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A Study on Intuitionistic Fuzzy Sets

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Abstract: Graph theory has numerous applications in modern science. Instuitionistic fuzzy set has shown advantages in handling vagueness and compared to fuzzy set. Now we discussed about the basic concept and decision making methods of fuzzy sets.

I. INTRODUCTION

In 1965, zadeh first introduced thereafter it has been developed by several authors through the contribution of the different articles on this concept and applied on different branches of pure and applied mathematics. The concept of fuzzy norm was introduced by katsaras in 1984 and in 1992, felbin introduced the idea of fuzzy norm on a linear space.

Cheng-moderson introduced another idea of fuzzy norm on a linear space whose associated metric is same as the associated metric of kramosil-michalek. Later on bag and samanta modified the defintition of fuzzy norm of cheng-moderson and established the concept of contiuity and boundedness of a function with respect to their fuzzy norm in the authors T.bag and s.k samanta introduced the definition of fuzzy norm over a linear space by the definition S.c cheng and J.N moorde-son and they have studied finite dimensional fuzzy nurmed linear spaces.

Also the definition of institionistic fuzzy n-normed linear space was introduced in the paper and established a sufficient condition for an instutionistic fuzzy n-normed linear space to be complete.

A. Basics Of Fuzzy Sets

1) Definitions

a) If X is a collection of objects denoted generically by x, then a fuzzy set Ã in X is a set of ordered pairs:

$$\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x) \right) \middle| x \in X \right\}$$

$\mu_{\tilde{A}}(x)$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in Ã that maps X to the membership space M (when M contains only the two points 0 and 1, Ã is non-fuzzy and $\mu_{\tilde{A}}(x)$ is identical to the characteristic function of a non-fuzzy set). The range of the membership function is a subset of the non-negative real numbers whose supremum is finite. Elements with a zero degree of membership are normally not listed.

b) The (crisp) set of elements that belong to the fuzzy set Ã at least to the degree α is called the α-level set:

$$A_\alpha = \left\{ x \in X \,\middle|\, \mu_{\tilde{A}}(x) \geq \alpha \right\}$$

$A'_\alpha = \left\{ x \in X \,\middle|\, \mu_{\tilde{A}}(x) > \alpha \right\}$ is called strong α-level set or strong α-cut.

c) A fuzzy set Ã is convex if

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\left\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\right\}, x_1, x_2 \in X,\ \lambda \in [0,1]$$

Alternatively, a fuzzy set is convex if all α-level sets are convex.

d) For a finite fuzzy set Ã, the cardinality |Ã| is defined as

$$\left|\tilde{A}\right| = \sum_{x \in X} \mu_{\tilde{A}}(x)$$

$||\tilde{A}|| = \frac{|\tilde{A}|}{|x|}$ is called the relative cardinality of Ã.

Obviously, the relative cardinality of fuzzy set depends on the cardinality of the universe. So the same universe has to be chosen to compare fuzzy sets by their relative cardinality.

e) The membership function $\mu_{\tilde{C}}(x)$ of the *intersection* $\tilde{C} = \tilde{A} \cap \tilde{B}$ is point wise defined by $\mu_{\tilde{C}}(x) = \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}, x \in X$

f) The membership function of the *complement* of a normalized fuzzy set Ã, $\mu_{\text{¢}\tilde{A}}(x)$ is defined by $\mu_{\text{¢}\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x), x \in X$

B. Basic Concept Of Intuitionistic Fuzzy Sets

Let X be the universe of discourse. An intuitionistic fuzzy set Ain Xis an object having the form

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle | \forall x \in X\}$$

Where $\mu_A(x): X \to [0,1]$ and $\gamma_A(x): X \to [0,1]$ denote membership function and non-membership function, respectively of A and satisfy $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for every $x \in X$.

$\mu_A(x)$ is the lowest bound of membership degree derived from proofs of supporting x; $\gamma_A(x)$ is the lowest bound of non-membership degree derived from proofs of rejecting x. It is clear that the membership degree of IF set Ahas been restricted in $[\mu_A(x), 1-\gamma_A(x)]$ which is a subinterval of [0,1].

Obviously, each fuzzy set A in X could be represented as the following IFS:

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle | \forall x \in X\}$$

For each IF set A in X, we call

$$\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x)$$

as the intuitionistic index of x in A. It is hesitation degree (or degree of indeterminacy) of X to A. It is obvious that $0 \le \mu_A(x) \le 1$ for each $x \in X$.

For example, let A be an IFS with membership function $\mu_A(x)$ and non-membership function $\gamma_A(x)$ respectively. If $\mu_A(x) = 0.5$ and $\gamma_A(x) = 0.3$, then we have $\mu_A(x) = 1 - 0.5 - 0.3 = 0.2$. It could be interpreted as the degree that the object x belongs to the IF set A is 0.5, the degree that the object x does not belong to the IF set A is 0.3 and the degree of hesitation is 0.2. Thus, IF set A in X can be expressed as

$$A = \{\langle x, \mu_A(x), \gamma_A(x), \pi_A(x)\rangle : x \in X\}$$

If A is an ordinary fuzzy set, then $\pi_A(x) = 1 - \mu_A(x) - (1 - \mu_A(x)) = 0$ for each $x \in X$.

