



IJRASET

International Journal For Research in
Applied Science and Engineering Technology



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: TPAM-2018 **Issue:** conference **Month of publication:** March 2018

DOI:

www.ijraset.com

Call: ☎ 08813907089

E-mail ID: ijraset@gmail.com

Anti- Homomorphism of Fuzzy Soft Subhemirings of a Hemiring

N.Anitha¹, J.Venkatesan²

¹Assistant Professor, Department of Mathematics, Periyar University PG Extension center, Dharmapuri- Tamilnadu, India.

²Assistant Professor, Department of Mathematics, Sri Vidya Mandir Arts and Science College, Uthangarai-635207, Tamilnadu, India.

Abstract: In this paper, we made an attempt to study the algebraic nature of an anti-homomorphism of fuzzy soft subhemirings of a hemiring. 2000 AMS Subject classification: 05C38, 15A15, 05A15, 15A18.

Keywords: Fuzzy soft set, fuzzy soft subhemiring, anti-fuzzy soft subhemiring, and pseudo Fuzzy soft coset.

I. INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring $(R; +; \cdot)$. Some of them in particular, nearrings and several kinds of semirings have been proven very useful. Semirings (called also halfrings) are algebras $(R; +; \cdot)$ share the same properties as a ring except that $(R; +)$ is assumed to be a semigroup rather than a commutative group. Semirings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra $(R; +, \cdot)$ is said to be a semiring if $(R; +)$ and $(R; \cdot)$ are semigroups satisfying $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(b + c) \cdot a = b \cdot a + c \cdot a$ for all a, b and c in R . A semiring R is said to be additively commutative if $a + b = b + a$ for all a, b and c in R . A semiring R may have an identity 1 , defined by $1 \cdot a = a = a \cdot 1$ and a zero 0 , defined by $0 + a = a = a + 0$ and $a \cdot 0 = 0 = 0 \cdot a$ for all a in R . A semiring R is said to be a hemiring if it is an additively commutative with zero. After the introduction of fuzzy sets by L.A. Zadeh [17], several researchers explored on the generalization of the concept of fuzzy sets. M. Borah, T. J. Neog and D. K. Sut,[5] were developed some operations of fuzzy soft sets, On operations of soft sets was developed by

A.Sezgin and A. O. Atagun,[13] and KumudBorgohain and ChittaranjanGohain,[7] was developed some New operations on Fuzzy Soft Sets, In this paper, we introduce some Theorems in Fuzzy soft subhemirings of a hemiring.

II. PRELIMINARIES

- 1) *Definition:* A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U . In other words, the soft set is a parameterized family of subsets of the set U . Every set $F(\varepsilon) (\varepsilon \in E)$ from this family may be considered as the set of ε -elements of the soft sets (F, E) or as the set of ε -approximate elements of the soft set.
- 2) *Definition:* Let (U, E) be a soft universe and $A \subseteq E$. Let $\mathcal{F}(U)$ be the set of all fuzzy subsets in U . A pair (\tilde{F}, A) is called a fuzzy soft set over U , where \tilde{F} , is a mapping given by $\tilde{F}: A \rightarrow \mathcal{F}(U)$.
- 3) *Definition:* Let R be a hemiring. A Fuzzy soft subset (F, A) of R is said to be an Fuzzy soft subhemiring (FSHR) of R if it satisfies the following conditions:
 - (i) $\mu_{(F,A)}(x + y) \geq \min \{ \mu_{(F,A)}(x), \mu_{(F,A)}(y) \}$,
 - (ii) $\mu_{(F,A)}(xy) \geq \min \{ \mu_{(F,A)}(x), \mu_{(F,A)}(y) \}$, for all x and y in R .
- 4) *Definition:* Let $(R, +, \cdot)$ be a hemiring. An Fuzzy soft subhemiring (F, A) of R is said to be an Fuzzy soft normal subhemiring (FSNSHR) of R if it satisfies the following conditions:
 - (i) $\mu_{(F,A)}(xy) = \mu_{(F,A)}(yx)$, (ii) $\nu_{(F,A)}(xy) = \nu_{(F,A)}(yx)$, for all x and y in R .
- 5) *Definition:* If $(R, +, \cdot)$ and $(R', +, \cdot)$ are any two hemirings, then the function $f: R \rightarrow R'$ is called a **homomorphism** if $f(x+y) = f(x)+f(y)$ and $f(xy)=f(x)f(y)$, for all x and y in R .
- 6) *Definition:* If $(R, +, \cdot)$ and $(R', +, \cdot)$ are any two hemirings, then the function $f: R \rightarrow R'$ is called an **anti-homomorphism** if $f(x+y) = f(y)+f(x)$ and $f(xy)=f(y)f(x)$, for all x and y in R .
- 7) *Definition:* Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Then the function $f: R \rightarrow R'$ be a hemiring homomorphism. If f is one-to-one and onto, then f is called a **hemiring isomorphism**.
- 8) *Definition:* Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Then the function $f: R \rightarrow R'$ be a hemiring anti-homomorphism. If f is one-to-one and onto, then f is called a **hemiring anti-isomorphism**.

