INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Suction/Injection Effect on MHD Flow of a Nanofluid and Heat Transfer Over a Nonlinear Stretching Plate

S. Ranjitha[1], M. Dinesh Kumar[2]

[1]*Department of Mathematics-PG, K.S.Rangasamy College of Arts and Science, Tiruchengode-637215, INDIA*
[2]*Department of Mathematics, PSG College of Arts and Science, Coimbatore-14. INDIA*

Abstract: In this paper, we investigate the suction/injection effect on hydromagnetic flow and heat transfer characteristics of a viscous nanofluid over a nonlinearly stretching sheet in the presence of transverse uniform magnetic field. The fluid is viscous assumed to be viscous, incompressible and electrically conducting. Governing nonlinear Partial differential equations are transformed to nonlinear ordinary differential equations by using similarity transformation and numerical solution is obtained by using Runge-Kutta-Merson method with shooting technique. These numerical solutions are shown graphically by means of graphs. The effects of nanoparticle volume fraction, magnetic interaction parameter, nonlinearly stretching sheet parameter, suction/injection parameter, Eckert number and Prandtl number on velocity, temperature, skin friction and rate of heat transfer are thoroughly discussed.
Keywords-- Nanofluid; Viscous dissipation; MHD; Non linearly stretching sheet.

I. INTRODUCTION

The heat transfer in the flow due to a stretching sheet is very important in practical point of view. This type of flow is frequently appears in many industrial and engineering processes and in those cases, the qualities of the final products depend to a great extent on the rate of cooling. This sheet plays an important role in aerodynamic, extrusion of plastic sheet, metal-spinning, manufacture of plastic and rubber sheets, paper production etc and thus, remains at the leading edge of technology development. In the industrial operation, metal or more commonly an alloy, is heated until it is molten, where upon it is poured into a mould or dies which contains a cavity, of required shape. The hot metal issue from the die is subsequently stretched to achieve the desired product. When the super heated melt issue comes out from the die, it loses its heat and contract as it cools, this is referred as liquid state contraction. With further cooling and loss of latent heat of fusion, the atoms of the metal lose energy and become closely bound together in a regular structure. The quality of the final product greatly depends on the rate of cooling and the process of stretching. The rate of heat transfer between the stretching surface and fluid flow is important for the end product's desired quality. The boundary layer flow generated by a stretching sheet was first studied by Crane [1]. He constructed an exact solution for the arising problem. Afterwards, the boundary layer flows by linear and nonlinear stretching surfaces have attracted a great deal of attention of the researchers [2]–[5]. The heat and mass transfer on a stretching sheet with suction or blowing was studied by Gupta and Gupta [6]. MHD boundary layer flow due to an exponentially stretching sheet with a radiation effect has been obtained by Anuar Ishak [7]. Ali [8] has investigated the thermal boundary layer flow by considering the nonlinear stretching surface. A few years later, Magyari and Keller [9] also focused on heat and mass transfer on boundary layer flow due to an exponentially continuous stretching sheet.

Magnetohydrodynamic (MHD) flow when especially associated with heat transfer has received considerable attention in the recent years because of their wide variety of applications in engineering areas, such as crystal growth in liquid, cooling of nuclear reactor, electronic package, microelectronic devices and solar technology. The Magnetohydrodynamic flows add a new dimension for the study of flow and heat transfer in a viscous fluid over a stretching surface. Slip MHD viscous flow over a stretching sheet – An exact solution was examines by Fang and Zhang [10]. Mukhopadhyay [11] investigated MHD boundary layer flow and heat transfer over an exponentially stretching sheet embedded in a thermally stratified medium.

Heat transfer analysis for a hydromagnetic viscous fluid over a nonlinear shrinking sheet was investigated by Javed et al. [12]. Takhar and Ram [13] have studied MHD forced and free convection flow through a porous medium in the presence of a uniform transverse magnetic field. Salem [14] investigated variable viscosity and thermal conductivity effects on MHD flow and heat transfer in viscoelastic fluid over a stretching sheet.

In recent decades, Nanofluids have attracted the attention of scientists as a new generation of fluids in heating of building, heat exchangers, technological plants, automotive cooling applications and many other applications. By employing nanofluids, it is

possible to reduce the dimensions of heat transfer equipments due to the improved thermo physical properties of the working fluid. Inclusion of nanoparticles into the base fluid such as water is known to increase the heat transfer capability of the fluid. The use of additives is a technique applied to enhance the heat transfer performance of base fluids. The thermal conductivity of ordinary heat transfer fluids is not adequate to meet today's cooling rate requirements.

The heat conduction has a great importance in many industrial heating or cooling equipments. In these days, there is a great advancement in the study of the flow of nanofluids with convective heat transfer. Laminar boundary layer flow of nanofluid over a flat plate was investigated by Anjali Devi and Julie Andrews [15] and it was found out that suspended nanoparticles enhance the heat transfer capacity of the fluids. Heat transfer performance of the conventional fluid is expected to enhance with the addition of small nanoparticles since the nanofluid has high thermophysical properties in thermal conductivity and convective heat transfer coefficient compared to conventional fluids by Murshed [16].

