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A Study on Fuzzy Pre-Continuity in Fuzzy Lindel of Closed Space

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Abstract: Fuzzy Topology is one such branch, combining ordered structure with Topological structure. It is used to analyze complex networks. E.g. social networks, biological networks, internet, etc… A Lindelof space is a topological space in which every open cover has a countable sub cover. The Lindelof property is weakening of the more commonly used notions of compactness, which requires the existence of a finite subcover. This paper deals with fuzzy pre-continuity in Fuzzy Lindelof closed space and the results.
Keywords: Fuzzy topology, Fuzzy Lindelof space, Fuzzy continuous, Fuzzy pre-Continuous, fuzzy pre* continuous, fuzzy g* pre-continuous.

I. INTRODUCTION

The term topology was introduced by Johann Benedict Listing in the 19th century. Topology-topos + logy. Topos means 'place'. Topolgy is a modern version of geometry. Topology called also 'rubber- sheet geometry' because the objects can be stretched and contracted like rubber but cannot be broken. Chang introduced the notion of Fuzzy topology of a set in 1968.
Lindelof space have been introduced and studied by several authors. In 1959 Z.Frolite introduced and studied the notion of weakly Lindelof Space. In 1982 G. Balasubamian introduced and studied the notion of nearly Lindelof space and In 1996 F. Cammaroto and G. Santors introduced and notion of weakly regular Lindelof space on using regular covers. A.S Bin shahna introduced the notion of Fuzzy Lindelof space and investigated by some of their properties. Here I discuss about fuzzy pre-continuous, fuzzy pre*-continuous and fuzzy g* pre-continuous function in Fuzzy Lindelof closed space and their properties.

II. PRELIMINARIES

A. Definition1. 1[2]

Let X be a non-empty set, a collection δ of subset of X is said to be a topology on X if

1) X and the empty set φ belong to δ,
2) The union of any numbers of elements in δ belongs to δ,
3) The intersection of any two sets in δ belongs to δ

The members of δ are said to be open sets. The complement of members of δ are said to be closed sets. A subset of a topology δ is said to be clopen if it is both open and closed in δ. The pair (X, δ) is called topological spaces.

B. Definition 1.2[3]

Let $A_{1,\ldots\ldots,}A_n$ be fuzzy sets in E. We define $A_1 \times \ldots. \times A_n$ to be the fuzzy set A in En whose membership function is given by $\mu_A(x_1,\ldots..,x_n) = \min\{ \mu_{A1}(x_1),\ldots., \mu_{An}(x_n)\}$.
Let $f : E^n \quad E$, $f(x_1,\ldots..,x_n) = x_1 + \ldots.. + x_n$. We define $A_{1\,+} \ldots. + A_n = f(A)$. For λa scalar and B a fuzzy set in E, we define $\lambda B = g(B)$ Where g: $E \rightarrow E$, $g(x) = \lambda x$.

C. Definition 1.3[4]

Let X and Y be fuzzy spaces. Then a mapping f: $X \rightarrow Y$ is called fuzzy continuous if $f^{-1}(\lambda) \in \tau X$, for each $\lambda \in \tau Y$, or equivalently $f^{-1}(\mu)$ is a fuzzy closed set of X for each fuzzy closed set μ of Y.

D. Definition1.4[4]

Let X and Y be fuzzy spaces. Then a mapping f: $X \rightarrow Y$ is called fuzzy homeomorphism if f is one-one onto and both f and f^{-1} are fuzzy continuous.

E. Definition1.5[4]

Let X and Y be fuzzy spaces. Then a mapping f: $X \rightarrow Y$ is called fuzzy open (closed) if $f(\lambda)$ is a fuzzy open (closed) set of Y for each fuzzy open (closed) set λ of X.

F. Definition1.6[4]

A fuzzy continuous mapping f : X → Y on a fuzzy space X to another fuzzy space Y, is called a perfect mapping (or a fuzzy proper mapping) if for any fuzzy space Z, the mapping $f \times I_z : X \times Z \to Y \times Z$ is fuzzy closed.

III. FUZZY PRE- CONTINUITY

A. Fuzzy Pre- Continuous

1) *Definition:2.1.1[5]:* A function f: (A, T) → (B, T) is said to be a fuzzy pre- continuous if and only if the inverse image of any open set in A of T is fuzzy pre-open set B of T.
2) *Result 2.1.2:* The fuzzy pre– continuous image of a fuzzy pre- compact space is fuzzy compact space.
3) *Result:2.1.3:* Let f: (A, T) → (B, T) be a function, then following are equivalent

a) f is fuzzy pre-continuous
b) f (p-cl (C) ⊆ cl-(f(C)), for every fuzzy set C in A.