9) *Definition:* Let R and R' be any two hemirings. Let $f: R \rightarrow R'$ be any function and let A be an Fuzzy soft subhemiring in R, V be an Fuzzy soft subhemiring in $f(R) = R'$, defined by

$$\mu_{V(y)} = \sup_{x \in f^{-1}(y)} (\mu_{(F,A)}(x)) \text{ for all } x \text{ in } R \text{ and } y \text{ in } R'. \text{ Then } A \text{ is called a preimage of } V \text{ under } f \text{ and is denoted by } f^{-1}(V).$$

10) *Definition:* Let (F,A) be an Fuzzy soft subhemiring of a hemiring (R, +, ·) and a in R. Then the pseudo Fuzzy soft coset $(a, (F,A))^p$ is defined by $((a\mu_{(F,A)})^p)(x) = p(a)\mu_{(F,A)}(x)$, for every x in R and for some p in P.

III. FUZZY SOFT SUBHEMIRINGS OF A HEMIRING

1) *Theorem:* If (F, A) is an Fuzzy soft subhemiring of a hemiring (R, +, ·), then (F, □A) is an Fuzzy soft subhemiring of R.

a) *Proof:* Let (F, A) be an fuzzy soft subhemiring of a hemiring R. Consider $(F,A) = \{ \langle x, \mu_{(F,A)}(x) \rangle \}$, for all x in R, we take $(F, \square A) = (F,B) = \{ \langle x, \mu_{(F,B)}(x) \rangle \}$, where $\mu_{(F,B)}(x) = \mu_{(F,A)}(x)$. Clearly, $\mu_{(F,B)}(x+y) \geq \min \{ \mu_{(F,B)}(x), \mu_{(F,B)}(y) \}$, for all x and y in R and $\mu_{(F,B)}(xy) \geq \min \{ \mu_{(F,B)}(x), \mu_{(F,B)}(y) \}$, for all x and y in R. Since A is an fuzzy soft subhemiring of R, we have $\mu_{(F,A)}(x+y) \geq \min \{ \mu_{(F,A)}(x), \mu_{(F,A)}(y) \}$, for all x and y in R, And $\mu_{(F,A)}(xy) \geq \min \{ \mu_{(F,A)}(x), \mu_{(F,A)}(y) \}$, for all x and y in R, for all x and y in R. Hence $(F,B) = (F, \square A)$ is an fuzzy soft subhemiring of a hemiring R.

2) *Theorem:* If (F,A) is a fuzzy soft subhemiring of a hemiring (R, +, ·), then (F, △A) is an fuzzy soft subhemiring of R.

a) *Proof:* Let (F,A) be an fuzzy soft subhemiring of a hemiring R. That is

$(F,A) = \{ \langle x, \mu_{(F,A)}(x) \rangle \}$, for all x in R. Let $(F,\triangle A) = (F,B) = \{ \langle x, \mu_{(F,B)}(x) \rangle \}$, for all x and y in R. Since (F,A) is an fuzzy soft subhemiring of R, which implies that $1 - \mu_{(F,B)}(xy) \leq \max \{ (1 - \mu_{(F,B)}(x)), (1 - \mu_{(F,B)}(y)) \}$, which implies that $\mu_{(F,B)}(xy) \geq 1 - \max \{ (1 - \mu_{(F,B)}(x)), (1 - \mu_{(F,B)}(y)) \} = \min \{ \mu_{(F,B)}(x), \mu_{(F,B)}(y) \}$. Therefore, $\mu_{(F,B)}(xy) \geq \min \{ \mu_{(F,B)}(x), \mu_{(F,B)}(y) \}$, for all x and y in R. Hence $(F,B) = (F,\triangle A)$ is an fuzzysoftsubhemiring of a hemiring R.