N. Bachok [17] discussed stagnation point flow over a stretching/shrinking sheet in a nanofluid. The buoyancy effects on MHD stagnation point flow and heat transfer of a Nanofluid past a convectively heated stretching/shrinking sheet was studied by Makinde & Khan [18]. Injection or withdrawal of fluid through a porous bounding heated or cooled wall is of general interest in practical problems involving boundary layer control applications such as film cooling, polymer fiber coating, coating of wires, etc. Ahmad and Pop [19] investigated mixed convection boundary layer flow from a vertical flat plate embedded in a porous medium filled with nanofluids. Hamad and Ferdows [20] discussed the heat and mass transfer analysis for boundary layer stagnation-point flow over a stretching sheet in a porous medium saturated by a nanofluid with internal heat generation/absorption and suction/blowing.   The purpose of this work is to analyse the thermal radiation on MHD flow of a nanofluid and heat transfer over a nonlinear stretching porous plate. Due to these practical importance, no studies have thus far been made with regard to Suction/injection on MHD flow of a nanofluid and heat transfer over a nonlinear stretching plate and hence the present investigation is concerned with such study. Using similarity transformations, the governing nonlinear partial differential equations are transformed into nonlinear ordinary differential equations. Then the resulting nonlinear differential equations which form a non-linear Boundary value problem is solved numerically utilizing efficient Nachtsheim Swigert shooting iteration scheme for the satisfaction of asymptotic boundary conditions along with Runge Kutta - Mersonn method. The effects of different physical parameters governing the flow and heat transfer characteristics such as Magnetic interaction parameter, Suction/injection parameter, Prantdl number, Eckert number, Volume fraction are investigated. Favourable comparisons with previous published work of the problem are obtained. The quantities of engineering interest such as Skin friction coefficient & non-dimensional Rate of heat transfer are obtained numerically and are thoroughly analyzed.

II. FORMULATION OF THE PROBLEM

The steady two dimensional, laminar, nonlinear boundary layer flow of an incompressible, viscous, electrically conducting nanofluid over a nonlinear porous stretching sheet has been considered. The sheet is coincident with the plane $y = 0$ and the flow being confined to $y > 0$ as given in Fig 1. The sheet is assumed to be permeable such that possible suction/injection effect occurs at the surface. A variable magnetic field of strength $B(x)$ is applied normal to the sheet and parallel to y-axis as shown in the figure.

The x-axis runs along the stretching sheet and the y-axis is perpendicular to it. The stretching sheet velocity is $U_w(x) = Cx^n$ where C is a positive constant and n is the nonlinear stretching parameter and the sheet is subjected to variable suction/injection such that $v = -V_w(x)$, the value of which will be defined later.
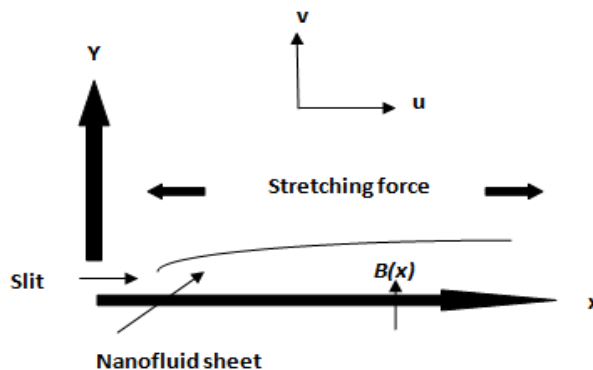

Fig. 1 A Schematic diagram of the physical model

The magnetic Reynolds number of the flow is taken to be sufficiently small enough, so that the induced magnetic field can be neglected in comparison with the applied magnetic field. Since the flow is steady, $Curl\,\vec{E}=0$, Also $div\,\vec{E}=0$ in the absence of surface change density and hence $\vec{E}=0$ is assumed. It is also assumed that the ambient temperature is constant, denoted by T_∞ and $T_w=T_\infty+bx^m$ is the surface temperature with m being the surface temperature parameter and b is a positive constant. The thermo physical properties of fluid and nanoparticles used in this study are taken from Abu-Nada and Oztop [21]. The fluid is a water based nanofluid containing nanoparticles such as copper. Thermo-physical properties of fluid and nanoparticles at 25°C (Oztop and Abu-Nada [21])

TABLE 1

Physical properties	Fluid Phase (water)	Cu
$C_p\,(J\,/\,kgK)$	4179	385
$\rho\left(kg\,/\,m^3\right)$	997.1	3970
$k\,(W\,/\,m\,K)$	0.613	40
$\beta\times10^5\,(K^{-1})$	21	0.85

For the present study, water is considered as the base fluid with Pr = 6.2. The nanofluid considered is water mixed with solid spherical copper nanoparticles.