4) *Result: 2.1.4*

Let if f: (A, T) → (B, T) be a fuzzy pre-continuous subjective function of a fuzzy pre- compact a space A onto a space B then B is fuzzy pre-compact.

B. Fuzzy Pre- Continuous*

1) *Definition:2.2.1[5]:* A function f: (A, T) → (B, T) is said to be a fuzzy pre* - continuous if and only if the inverse image of any fuzzy pre-open set in T is fuzzy pre-open set in T.
2) *Result: 2.2.2:* If f: (A, T) → (B, T) is fuzzy continuous function, then f is fuzzy pre*- continuous.
3) *Result:2.2.3:* Let f: (A, T) → (B, T) be a function, then following are equivalent

a) f is fuzzy pre*-continuous
b) f (p-cl (C) ⊆ p-cl-(f(C)), for every fuzzy set C in A.

4) *Result: 2.2.4:* If a function f: (A, T) → (B, T) is fuzzy pre*-continuous and C is a fuzzy pre- compact relative to A then so is f(C) is fuzzy pre-compact.

C. Fuzzy G-Pre Continuous Function*

1) *Definition: 2.3.1[6]:* A fuzzy set A of a fuzzy topological spaces (X, T) is called a g*-pre-closed fuzzy set if pcl(A) ≤ U whenever A ≤ U and U is g-open fuzzy set in (X, T).
2) *Definition: 2.3.2[6]:* A fuzzy set A of a fuzzy topological spaces (X, T) is called a g*-pre-open fuzzy set if its compliments 1-A is g*p-closed fuzzy set.
3) *Definition: 2.3.3[6]:* Let X,Y be two fuzzy topological spaces. A function f: X→Y is called fuzzy g Pre-continuous functions if $f^{-1}(A)$ is g pre-closed fuzzy set in X, for every closed fuzzy set A of Y.
4) *Definition: 2.3.4[6]:* Let X and Y be fuzzy topological spaces. A map f: X→Y is said to be fuzzy g*p-continuous if the inverse image of every open fuzzy set in Y is g*p-open fuzzy set in X.
5) *Result: 2.3.5:* Every f continuous function is fuzzy g* pre-continuous function.
6) *Result: 2.3.6*: Every fuzzy g* pre-continuous function is fuzzy g pre-continuous function.
7) *Definition: 2.3.7[7]:* A function f : X → Y is said to be strongly g* pre-continuous if the inverse image of every g* p-closed set in Y is closed in X.
8) *Result: 2.3.8:* Every strongly g*p-continuous function is continuous and thus pre-continuous and g*p-continuous.

IV. FUZZY LINDELOF CLOSED SPACE

A. Definition:3.1[8]

A fuzzy topological space (X, T) is said to be fuzzy Lindelof if every fuzzy open cover of X has a countable subcover. That is, for every fuzzy open cover $\{ \lambda_\alpha\}_{\alpha \in \Delta}$ of X, there exist $\{ \lambda \alpha_n \}_{n \in N}$ of fuzzy open sets in (X, T) such that $V_{n \in N} \{ \lambda \alpha_n \}=1$.

B. Definition:3.2[8]

A fuzzy topology space (X, T) is said to be fuzzy simply Lindelof if each cover of X by fuzzy simply open sets has a countable subcover. That is, (X, T) is a fuzzy Lindelof α_n space if $V_{\alpha \in \Delta} \{ \lambda_\alpha\}=1$ where intcl [bd(λ_α)] = 0 in (X, T), then $V_{n \in N} \{ \lambda \alpha_n \}=1$ in (X, T).

C. Result:3.3

If (X,T) is a fzzy simply Lindelof space and if $V_{\alpha \in \Delta}$ { λ_α}=1 where { λ_α}'s are fuzzy closed set with int (λ_α) = 0 in (X, T), then $V_{n \in N}$ { $\lambda \alpha_n$ }=1 in (X, T).

D. Result:3.4

If (X, T) is a fuzzy simply Lindelof space and if $V_{\alpha \in \Delta}$ { λ_α}=1 where $\lambda \alpha_n \in$ T and cl(λ_α) = 1 in (X, T), then $\Lambda_{n \in N}$ { $\mu \alpha_n$ }= 0, where ($\mu \alpha_n$)'s are fuzzy nowhere dense sets in (X, T).