3) *Theorem:* Let (R, +, ·) be a hemiring and (F,A) be a non-empty subset of R. Then (F,A) is a subhemiring of R if and only if $(F,B) = \langle \chi_{(F,A)}, \bar{\chi}_{(F,A)} \rangle$ is a fuzzy soft subhemiring of R, where $\chi_{(F,A)}$ is the characteristic function.

a) *Proof:* Let (R, +, ·) be a hemiring and (F,A) be a non-empty subset of R. First let (F,A) be a subhemiring of R. Take x and y in R.

Case (i): If x and y in (F,A), then $x+y, xy$ in (F,A), since (F,A) is a subhemiring of R, $\chi_{(F,A)}(x) = \chi_{(F,A)}(y) = \chi_{(F,A)}(x+y) = \chi_{(F,A)}(xy) = 1$ and $\chi_{(F,A)}(x) = \chi_{(F,A)}(y) = \chi_{(F,A)}(x+y) = \chi_{(F,A)}(xy) = 0$. So, $\chi_{(F,A)}(x+y) \geq \min \{ \chi_{(F,A)}(x), \chi_{(F,A)}(y) \}$, for all x and y in R, $\chi_{(F,A)}(xy) \geq \min \{ \chi_{(F,A)}(x), \chi_{(F,A)}(y) \}$, for all x and y in R. So, $\chi_{(F,A)}(x+y) \leq \max \{ \chi_{(F,A)}(x), \chi_{(F,A)}(y) \}$, for all x and y in R, $\chi_{(F,A)}(xy) \leq \max \{ \chi_{(F,A)}(x), \chi_{(F,A)}(y) \}$, for all x and y in R.

Case (ii): If x in (F,A), y not in (F,A) (or x not in (F,A), y in (F,A)), then $x+y, xy$ may or may not be in (F,A), $\chi_{(F,A)}(x) = 1, \chi_{(F,A)}(y) = 0$ (or) $\chi_{(F,A)}(x) = 0, \chi_{(F,A)}(y) = 1$, $\chi_{(F,A)}(x+y) = \chi_{(F,A)}(xy) = 1$ (or 0) and $\chi_{(F,A)}(x) = 0, \chi_{(F,A)}(y) = 1$ (or) $\chi_{(F,A)}(x) = 1, \chi_{(F,A)}(y) = 0$) = $\chi_{(F,A)}(x+y) = \chi_{(F,A)}(xy) = 0$ (or 1). Clearly $\chi_{(F,A)}(x+y) \geq \min \{ \chi_{(F,A)}(x), \chi_{(F,A)}(y) \}$, for all x and y in R $\chi_{(F,A)}(xy) \geq \min \{ \chi_{(F,A)}(x), \chi_{(F,A)}(y) \}$, for all x and y in R, and $\chi_{(F,A)}(x+y) \leq \max \{ \chi_{(F,A)}(x), \chi_{(F,A)}(y) \}$, for all x and y in R $\chi_{(F,A)}(xy) \leq \max \{ \chi_{(F,A)}(x), \chi_{(F,A)}(y) \}$, for all x and y in R.