III.   GOVERNING EQUATIONS OF THE FLOW

Using the boundary layer approximations, the simplified steady two dimensional non-linear boundary layer equations governing the flow and heat transfer using the nanofluid model introduced by Tiwari and Das [22] are considered. Under these conditions, the governing boundary layer equations of continuity, momentum and energy equations are,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon_{nf}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2\left(x\right)u}{\rho_{nf}} \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon_{nf}}{\left(c_p\right)_{nf}}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (3)$$

The boundary conditions are

$$u=u_w\left(x\right)=Cx^n,$$
$$v=-V_w,\ T=T_w\left(x\right)=T_\infty+bx^m\ at\ y=0; \qquad (4)$$
$$u\to0;\,T\to T_\infty \qquad\qquad as\ y\to\infty$$

where u and v are the velocity components in the x and y directions, $V_w\left(x\right)$ is the variable suction/injection velocity and $B(x)$ is the variable magnetic field. Variable suction/injection and variable magnetic field are considered as

$$V_w(x)=\sqrt{\frac{C(n+1)v_f}{2}}x^{\frac{n-1}{2}};\quad B(x)=B_0x^{\frac{n-1}{2}}$$

where B_o is a constant magnetic field. n and m are the nonlinear stretching parameter and the surface temperature parameter, respectively. The temperature on the wall is T_w, and the ambient is held at constant temperature T_∞. ρ_{nf} and μ_{nf} are the density and effective viscosity of the nanofluid, and α_{nf} and υ_{nf} are the thermal diffusivity and the kinematic viscosity, respectively, which are defined as (see Khanafer et al.[23]);

$$\upsilon_{nf} = \frac{\mu_{nf}}{\rho_{nf}}, \qquad \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s,$$

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \qquad \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}$$

$$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s,$$

$$\frac{k_{nf}}{k_f} = \frac{(k_f + 2k_f) - 2\phi(k_f - k_s)}{(k_f + 2k_f) + 2\phi(k_f - k_s)} \tag{5}$$

Here, ϕ is the solid volume fraction, where μ_f is the viscosity of the basic fluid, ρ_f and ρ_s are the densities of the pure fluid and nanoparticle, respectively, $(\rho c_p)_f$ and $(\rho c_p)_s$ are the specific heat parameters of the base fluid and nanoparticle, respectively, and k_f and k_s are the thermal conductivities of the base fluid and nanoparticle, respectively. The relation for effective dynamic viscosity μ_{nf} used in the present study is taken from the equation by Brinkman [24].

IV. SIMILARITY TRANSFORMATIONS

In order to seek the solution of the problem, the following dimensionless variables are introduced:

$$\eta = y\sqrt{\frac{C(n+1)}{2\nu_f}}x^{\frac{n-1}{2}} \quad ; \quad u = Cx^n f'(\eta)$$

$$\theta(\eta) = \frac{(T - T_\infty)}{(T_w - T_\infty)} \quad ; \tag{6}$$

$$v = -\sqrt{\frac{C(n+1)\nu_f}{2}}x^{\frac{n-1}{2}}\left[f(\eta) + \frac{n-1}{n+1}\eta f'(\eta) \right]$$

Employing the similarity variables (9), Eqs. (1), (2) and (8) reduces to the following nonlinear system of ordinary differential equations:

$$f''' + (1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\left(ff'' - \frac{2n}{n+1}f'^2\right) - (1-\phi)^{2.5}M^2 f' = 0 \tag{7}$$

$$\frac{1}{\Pr}\left(\frac{k_{nf}}{k_f}\right)\theta'' + \frac{Ec}{(1-\phi)^{2.5}}x^{2n-m}f''^2 + \left((1-\phi)+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)\left(f\theta' - \frac{2m}{n+1}f'\theta\right) = 0 \tag{8}$$

so that all similar solutions put $m = 2n$ in Equation 11, which becomes:

$$\frac{1}{\Pr}\left(\frac{k_{nf}}{k_f}\right)\theta'' + \frac{Ec}{(1-\phi)^{2.5}}f''^2 + \left((1-\phi)+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)\left(f\theta' - \frac{4n}{n+1}f'\theta\right) = 0 \tag{9}$$

and the transformed boundary conditions are:

$$f(0) = \lambda, f'(0) = 1, \theta(0) = 1,$$
$$f'(\infty) \to 0, \theta(\infty) \to 0, \tag{10}$$

where $\Pr = \upsilon_f/\alpha_f$ the Prandtl is number, $Ec = u_w^2/\left[(c_p)_f(T_w - T_\infty)\right]$ is the Eckert number, $M^2 = 2\sigma B_0^2/\rho_f C(n+1)$ is the Magnetic interaction parameter and λ is the suction/injection parameter. In the above equations, primes denote differentiation with respect to η.

