closed set of (X, τ) for every closed set V of (Y, σ).
To prove: f is strongly ($\hat{g}$)*-continuous.
Let V be a closed set in (Y, σ).
Since, f is g-continuous; there exists a g-closed set f^{-1}(V) in (X, τ).
By theorem 3.3,
"Every g-closed set is strongly ($\hat{g}$)*-closed."
Hence, f^{-1}(V) is a strongly ($\hat{g}$)* -closed set in (X, τ).
$\therefore f$ is strongly ($\hat{g}$)*-continuous.
The converse of the above theorem need not be true as shown in the following example.

E. Example 4.5
Let X = Y = {a, b, c}
And τ = {ϕ, X, {a, b}}
Closed sets in (X, τ) are X, ϕ, {c}.
And σ = {ϕ, Y, {b, c}}
Closed sets in (Y, σ) are Y, ϕ, {a}.
Let f : (X, τ) → (Y, σ) be an identity map.
g- closed sets are {c}, {b, c}, {a, c}, ϕ, X.
Strongly ($\hat{g}$)*-closed sets in (X, τ) are {a}, {b}, {c}, {b, c}, {a, c}, ϕ, X.
$\therefore f^{-1}\{b\} = \{b\}$ is a strongly ($\hat{g}$)*-closed set in (X, τ) but not a g-closed set in (X, τ).
Thus, the converse of the above theorem is not true.
Hence, every strongly ($\hat{g}$)*-continuous map need not be g-continuous.

F. Theorem 4.6
Every g*-continuous map is strongly ($\hat{g}$)*-continuous.
1) Proof: Let f : (X, τ) → (Y, σ) be a continuous map.
By the definition 2.5.2,
f^{-1}(V) is a g*-closed set of (X, τ) for every closed set V of (Y, σ).
To prove: f is strongly ($\hat{g}$)*-continuous.

Let V be a closed set in (Y, σ).
Since, f is g*-continuous; there exists a g*-closed set f^{-1}(V) in (X, τ).
By theorem 3.5,
"Every g*-closed set is strongly ($\hat{g}$)*-closed."
Hence, f^{-1}(V) is a strongly ($\hat{g}$)* -closed set in (X, τ).
∴ f is strongly ($\hat{g}$)*-continuous.
The converse of the above theorem need not be true as shown in the following example.

G. Example 4.7
Let X = Y = {a, b, c}
And τ = {ϕ, X, {a, b}}
Closed sets in (X, τ) are X, ϕ, {c}.
And σ = {ϕ, Y, {b, c}}
Closed sets in (Y, σ) are Y, ϕ, {a}.
Let f : (X, τ) → (Y, σ) be an identity map.
g-open sets in (X, τ) are {a, b}, {a}, {b}, ϕ, X.
g*-closed sets are {c}, {b, c}, {a, c}, ϕ, X.
Strongly ($\hat{g}$)*-closed sets in (X, τ) are {a}, {b}, {c}, {b, c}, {a, c}, ϕ, X.
∴ f^{-1}{b} = {b} is a strongly ($\hat{g}$)*-closed set in (X, τ) but not a g*-closed set in (X, τ).
Thus, the converse of the above theorem is not true.
Hence, every strongly ($\hat{g}$)*-continuous map need not be g*-continuous.

H. Theorem 4.8
Every ($\hat{g}$)*-continuous map is strongly ($\hat{g}$)*-continuous.
1) Proof: Let f : (X, τ) → (Y, σ) be a continuous map.
By the definition 2.5.3,
f^{-1}(V) is a ($\hat{g}$)*-closed set of (X, τ) for every closed set V of (Y, σ).
To prove: f is strongly ($\hat{g}$)*-continuous.
Let V be a closed set in (Y, σ).
Since, f is ($\hat{g}$)*-continuous; there exists a ($\hat{g}$)*-closed set f^{-1}(V) in (X, τ).
By theorem 3.7,
"Every ($\hat{g}$)*-closed set is strongly ($\hat{g}$)*-closed."
Hence, f^{-1}(V) is a strongly ($\hat{g}$)* -closed set in (X, τ).
∴ f is strongly ($\hat{g}$)*-continuous.
The converse of the above theorem need not be true as shown in the following example.

I. Example 4.9
Let X = Y = {a, b, c}
And τ = {ϕ, X, {a, b}}
Closed sets in (X, τ) are X, ϕ, {c}.
And σ = {ϕ, Y, {b, c}}
Closed sets in (Y, σ) are Y, ϕ, {a}.
Let f : (X, τ) → (Y, σ) be an identity map.
$\hat{g}$ -closed sets are {c}, {b, c}, {a, c}, ϕ, X.
$\hat{g}$ -open sets are {a, b}, {a}, {b}, ϕ, X.
($\hat{g}$)*-closed sets are {c}, {b, c}, {a, c}, ϕ, X.
Strongly ($\hat{g}$)*-closed sets in (X, τ) are {a}, {b}, {c}, {b, c}, {a, c}, ϕ, X.
∴ f^{-1}{b} = {b} is a strongly ($\hat{g}$)*-closed set in (X, τ) but not a ($\hat{g}$)*-closed set in (X, τ).
Thus, the converse of the above theorem is not true.
Hence, every strongly ($\hat{g}$)*-continuous map need not be ($\hat{g}$)*-continuous.