Case (iii): If x and y not in (F,A) , then $x+y, xy$ may or may not be in (F,A) , $\chi_{(F,A)}(x) = \chi_{(F,A)}(y) = 0, \chi_{(F,A)}(x+y) = \chi_{(F,A)}(xy) = 1$ or 0 and $\chi_{(F,A)}(x) = \chi_{(F,A)}(y) = 1, \chi_{(F,A)}(x+y) = \chi_{(F,A)}(xy) = 0$ or 1. Clearly $\chi_{(F,A)}(x+y) \geq \min \{ \chi_{(F,A)}(x), \chi_{(F,A)}(y) \}$, for all x and y in R $\chi_{(F,A)}(xy) \geq \min \{ \chi_{(F,A)}(x), \chi_{(F,A)}(y) \}$, for all x and y in R, and $\chi_{(F,A)}(x+y) \leq \max \{ \chi_{(F,A)}(x), \chi_{(F,A)}(y) \}$, for all x and y in R $\chi_{(F,A)}(xy) \leq \max \{ \chi_{(F,A)}(x), \chi_{(F,A)}(y) \}$, for all x and y in R. So in all the three cases, we have B is a fuzzy soft subhemiring of (F,A) hemiring R. Conversely, let x and y in (F,A) , since (F,A) is (F,A) non empty subset of R, so, $\chi_{(F,A)}(x) = \chi_{(F,A)}(y) = 1, \chi_{(F,A)}(x) = \chi_{(F,A)}(y) = 0$. Since $B = \langle \chi_{(F,A)}, \bar{\chi}_{(F,A)} \rangle$ is a fuzzy soft subhemiring of R, we have $\chi_{(F,A)}(x+y) \geq \min \{ \chi_{(F,A)}(x), \chi_{(F,A)}(y) \} = \min \{ 1, 1 \} = 1, \chi_{(F,A)}(xy) \geq \min \{ \chi_{(F,A)}(x), \chi_{(F,A)}(y) \} = \min \{ 1, 1 \} = 1$. Therefore $\chi_{(F,A)}(x+y) = \chi_{(F,A)}(xy) = 1$, and, $\chi_{(F,A)}(x+y) \leq \max \{ \chi_{(F,A)}(x), \chi_{(F,A)}(y) \} = \max \{ 0, 0 \} = 0, \chi_{(F,A)}(xy) \leq \max \{ \chi_{(F,A)}(x), \chi_{(F,A)}(y) \} = \max \{ 0, 0 \} = 0$. Therefore $\chi_{(F,A)}(x+y) = \chi_{(F,A)}(xy) = 0$. Hence $x+y$ and xy in (F,A) , so (F,A) is a subhemiring of R.

In the following Theorem ° is the composition operation of functions:

4) *Theorem:* Let (F,A) be an fuzzy soft subhemiring of a hemiring H and f is an isomorphism from a hemiring R onto H. Then $(F,A)^\circ f$ is a fuzzy soft subhemiring of R.

a) *Proof:* Let x and y in R and (F,A) be an fuzzy soft subhemiring of a hemiring H. Then we have, $(\mu_{(F,A)}^\circ f)(x+y) = \mu_{(F,A)}(f(x+y)) = \mu_{(F,A)}(f(x)+f(y))$, as f is an isomorphism $\geq \min \{ \mu_{(F,A)}(f(x)), \mu_{(F,A)}(f(y)) \} = \min \{ (\mu_{(F,A)}^\circ f)(x), (\mu_{(F,A)}^\circ f)(y) \}$, which implies that $(\mu_{(F,A)}^\circ f)(x+y) \geq \min \{ (\mu_{(F,A)}^\circ f)(x), (\mu_{(F,A)}^\circ f)(y) \}$. And, $(\mu_{(F,A)}^\circ f)(xy) = \mu_{(F,A)}(f(xy)) = \mu_{(F,A)}(f(x)f(y))$, as f is an isomorphism $\geq \min \{ \mu_{(F,A)}(f(x)), \mu_{(F,A)}(f(y)) \} = \min \{ (\mu_{(F,A)}^\circ f)(x), (\mu_{(F,A)}^\circ f)(y) \}$, which implies that $(\mu_{(F,A)}^\circ f)(xy) \geq \min \{ (\mu_{(F,A)}^\circ f)(x), (\mu_{(F,A)}^\circ f)(y) \}$. Therefore $(F,A)^\circ f$ is a Fuzzy soft subhemiring of a hemiring R.

5) *Theorem:* Let (F,A) be an fuzzy soft subhemiring of a hemiring h and f is an anti-isomorphism from a hemiring r onto h . then $(F,A)^{\circ f}$ is a fuzzy soft subhemiring of R .

a) *Proof:* Let x and y in R and (F,A) be an fuzzysoftsubhemiring of a hemiring H . Then we have,