V. NUMERICAL SOLUTION

The Numerical solution to system of nonlinear ordinary differential equations (7) and (9) with the boundary conditions (10) were obtained by using Nachtsheim-Swigert Shooting iteration technique along with Runge Kutta- Merson method. The most important

thing to be considered here is that the initial guesses for values $f''(0)$ and $\theta'(0)$ to initiate the shooting process are to be made. The success of the procedure depends very much on how good these guesses are. Initial guesses are made taking into account of convergency and numerical solutions for velocity and temperature are obtained for several values of the physical parameters. The quantities of practical interest in this study are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as:

$$C_f = \frac{2\mu_{nf}}{\rho_f \left(U_w(x)\right)^2}\left(\frac{\partial u}{\partial y}\right)_{y=0} , Nu_x = \frac{-xk_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0}}{k_f (T_w - T_\infty)} \quad (11)$$

Using equation (9), it is obtained as

$$C_f \, \text{Re}_x^{1/2} = \frac{\sqrt{2}\sqrt{n+1}}{(1-\phi)^{2.5}} f''(0) , Nu_x \, \text{Re}_x^{-1/2} = -\frac{k_{nf}\sqrt{n+1}}{\sqrt{2}k_f}\theta'(0) \quad (12)$$

where $\text{Re}_x = \dfrac{U_w x}{\nu_f}$ is the local Reynolds number.

VI.   RESULTS AND DISCUSSION

Similarity solutions are obtained utilizing the efficient shooting method such as Nachtsheim Swigert shooting iteration scheme for satisfaction of asymptotic boundary conditions together with Runge-Kutta-Merson method. Nachtsheim Swigert [25] shooting iteration scheme justifies the unique solution. In order to have the physical insight of the problem, numerical solutions are obtained for various values of the physical parameters and are illustrated graphically.
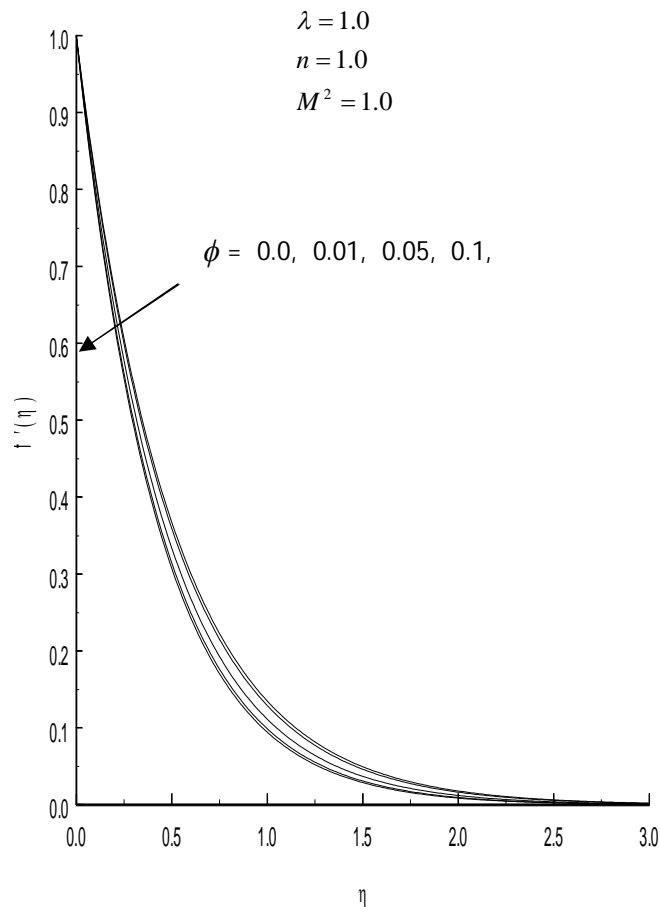

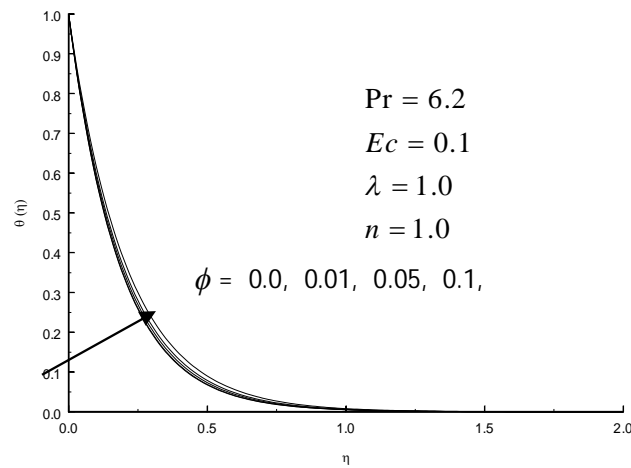Fig. 2 Effects of nanoparticle volume fraction ϕ on velocity distribution $f'(\eta)$

