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A Study on Magma and Medial Magma in Category Theory

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Abstract: This paper includes a contribution to explaining category theory using magma and medial magma and vice versa. First, basics of category theory are presented. In addition, theorem follows the categories definition and how it was applied in various fields.
Keywords: Magma, medial magma, free magma, subcategories and magma morphisms.

I. INTRODUCTION

Category theory was invented in the middle of the last century with the goal of better connecting algebra with topology. It has since produced a network of connections between all branches of mathematics as well as between mathematics, sciences, and engineering .Category theory has long been recognized as a powerful too for unifying different branches of pure mathematics, but its potential for applications has barely begun to be tapped. Category theory is the study of categories, which are collection of objects and morphisms (or arrows') ,from one object to another. A category has two basic properties: the ability to compose the arrows associatively and the existence of an identity arrow for each object. The language of category theory has been used to formalize concepts of other high-level abstractions such as sets , rings ,and groups. It generalizes many common notions in algebra , such as different kinds of products ,the notion of kernel , etc. A systematic study of category theory allows us to prove general results about any of mathematical structures from axioms of a category.

II. PRELIMINARIES

A. Definition 2.1

1) *MAGMA:* A magma is a set M matched with an operation,*, that sends any two elements a, b M to another element a*b. The symbol,*, is a general placeholder for a properly defined operation.

The set and operation (M,*) must satisfy the following requirement for all a. b in M. The result a*b is also in M. In notation, a , b M such that a*b M.

If * is a partial operator, then S is called partial magma or more often a partial groupoid.

B. Definition 2.2

1) *Free Magma:* A free magma, Mx on a set X ,is the "most general possible" magma generated by X. i.e, no relations or axioms imposed on the generators ; see free object.

It can be described as the set of non-associative words on X with parantheses retained.

A free magma has the universal property such that, if f : X→N is a function from X to any magma,N,then there is a unique extension of f to a morphism of magmas ,f'.

$f' : Mx \rightarrow N$

C. Definition 2.3

A morphism of magmas is a function, f : M→ N ,mapping magma M to magma N that preserves the binary operation $f(X\cdot_M Y)=f(X)\cdot_N f(Y)$ operation where $\cdot_M$ and $\cdot_N$ denote the binary operation on M and N respectively.

D. Definition 2.4

1) *Medial Magma:* In abstract algebra, a medial magma or medial grouped ,is a with a binary operation which satisfies the identity

$(x.y).(u.v)=(x.u).(y.v)$ or more simply $xy.uv=xu.yv$

E. Definition 2.5

1) *Locally Small:* A (locally small) category C consists of

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a) A collection obj(c) of objects
b) A collection Arr(c) of morphisms
For any X, Yobj(c) , Hom(X,Y) is the subcollection of Arr(c) of morphisms from X to Y, where each Hom(X, Y) is required to be set.

F. Definition 2.6
1) Initial: An object I in a category is called initial or co final ,if any object X there exists a unique morphism f: I →X .

G. Definition 2.7
1) Final: An object F in a category is called final or coinitial, if any object X there exists a unique morphism f: X→F

H. Definition 2.8
1) Subcategories
A subcategory S of a category C is a category in which:
a) The class of objects of S is contained in the class of objects of C.
b) The class of arrows of S is contained in the class of arrows of C.
c) For every arrow f in S, the domain and co domain of f are in S.
d) For every object s in S, the identity arrow 1s is in S.
e) For every pair of arrows f , g in S ,the arrow g ○f is in S where it is defined.

I. Definition 2.9
1) Opposite Category: Given a category C, the opposite(or dual) category C° has the same objects as C and for every arrow f :a b in C ,the arrow f :b→ a is in C .In other words, it has the same objects, and arrows are reversed.

J. Definition 2.10
1) Product Category: Given two categories B and C /the product category , denoted B x Cis givenby the following data:
a) The objects of B x C are (b,c) where b is an object of B and C is an object of C .
b) The arrows of B x C are (f,g) where f isan arrow of B and g is an arrow of C.
c) Composition is given by (f′,g′) ○(f ,g)=(f′○ f, g′○ g).
The product C x 2 is called the cylinder category, denoted Cyl(c).

K. Definition 2.11
A magma morphism or morphism of magmas in C is a morphism h :M→N such that

$$\begin{tikzcd} F(A) \arrow[r, "\alpha"] \arrow[d, "F(f)"'] & A \arrow[d, "f"] \\ F(B) \arrow[r, "\beta"] & B \end{tikzcd}$$

III. THEOREM

Let M be an object in ج and suppose that for every object c in ج a binary operation **∇** , is defined on Hom(C , M) in such a way that for every morphism h :C D the function h* : Hom (D ,M)→ Hom(C , D) is a magma homomorphism. Then there is a unique binary operation μ : M x M → M so f **∇** g =μ (f, g) for all f and g in
Hom (C,M).