E. Definition: 3.5[9]

A fuzzy topological space (X,Y) is said to be fuzzy Lindelof closed space if and only if for every family ψ of fuzzy open sets of X and for every α∈I such that v{U:U∈ ψ} ≥α and for every ε∈ (0, α] there exist a countable subfamily ψ* of ψ whose closure such that v{U:U∈ ψ*} ≥α- ε.

F. Result:3.6

If (X, w(δ)) is fuzzy Lindelof closed, then (X, δ) is Lindelof closed.

G. Definition:3.7[9]

A fuzzy topology space (X, δ), λ is said to be fuzzy para compact if for every open cover in the sense of lowen H of λ and for every ε ∈ (0, α), there exist an open refinement D of H whichis both locally finite in X and cover of λ – ε in the sense of Lowen.

H. Result:3.8

Let (X, δ) be a fuzzy para compact seperable topological space, then (X, δ) is fuzzy Lindelof closed.

V. FUZZY PRE-CONTINUITY IN FUZZY LINDELOF CLOSED SPACE

Let us note here the properties of fuzzy pre-continuous, pre* continuous, g* pre continuous in fuzzy Lindelof closed space.
Before discussing that let's see an important result relating fuzzy topological space and Lindelof space.

A. Result 4.1

If (X, T) is a fuzzy topological spaces, it is also Lindelof
1) Proof: Given (X, T) is a fuzzy topological spaces.
Let A= {A_i}, i ∈ I, be an open cover of X.
By the theorem
"A fuzzy topological spaces (X, T) is Lindelof if and only if every open cover of X as a countable subcover."
To prove A_I is a countable sub cover of A.
Let B is a countable sub family of T.
A_i is the union of members of a countable sub family B = {B_n}, n=1,2,… of T.
Now let $A_i = \cup B\, i_k$, here k= 1 to i_j
May i_0 be infinity.
B_0 = {B i_k }, i ∈ I
Here, $1 \leq k \leq i_0$,
It forms an open cover of X.
B_0 is countable,
Since it is a sub family of B.
Each member of B_0 is contained in one A_i.
Therefore A_i form a countable sub cover of A.
So A is a Lindelof.
Therefore a fuzzy topological spaces (X, T) is also Lindelof.
Hence proved.

B. Theorem 4.2

Let f : (X, δ) → (Y, π) be a fuzzy continuous surjection map. If (X, δ) is a fuzzy Lindelof closed then the (Y, π) is also fuzzy Lindelof closed.

1) Proof

Given f : (X, δ) → (Y, π) be a fuzzy continuous surjection map.
i.e., f is an fuzzy continuous onto map.
For y ∈ (Y, π) there exist x ∈ (X, δ) such that f (x) = y
We know f continuous
"A function f: (A, T) → (B, T) is fuzzy continuous (f-continuous) if and only if the inverse image of any fuzzy open set in T is fuzzy open set in T. "
For A be closed in (Y, π)
There exist f^{-1}(A) is closed in (X, δ)
Since (X, δ) is fuzzy Lindelof closed.
And f^{-1}(A) closed for A is closed in (Y, π)
Therefore (Y, π) is also fuzzy Lindelof closed.

C. Theorem 4.3

Let f : (X, δ) → (Y, π) be a fuzzy pre-continuous surjection map. If (X, δ) is a fuzzy Lindelof closed then the (Y, π) is also fuzzy Lindelof closed.

1) Proof Given f : (X, δ) → (Y, π) be a fuzzy pre-continuous surjection map.
i.e., f is an fuzzy pre-continuous onto map.
For y ∈ (Y, π) there exist x ∈ (X, δ) such that f (x) = y
We know fuzzy pre-continuous
"A function f: (A, T) → (B, T) is said to be a fuzzy pre- continuous if and only if the inverse image of any set in A of T is fuzzy pre-open set B of T."
For A be closed in (Y, π)
There exist f^{-1}(A) is pre-closed in (X, δ)
Since (X, δ) is fuzzy Lindelof closed.
And f^{-1}(A) pre-closed for A is closed in (Y, π)
Therefore (Y, π) is also fuzzy Lindelof closed.

D. Theorem 4.4

Let f : (X, δ) → (Y, π) be a fuzzy pre*-continuous surjection map. If (X, δ) is a fuzzy Lindelof closed then the (Y, π) is also fuzzy Lindelof closed.