Fig. 3 Effects of nanoparticle volume fraction ϕ on temperature profiles $\theta(\eta)$
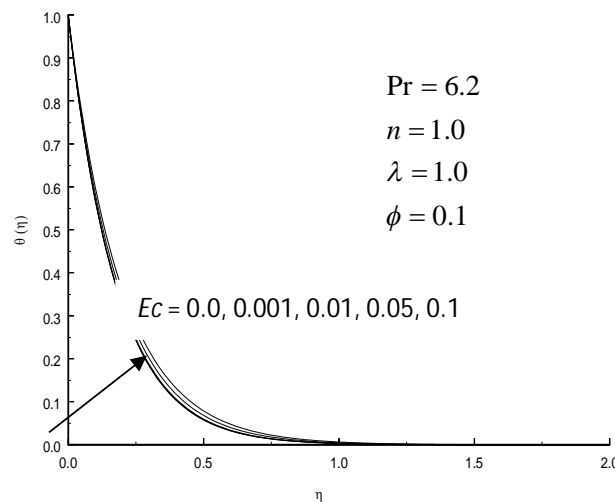

Fig. 4 Effects of Eckert number Ec on temperature profiles $\theta(\eta)$
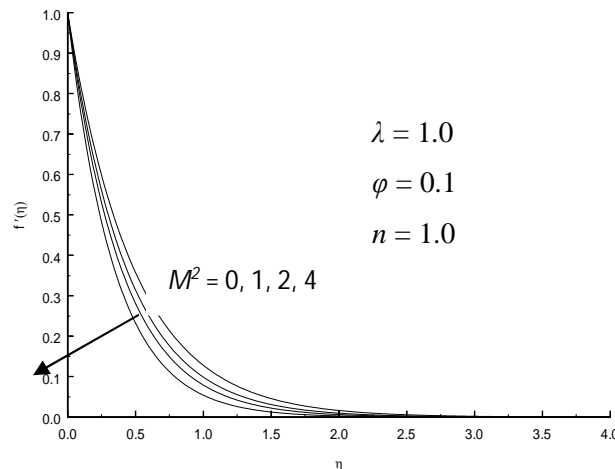

Fig. 5 Effects of magnetic interaction parameter M^2 on velocity profiles $f'(\eta)$

In Figure 2, effect of volume fraction over the velocity for copper water nanofluid, is portrayed in the figure. The velocity of the nanofluid gets decelerated respectively for increasing values of volume fraction. It is vivid from the figures that there is shrinkage in momentum boundary layer thickness due to volume fraction φ. Figure 3 elucidates that the temperature distribution for various values of volume fraction φ for copper water nanofluid. The fact that the temperature and as well as the thermal boundary layer thickness are enhanced for copper water nanofluid due to the effect of increasing volume fraction are visualized in these figures. An

increase in Eckert number increases the temperature distribution in the flow region which is depicted through Figure 4. It is obviously seen that the temperature is enhanced together with an increase in the thermal boundary layer thickness due to increasing Eckert number. In Figure 5, effect of Magnetic interaction parameter on dimensionless velocity for cu-water nanofluid respectively. It is observed that a growing magnetic interaction parameter accelerates the velocity of the flow field at all points due to the flow field. The velocity of the nanofluid is decreased for increase in the value of M^2.
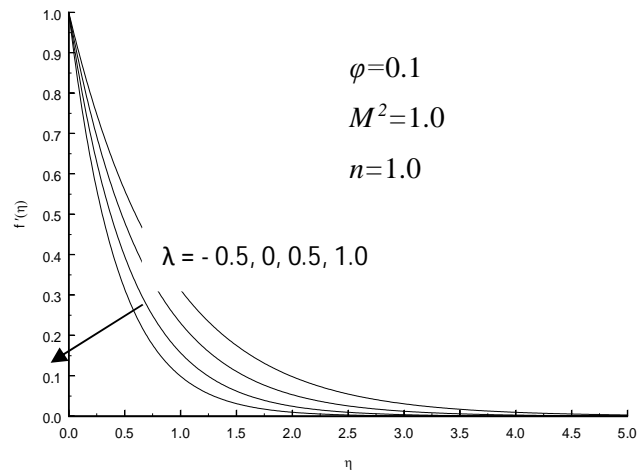