A. Proof
Consider $\mu = \Pi_1 \quad \Pi_2 : M \times M \to M$ .Then for f and g in Hom (C , M)
We have < f , g> : C x C → M and
$\mu < f, g> = (\Pi_1 \nabla \ \Pi_2) < f, g >$
$= (f , g)^* < \Pi_1 \ \nabla \Pi_2 >$
$= \ < f , g > ^* (\Pi_1) \quad \nabla < f , g > ^* (\Pi_2)$
$= \Pi_1 < f ,g > \nabla \ \ \Pi_2 < f , g >$

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$= f \;\; \nabla g$

And if υ is some binary operation on M that induces ,then

$\mu = \pi_1 \nabla \pi_2$

$= (\pi_1 . \pi_2)$

$= \upsilon 1$

$= \upsilon.$

IV. APPLICATION OF CATEGORIES

1) Category now appear in many branches of mathematics some areas of theoretical computer science where they can correspond to types or to database schemas.
2) Category theory can spot inconsistence and errors similar to the way dimensional analysis does in engineering , or type checking in software development. It can help you ask right question .It can guide you to including the right things , and leaving the right things out.
3) Category theory isn't the solution to life , the universe ,and everything . It's a tool , like dimensional analysis and type checking.
4) Category theory has been used to study grammar and human language.
5) In building a spreadsheet application.
6) As a descriptive tool in neuroscience.
7) In the analysis and design of cognitive neural network architechtures.
8) Many application in graduate level mathematics and physics.
9) In programming languages , especially Haskell and most famously monads , but also , for instance , a typed assembly language and work on the typed lamda calculus.
10) Generating program optimizations
11) To model systems of interacting agents.
12) To generalize sorting algorithms.
13) In the study of analogy.
14) In definition of emergence and discussion of biology
15) Generally speaking , there seems to be a cabal of radical category theorists , led by John Baez , who are reinterpreting anything interesting in category theoretic terms.

V. CONCLUSION

Here we have discussed about how magma and medial magma apply in category theory and their applications.

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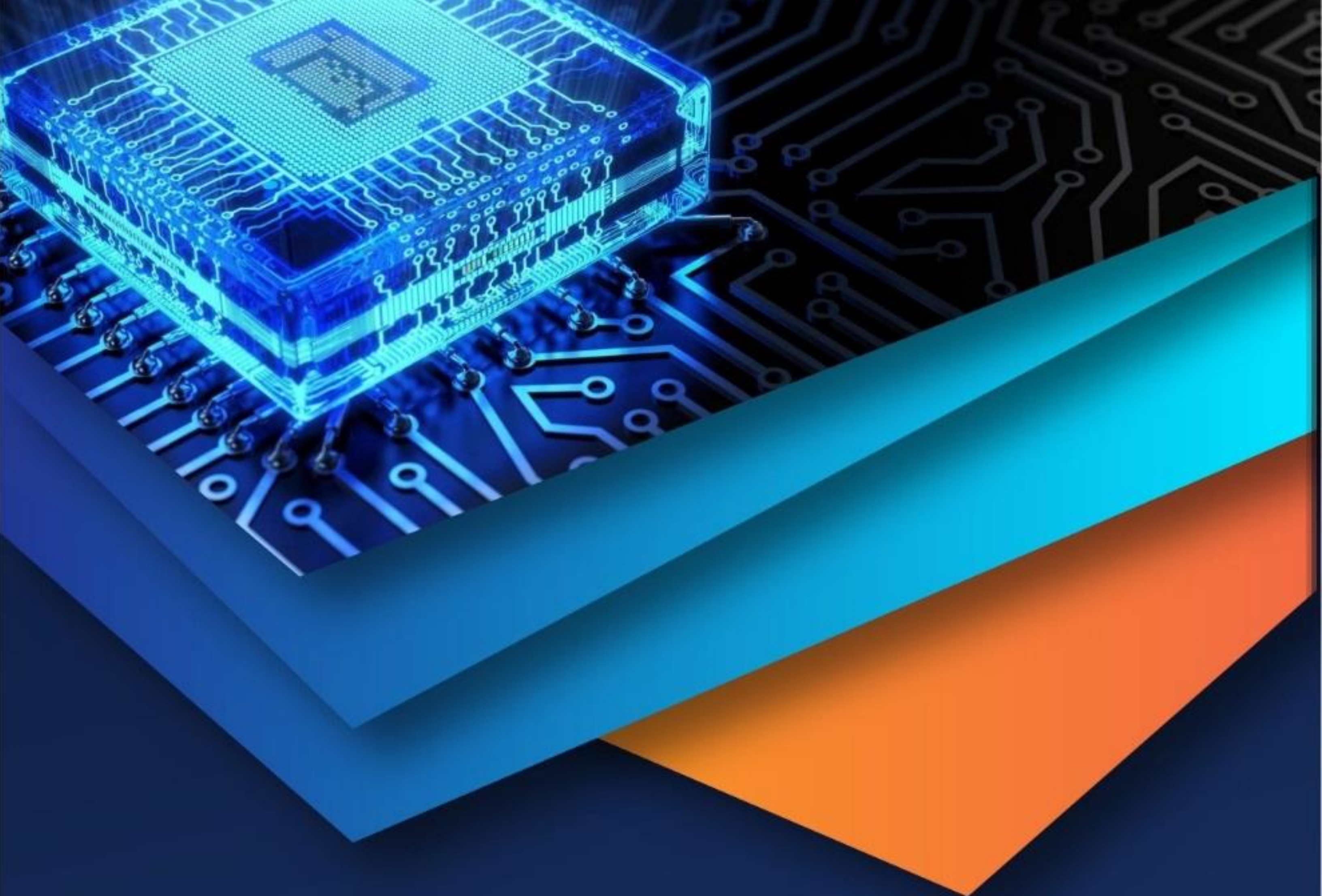

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