1) Proof

Given f : (X, δ) → (Y, π) be a fuzzy pre*-continuous surjection map.
i.e., f is an fuzzy pre*-continuous onto map.
For y ∈ (Y, π) there exist x ∈ (X, δ) such that f (x) = y
We know fuzzy pre*-continuous
"A function f: (A, T) → (B, T) is said to be a fuzzy pre* - continuous if and only if the inverse image of any fuzzy pre-open set in T is fuzzy pre-open set in T "
For A be pre-closed in (Y, π)
There exist f^{-1}(A) is pre-closed in (X, δ)
Since (X, δ) is fuzzy Lindelof closed.
And f^{-1}(A) pre-closed for A is pre-closed in (Y, π)
Therefore (Y, π) is also fuzzy Lindelof closed.

E. Theorem 4.5

Let f : (X, δ) → (Y, π) be a fuzzy g*pre-continuous surjection map. If (X, δ) is a fuzzy Lindelof closed then the (Y, π) is also fuzzy Lindelof closed.

1) Proof

Given f : (X, δ) → (Y, π) be a fuzzy g*pre-continuous surjection map.
i.e., f is an fuzzy g*pre-continuous onto map.
For y ∈ (Y, π) there exist x ∈ (X, δ) such that f (x) = y

We know fuzzy g*pre-continuous

"Let X and Y be fuzzy topological spaces. A map f: X→Y is said to be fuzzy g*p-continuous if the inverse image of every open fuzzy set in Y is g*p-open fuzzy set in X."

For A be closed in (Y, π)

By the definition,

"A fuzzy set A of a fuzzy topological spaces (X, T) is called a g*-pre-closed fuzzy set if $pcl(A) \leq U$ whenever $A \leq U$ and U is g-open fuzzy set in (X, T)"

There exist $f^{-1}(A)$ is g*pre-closed in (X, δ)

Since (X, δ) is fuzzy Lindelof closed and $f^{-1}(A)$ g*pre-closed for A is closed in (Y, π)

We get (Y, π) is also fuzzy Lindelof closed.

F. Theorem 4.5

Let (X, δ) be a fuzzy topological space, then the fuzzy continuous image of fuzzy para-compact is fuzzy Lindelof closed.

1) Proof

Let f: $(X, \delta) \rightarrow (Y, \pi)$ be fuzzy continuous.

Let B a fuzzy para compact subset of X.

By the theorem,

"A fuzzy topology space (X, δ), λ is said to be fuzzy para compact if for every open cover in the sense of lowen H of λ and for every $\varepsilon \in (0, \alpha)$, there exist an open refinement D of H which is both locally finite in X and cover of $\lambda - \varepsilon$ in the sense of Lowen."

Then there exist countable base $\{L_i\}$ for i=1,2,3….such that

$\{W \cap L_i\} \subset \delta_w$ where (W, δ_w) is the subspace of (X, δ).

Since the (W, δ_w) is the subspace which is seperable.

By the definition,

"A fuzzy topological spaces (X, T) is said to be seperable if and only if there exist a countable family of fuzzy points in X which is dense in (X, T)"

We can say B is seperable.

Then by theorem

"Let (X, δ) be a fuzzy para compact seperable topological space then (X, δ) is fuzzy Lindelof space"

Let $H = \{\lambda_j\}_{j\in J} \subset \delta$ be a family such that $\vee_{k\in N} \{\lambda_j\} \geq \alpha$ for each $\alpha \in I$.

Let H^* be a δ – open refinement of H which is locally finite and

so, $\vee_{k\in N} h \geq \alpha - \varepsilon, \varepsilon \in (0, \alpha)$

X has countable sequence of fuzzy points $\{p_i, i=1,2,….\}$ such that for every $h \neq 0$ there exist a $p_i \in h$. Then the family $\{h; h \in H^*\}$ is at most countable, otherwise since each h contains at least one p_i. Hence, there would be some p_n contained in uncountable many $h \in H$ which contradicts locally finite property. Choose for each $h_i \in H^*$ an element $\lambda_i \in H$ such that $h_i < \lambda_i \leq h$ Then there exists an open countable H' subset of H such that

$\vee_{\lambda\in H'} \{\lambda\} \geq \alpha - \varepsilon.$

Therefore (X, δ) is fuzzy Lindelof closed.

By the above, B is a fuzzy Lindelof closed.

Hence proved.

G. Theorem 4.6

Let (X, δ) be a fuzzy topological space, then the fuzzy pre-continuous image of fuzzy para-compact is fuzzy Lindelof closed.

1) Proof

Let f: $(X, \delta) \rightarrow (Y, \pi)$ be fuzzy pre-continuous.

Let B a fuzzy para compact subset of X.