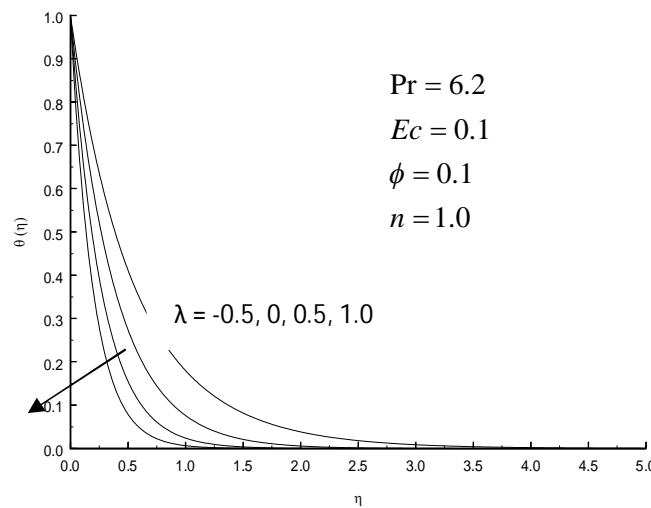Fig. 6 Effects of suction/injection parameter λ on velocity profiles $f'(\eta)$

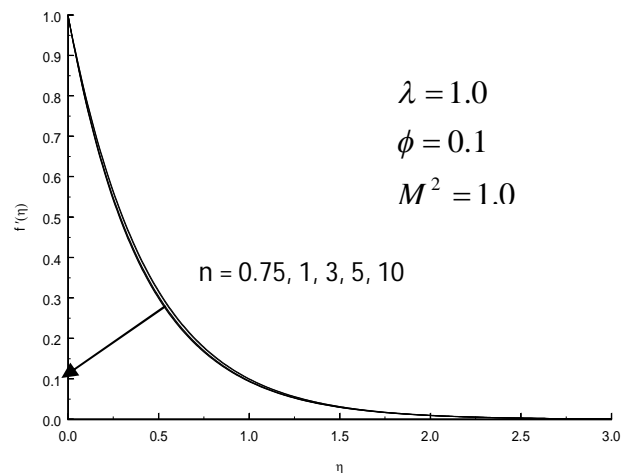Fig. 7 Effects of suction/injection parameter λ on temperature profiles $\theta(\eta)$

Fig. 8 Effects of stretching parameter n on velocity distribution $f'(\eta)$

It is describe the effect of suction parameter on the flow field when the magnetic parameter is fixed in displayed Figure 6. It is noted that, a steady raise in the velocity accompanies a rise in suction parameter, with all profiles tending asymptotically to horizontal axis. In Fig. 6, the dimensionless velocity for different values of suction parameter λ= -0.5, 0, 0.5, 1.0 for Pr = 6.2 are portrayed. It is inferred that as λ increases, the velocity of the copper-water nanofluid is decrease. The effect of λ on the temperature profiles $\theta(\eta)$ for copper-water nanofluid are illustrated in Figure 7. It is displays that an increase in the suction parameter recorded a decrease in temperature $\theta(\eta)$ in the copper-water nanofluid. Figure 8 depicts the effect of nonlinearly stretching sheet parameter n on velocity distribution $f'(\eta)$. The velocity of the copper water nanofluid gets decelerated respectively for increasing the values of stretching sheet parameter n tends to decrease the nanofluid velocity.

Figure 9 discloses the temperature distribution for various values of nonlinearly stretching sheet parameter n. The thermal boundary thickness becomes thicker for the nanofluid for increasing the stretching sheet parameter n. Further, there is an increase in temperature for copper water nanofluid due to decreasing in stretching sheet parameter n. Figures 10 & 11 display the behavior of the heat transfer rates under the effects of Ec and n respectively, using different nanofluids for Pr = 6.2 and ϕ = 0.1. These figures show that, when using different kinds of nanofluids, the heat transfer rates change, which means that the nanofluids will be important in the cooling and heating processes.
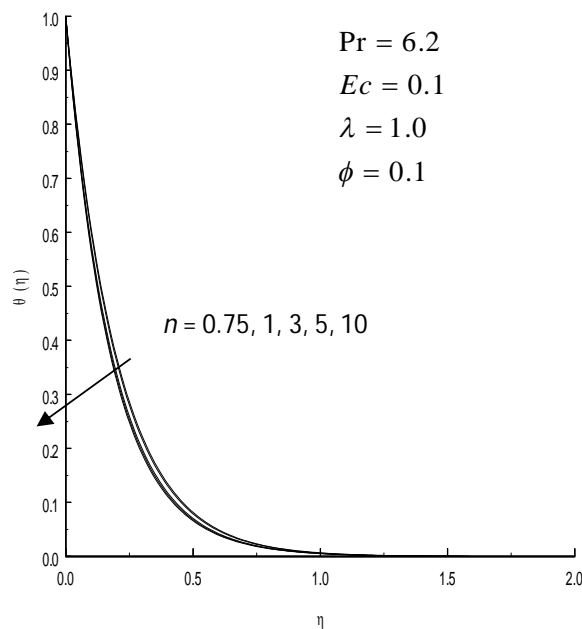