III. CONCLUSION

Hence, I would like to conclude my paper by giving the properties of strongly ($\hat{g}$)*-closed set and strongly ($\hat{g}$)*- continuous function. And also with further results and solutions we can bring in the comparison of strongly ($\hat{g}$)*-closed set and function with other sets and functions as well in a given topological space.

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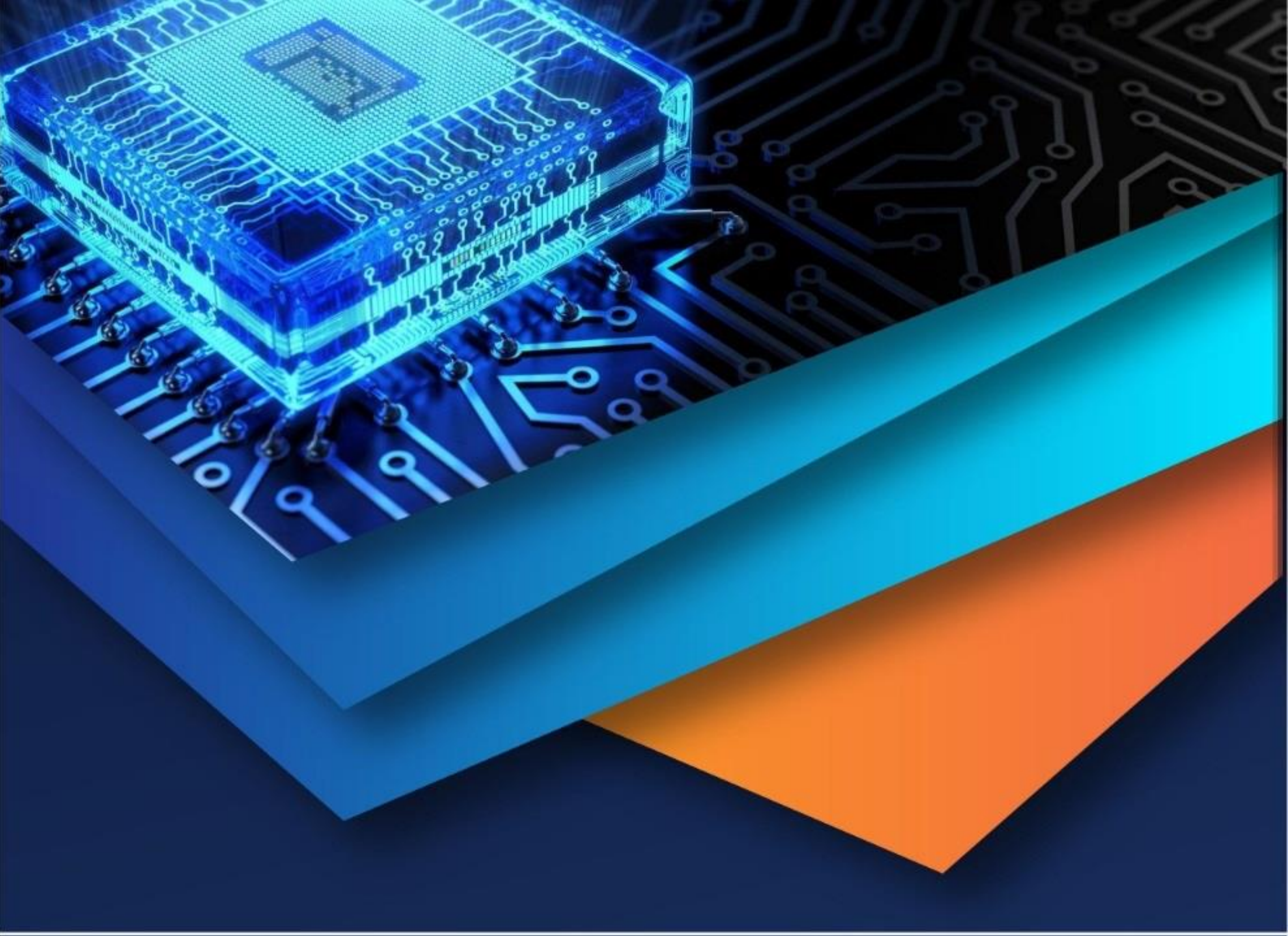

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