By the theorem,

"A fuzzy topology space (X, δ), λ is said to be fuzzy para compact if for every open cover in the sense of lowen H of λ and for every $\varepsilon \in (0, \alpha)$, there exist an open refinement D of H which is both locally finite in X and cover of $\lambda - \varepsilon$ in the sense of Lowen."

Then there exist countable base $\{L_i\}$ for i=1,2,3….such that

$\{W \cap L_i\} \subset \delta_w$ where (W, δ_w) is the subspace of (X, δ).

Here there exist open cover $\{L_i\}$ with $\{W \cap L_i\}$ as open sub cover.
Also f: $(X, \delta) \rightarrow (Y, \pi)$ is fuzzy pre-continuous.
Which implies A^c pre open in (Y, π).
$f^{-1}(A^c)$ is open in (X, δ).
$(A^c)^c$ is pre closed in (Y, π).
$f^{-1}((A^c)^c)$ is closed in (X, δ).
A is pre closed in (Y, π).
Then $f^{-1}(A)$ is closed in (X, δ).
So (X, δ) fuzzy Lindelof closed.
Hence proved.

H. Theorem 4.7

Let (X, δ) be a fuzzy topological space, then the fuzzy pre*-continuous image of fuzzy para-compact is fuzzy Lindelof closed.

1) Proof

Let f: $(X, \delta) \rightarrow (Y, \pi)$ be fuzzy pre*-continuous.
Let B a fuzzy para compact subset of X.
By the theorem,
"A fuzzy topology space (X, δ), λ is said to be fuzzy para compact if for every open cover in the sense of lowen H of λ and for every
$\varepsilon \in (0, \alpha)$, there exist an open refinement D of H which is both locally finite in X and cover of $\lambda - \varepsilon$ in the sense of Lowen."
Then there exist countable base $\{L_i\}$ for i=1,2,3….such that
$\{W \cap L_i\} \subset \delta_w$ where (W, δ_w) is the subspace of (X, δ).
Here there exist open cover $\{L_i\}$ with $\{W \cap L_i\}$ as open sub cover.
Also f: $(X, \delta) \rightarrow (Y, \pi)$ is fuzzy pre*-continuous. Which implies A^c pre open in (Y, π).
$f^{-1}(A^c)$ is pre open in (X, δ).
$(A^c)^c$ is pre closed in (Y, π).
$f^{-1}((A^c)^c)$ is pre closed in (X, δ).
A is pre closed in (Y, π).
$f^{-1}(A)$ is pre closed in (X, δ).
So (X, δ) fuzzy Lindelof closed.
Hence proved.

I. Theorem 4.8

Let (X, δ) be a fuzzy topological space, then the fuzzy g*pre-continuous image of fuzzy para-compact is fuzzy Lindelof closed.

1) Proof: Let f: $(X, \delta) \rightarrow (Y, \pi)$ be fuzzy g*pre-continuous.
Let B a fuzzy para compact subset of X.
By the theorem, "A fuzzy topology space (X, δ), λ is said to be fuzzy para compact if for every open cover in the sense of lowen H of λ and for every $\varepsilon \in (0, \alpha)$, there exist an open refinement D of H which is both locally finite in X and cover of $\lambda - \varepsilon$ in the sense of Lowen."
Then there exist countable base $\{L_i\}$ for i=1,2,3….
Such that $\{W \cap L_i\} \subset \delta_w$ where (W, δ_w) is the subspace of (X, δ).
Here there exist open cover $\{L_i\}$ with $\{W \cap L_i\}$ as open sub cover.
Also f: $(X, \delta) \rightarrow (Y, \pi)$ is fuzzy g*pre-continuous.
Which implies A^c pre open in (Y, π).
$f^{-1}(A^c)$ is g*pre-open in (X, δ).
$(A^c)^c$ is pre closed in (Y, π).
$f^{-1}((A^c)^c)$ is g*pre-closed in (X, δ).
A is pre closed in (Y, π).
Then $f^{-1}(A)$ is g*pre-closed in (X, δ).
So (X, δ) fuzzy Lindelof closed.
Hence proved.

VI. CONCLUSION

We all know every closed space is bounded and hence compact. Hence our fuzzy Lindelof closed space is compact and so its satisfies the pre-continuity properties with para compactness. Also fuzzy pre-continuous, fuzzy pre*-continuous, fuzzy g*pre-continuous are inter-related with each other based on its definitions and properties. Therefore fuzzy Lindelof closed space satisfies all the pre-continuity properties.

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