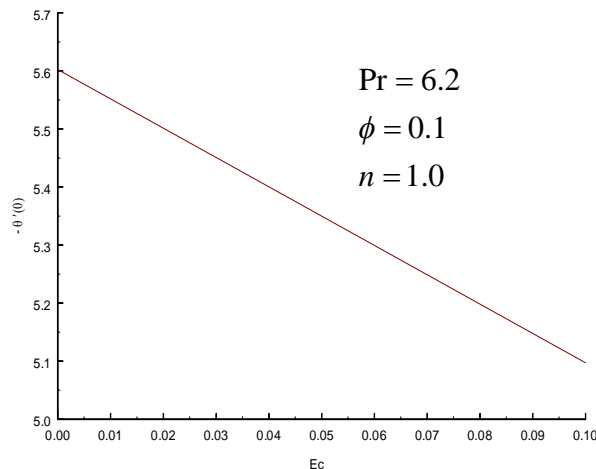Fig. 9 Effects of stretching parameter *n* on temperature profiles $\theta(\eta)$

Fig. 10 Effects of viscous dissipation parameter Ec on heat transfer rate for different types of nano particles
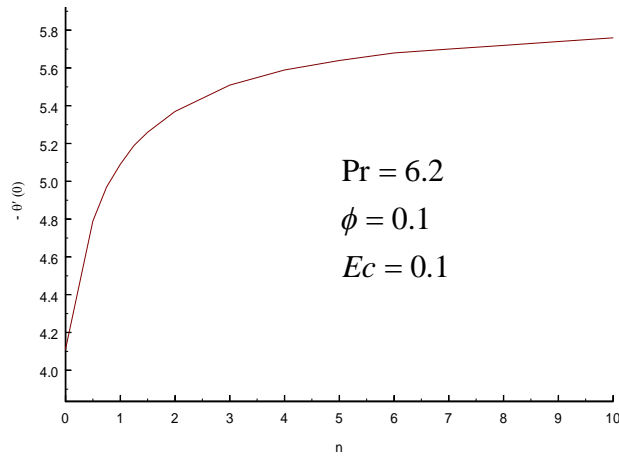
Fig. 11 Effects of stretching parameter *n* on heat transfer rate for different types of nanoparticles

The numerical values of non-dimensional rate of heat transfer for different values of *n*, φ, λ, and Ec for copper water nanofluid when Pr=6.2 is provided in Table 2. It is noted that for increasing values of stretching parameter, volume fraction and suction parameter, Non-dimensional rate of heat transfer increases in case of Copper- water nanofluid. Non-dimensional rate of heat transfer decreases for increasing Eckert number for the Copper- water nanofluid. Table 3 inferred the skin friction coefficient for different values of magnetic interaction parameter, stretching parameter, suction parameter and volume fraction for Copper-water nanofluid respectively. It is seen that the skin friction coefficient increases for decreases values of all these parameters.

TABLE 2

Non-dimensional rate of heat transfer for various λ, *n*, φ and *Ec* (Copper-water Nanofluid)

λ	n	ϕ	Ec	$\theta'(0)$	$-\dfrac{k_{nf}}{k_f}\dfrac{\sqrt{n+1}}{\sqrt{2}}\theta'(0)$
-0.5				-1.84098	2.75594
0.0	1.0	0.1	0.1	-2.61472	3.91423
0.5				-3.72495	5.57625
1.0				-5.09731	7.63067
	0.75			-4.97044	6.96007
1.0	1.0	0.1	0.1	-5.09731	7.63067
	3.0			-5.51462	11.6748
	5.0			-5.64561	14.6380
	10.0			-5.76159	20.2250
		0.0		-5.40646	5.40646
		0.01		-5.37398	5.53573
1.0	1.0	0.05	0.1	-5.24768	6.40794
		0.1		-5.09732	7.63067
		0.2		-4.81703	11.19092
			0.0	-5.60267	8.38719
			0.001	-5.59761	8.37962
1.0	1.0	0.1	0.01	-5.55213	8.31153
			0.05	-5.34999	8.00893
			0.1	-5.09731	7.63067

TABLE 3

Skin friction coefficient for various M^2, λ, n and ϕ with Pr=6.2 (Copper-water Nanofluid)

M^2	λ	n	ϕ	$f''(0)$	$\dfrac{\sqrt{2}\sqrt{n+1}}{(1-\phi)^{2.5}} f''(0)$
0				-2.05242	-5.3420
1				-2.31007	-6.0127
2	1.0	1.0	0.1	-2.53203	-6.5904
4				-2.91035	-7.5751
1	-0.5	1.0	0.1	-1.16081	-3.0213
	0.0			-1.46576	-3.8150
	0.5			-1.85083	-4.8173
	1.0			-2.31007	-4.7652
1	1.0	0.75	0.1	-2.27512	-5.5392
		1.0		-2.31007	-6.0127
		3.0		-2.42807	-8.9376
		5.0		-2.46603	-11.1173
		10.0		-2.49990	-15.2597
1	1.0	1.0	0.0	-2.00000	-4.0000
			0.01	-2.04465	-4.1933
			0.05	-2.19200	-4.9841
			0.1	-2.31007	-6.0127
			0.2	-2.35624	-8.2328