$(\mu_{(F,A)^{\circ f}})(x+y) = \mu_{(F,A)}(f(x+y)) = \mu_{(F,A)}(f(y)+f(x))$, as f is an anti-isomorphism $\geq \min\{\mu_{(F,A)}(f(x)), \mu_{(F,A)}(f(y))\} = \min\{(\mu_{(F,A)^{\circ f}})(x), (\mu_{(F,A)^{\circ f}})(y)\}$, which implies that $(\mu_{(F,A)^{\circ f}})(x+y) \geq \min\{(\mu_{(F,A)^{\circ f}})(x), (\mu_{(F,A)^{\circ f}})(y)\}$. And, $(\mu_{(F,A)^{\circ f}})(xy) = \mu_{(F,A)}(f(xy)) = \mu_{(F,A)}(f(y)f(x))$, as f is an anti-isomorphism $\geq \min\{\mu_{(F,A)}(f(x)), \mu_{(F,A)}(f(y))\} = \min\{(\mu_{(F,A)^{\circ f}})(x), (\mu_{(F,A)^{\circ f}})(y)\}$, which implies that $(\mu_{(F,A)^{\circ f}})(xy) \geq \min\{(\mu_{(F,A)^{\circ f}})(x), (\mu_{(F,A)^{\circ f}})(y)\}$. Therefore $(F,A)^{\circ f}$ is an fuzzysoftsubhemiring of the hemiring R .

6) *Theorem:* Let (F,A) be an fuzzy soft subhemiring of a hemiring $(R, +, \cdot)$, then the pseudo fuzzy soft coset $(a(F,A))^p$ is an fuzzy soft subhemiring of a hemiring R , for every a in R .

a) *Proof:* Let (F,A) be an fuzzy soft subhemiring of a hemiring R . For every x and y in R , we have, $((a\mu_{(F,A)})^p)(x+y) = p(a)\mu_{(F,A)}(x+y) \geq p(a) \min\{\mu_{(F,A)}(x), \mu_{(F,A)}(y)\} = \min\{p(a)\mu_{(F,A)}(x), p(a)\mu_{(F,A)}(y)\} = \min\{((a\mu_{(F,A)})^p)(x), ((a\mu_{(F,A)})^p)(y)\}$. Therefore, $((a\mu_{(F,A)})^p)(x+y) \geq \min\{((a\mu_{(F,A)})^p)(x), ((a\mu_{(F,A)})^p)(y)\}$. Now, $((a\mu_{(F,A)})^p)(xy) = p(a)\mu_{(F,A)}(xy) \geq p(a) \min\{\mu_{(F,A)}(x), \mu_{(F,A)}(y)\} = \min\{p(a)\mu_{(F,A)}(x), p(a)\mu_{(F,A)}(y)\} = \min\{((a\mu_{(F,A)})^p)(x), ((a\mu_{(F,A)})^p)(y)\}$. Therefore, $((a\mu_{(F,A)})^p)(xy) \geq \min\{((a\mu_{(F,A)})^p)(x), ((a\mu_{(F,A)})^p)(y)\}$. Hence $(a(F,A))^p$ is an fuzzy soft subhemiring of a hemiring R .

REFERENCE

- [1] Akram.M and K.H.Dar On fuzzy d-algebras, Punjab university journal of mathematics, 37, 61- 76(2005).
- [2] Azriel Rosenfeld, Fuzzy Groups, Journal of mathematical analysis and applications, 35,512-517 (1971).
- [3] K. Atanassov, Operators over interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems. 64 (1994) 159-174
- [4] Biswas.R, Fuzzy subgroups and anti-fuzzy subgroups, Fuzzy sets and systems, 35; 121-124, (1990).
- [5] M. Borah, T. J. Neog and D. K. Sut, A study on some operations of fuzzy soft sets, International Journal of Modern Engineering Research (IJMER), Vol.2, Issue. 2 (2012) 219-225.
- [6] Feng, F., C. Li, B. Davvaz and M.I. Ali, 2010. Soft sets combined with fuzzy sets and rough sets: tentative approach, Soft Computing, 14: 899-911.
- [7] KumudBorgohain and Chittaranjan Gohain,2014. International Journal of Modern Engineering Research (IJMER), Some New operations on Fuzzy Soft Sets, Vol.4, Issue 4(2014),65-68.
- [8] Maji, P.K., R. Biswas and A.R. Roy, 2001. Fuzzy soft sets. The J. Fuzzy Math., 9: 589-602. [9] D. Molodtsov, Soft set theory results, Comput. Math. Appl. 37 (4-5) (1999) 19-31.
- [9] P. K. Maji, A. R. Roy, and R. Biswas, An application of soft sets in decision making problem, Comput. Math. Appl. 44 (8-9) (2002).



10.22214/IJRASET



45.98



IMPACT FACTOR:
7.129



IMPACT FACTOR:
7.429



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089  (24*7 Support on Whatsapp)