It means that the third parameter $\pi_A(x)$ cannot be casually omitted if A is a general IFS, not an ordinary fuzzy set. Therefore, the representation of IFS should consider all three parameter in calculating the degree of similarity between IFSs.

For $A, B \in IFS(X)$ Atanassov defined the notion of containment as follows

$A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$.

C. Basic Relations And Operations On Intuitionistic Fuzzy Sets

Inclusion:

$A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$.

Equality:

$A = B \Leftrightarrow \mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$ for all $x \in X$.

Complement:

$$A^c = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

Union:

$$A \cup B = \{x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x)) : x \in X\}$$

Intersection:

$A \cap B = \{x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x)) : x \in X\}$

Addition:

$A \oplus B = \{x, (\mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x)), (\nu_A(x).\nu_B(x)) : x \in X\}$

Multiplication:

$$A \otimes B = \{x, (\mu_A(x).\mu_B(x)), \nu_A(x) + \nu_B(x) - (\nu_A(x).\nu_B(x)) : x \in X\}$$

Difference:

$$A - B = \{x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x)) : x \in X\}$$

Symmetric Difference:

$$A\Delta B = \{x, \max(\min(\mu_A(x), \nu_B(x)), \min(\nu_A(x), \mu_B(x))), \min(max(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x))\}$$

Cartesian Product :

A X B = $\{(\mu_A(x)\mu_B(x), (\nu_A(x)\nu_B(x)) : x \in X\}$

1) Definition

Let X be a non empty universal set. The normalization of an intuitionistic fuzzy set A denoted by NORM (A) is defines as: NORM (A) = $\{(x, \mu_{NORM(A)}(x), \mu_{NORM(A)}(x)) : x \in X\}$ where $\mu_{NORM(A)} = \frac{\mu_{A}(x)}{\sup \mu_A(x)}$ and $\vartheta_{NORM(A)}(x) = \frac{\vartheta_A(x) - \inf(\vartheta_A(x))}{1 - \inf(\vartheta_A(x))}$ for X = {x}.

Incorporating

$\pi_{NORM(A)}(x)$, NORM(A)=$\{\langle x, \mu_{NORM(A)}(x), \vartheta_{NORM(A)}(x), \pi_{NORM(A)}(x)\rangle : x \in X$

$for\ \pi_{NORM(A)}(x) = 1 - \mu_{NORM(A)}(x) - \vartheta_{NORM(A)}(x)\}$

E.g :

Let X = $\{x_1, x_2, x_3\}$ and let IF set A be s.t. A = $\{\langle 0.5, 0.5\rangle, \langle 0.4, 0.1\rangle, \langle 0.25, 0.75\rangle\}$

Then sup $(\mu_A(x)) = 0.9$ *and* $\inf(\vartheta_A(x)) = 0.1$ *thus* $\mu_{NORM(A)}(x_1) = 0.56$ $\mu_{NORM(A)}(x_2) = 1$, $\mu_{NORM(A)}(x_3) = 0.28$

$\vartheta_{NORM(A)}(x_1) = 0.45$, $\vartheta_{NORM(A)}(x_2) = 0.0$, $\vartheta_{NORM(A)}(x_3) = 0.72$

NORM (A) = $\{\langle 0.56, 0.4\rangle, \langle 1.0, 0.0\rangle, \langle 0.28, 0.72\rangle\}$

Obviously , $\mu_{NORM(A)}(x_1) + \vartheta_{NORM(A)}(x_1) = 1$, $\mu_{NORM(A)}(x_2) + \vartheta_{NORM(A)}(x_2) = 1$

II. CONCLUSION

Currently IF sets have applications in various areas. There are applications IF sets in medical diagnosis and in decision making in medichine, developed by Anthony Shannon, Soon Ki - Kim, Eulalia Szmidt, Janusz Kacprzyk, Humebrto Bustince, Joseph Sorsich and others. Intuitionistic fuzzy systems and IF abstract systems are defined and studied by Valentina Radeva, Hristo Aladjov and the author. A first step to describe a theory of the IF graphs and temporal IF graphs is made by Anthony Shannon and the author. Application of IF graphs and IF relation methods are also developed. Intuitionistic fuzzy set theory is defined with detailed explaination of different operation applicable on it. In this paper we have introduced NORM of IF sets and studied several properties of these operations.

REFERENCE

[1] Atanassov.K, Intuitionistic Fuzzy sets,Fuzzy Sets and Systems, (1986).

[2] Atanassov.K, More on intuitionistic fuzzy sets,Fuzzy Sets and Systems,(1989).

[3] Atanassov.K,New operations defined over the intuitionistic fuzzy sets. Fuzzy sets and systems, (1994).

[4] Atanassov.K, Operators over interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems,(1994).

[5] Atanassov.K,Gargov.G, Interval-valued intuitionistic fuzzy sets. Fuzzy sets and Systems,(1989) .