VII.   CONCLUSION

Numerical Investigation of Suction/injection on MHD flow of a nanofluid and heat transfer over a nonlinear stretching surface for copper-water nanofluid has been made. Numerical solutions are obtained by Nachtsheim-Swigert shooting iteration scheme together with fourth order Runge-Kutta-Merson method scheme for various values of physical parameters. A parametric study is performed to illustrate the influence of physical parameters such as Magnetic interaction parameter, volume fraction, suction/injection parameter, stretching sheet parameter and Eckert number with Pr = 6.2 for the base fluid over velocity and temperature. The skin friction coefficient and the non dimensional rate of heat transfer are also obtained numerically.

Some of the important findings drawn from the present investigation are listed as follows:

A. In the absence of magnetic field, the author's results are identical to that of Hady et al. [26] of their physically meaningful results.

B. All the profiles tend to zero asymptotically which satisfies the far field boundary conditions.

C. Skin friction decreases with increase the magnetic parameter.

D. Velocity decreases with an increasing value of magnetic parameter.

REFERENCES

[1] Crane L J., "Flow past a stretching plate" Zeitschrift Fur Angewandte Mathematik Und Physik, (1970), **21**, 645.

[2] Cortell R., Phys. Lett. (2006) **A 357,** 298.

[3] Bhattacharyya K., Hayat T., Alsaedi A ., Chin. Phys. (2013), **B 22**, 024702

[4] Mukhopadhyay S., Chin. Phys. (2013) **B 22** 074701

[5] Rashidi M M., Mohimanian Pour S A., Nonlinear Analysis:Modelling and Control, (2010),**15**, 83.

[6] Gupta PS., Gupta AS., "Heat and mass transfer on a stretching sheet with suction and blowing", Can J Chem Eng (1977), **55**,744–6

[7] Ishak A ., Sains Malaysiana, (2011), **40**, 391.

[8] Ali M.E.., "On thermal boundary layer on a power law stretched surface with suction or injection", Int. J. Heat Fluid Flow **16**, (1995) 280–290.

[9] Magyari E., Keller B., "Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface", J. Phys. D: Appl. Phys. **32,** (2000) 577–585

[10] Fang T., Zhang Ji., Yao S., Communications in nonlinear Sci. and Numerical simulation, (2009). **14**, 11, 3731-3737.

[11] Mukhopadhyay S., Alexandria Engineering Journal, (2013), **52**, 3, 259-265.

[12] Javed T., et al. International Journal of Heat and Mass transfer, (2011), **54**, 2034

[13] Takhar H S., Ram P C., Int. Commun. Heat Mass Transfer, (1994), **21**, 371.

[14] Salem AM., "Variable viscosity and thermal conductivity effects on MHD flow and heat transfer in viscoelastic fluid over a stretching sheet". Phys Lett A, (2007), **369(4)**, 315–22.

[15] Anjali Devi SP., Julie Andrews., "Laminar flow of nanofluids over a horizontal flat plate", International Journal of Applied Mathematics and Mechanics, (2011) **7**, 52.

[16] Murshed, S. M. S. et al. Renew. Sust. Energ. Rev., (2011), **15**, 2354.

[17] Bachok, N., Ishak, A. Physica B: Consensed Matter, (2011), **406**, 1767.

[18] Makinde, O.D., Khan, W.A., Int. J. Heat Mass Transfer, (2013), **20**, 526.

[19] Ahmad S., and Pop I., "Mixed convection boundary layer flow from a vertical flat plate embedded in a porous medium filled with nanofluids", Int. Comm. in Heat and Mass Transfer, **37** (2010) 987-991.

[20] M.A.A. Hamad and M. Ferdows, "Similarity solution of boundary layer stagnation-point flow towards a heated porous stretching sheet saturated with a nanofluid with heat absorption/generation and suction/blowing a lie group analysis", Commun. Nonlinear Sci. Numer. Simulat., **17** (2011) 132 -140.

[21] Oztop. H. F., and E. Abu-Nada, Int. J. Heat Fluid Flow, (2008), **29**, 1326.

[22] Tiwari. R.K., and M. K. Das, Int. J. Heat Mass Transf, (2007), **50**, 2002.

[23] Khanafer K, Vafai K, Lightstone M: "Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids". Int J Heat Mass Transf, (2003), **46**: 3639-3653.

[24] Brinkman, H. C. The Journal of Chemical Physics, (1952), **20**, 571.

[25] Nachtsheim, P. R. and Swigert, P. "Satisfaction of Asymptotic boundary conditions in numerical solution of the system of nonlinear equations of boundary layer type", (1965), NASA TND-3004.

[26] Hady FM., Ibrahim FS., Abdel SM., Eid MR., Nanoscale Research Letters, (2012), **7**:229

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089 (24*7 Support on Whatsapp)