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# **Optimizing Portfolio by using Data Envelopment Analysis (DEA) and Cluster Analysis**

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Abstract: One of the most promising alternatives for investment is by investing funds in capital market. However, investors realize that their investments have the risk offered so that diversification or investment distribution is done by establishing stock portfolio. The problems in portfolio are that there are so various types of stock and how to select and allocate the amount of wealth in order to obtain optimal result. This article used cluster analysis to agglomerate Decision Making Unit (DMU), the stocks which productive characteristics were mostly similar to one another. Data Envelopment Analysis (DEA) was used to evaluate the efficiency of DMU. After the efficient DMUs were selected, optimal portfolio was established by using Single Index Model approach, and some fund proportions invested in each stock were determined.

Keywords: Portfolio Optimization, Data Envelopment Analysis, Cluster Analysis, Single Index Model

# I. INTRODUCTION

The problems in portfolio are that there are so many types of stock and how to select and allocate an amount of wealth owned in order to obtain optimal result [1,2,3]. Based on these problems, Cluster Analysis was used in this article in order to classify stocks into groups based on the similarity among stocks [4]. Data Envelopment Analysis (DEA) was used to compare input data with output data from each Decision Making Unit (DMU) [5]; this comparison was made to obtain efficient values. DEA has two models: DEA-CCR and DEA-BCC. Single Index Model (market model) related to return calculation of each asset in market index return was used in this article. This model was developed from Markowits model which had been found by William Sharpe. Single Index Model divides return or expected return from a stock into two components return which is influenced by market return and return which is not influenced by market return [9].

# II. DISCUSSION

Optimal portfolio is a portfolio which is selected by an investor from various selections existed in efficient portfolios. The selected portfolios were, of course, in accordance with the investors preference to return or their available risks [2,3]. A portfolio can be determined by selecting the level of the expected profit which can be minimized or maximized by their risks. Therefore, as portfolio can be efficient when 1) a higher level of profit can be obtained by the same risk, 2) the same level of profit can be obtained with the lower risk. On the other hand, an efficient portfolio can be established in one or more efficient portfolios so that choices by investors can be made in selecting the best portfolio which is in accordance with what has been expected. Optimal portfolio constitutes the selection from various securities of efficient portfolios. This optimal portfolio can be determined by selecting the level of return expectancy which risks are minimized, or its certain level of risk is determined by maximizing its return expectancy. A rational investor will select this optimal portfolio because it is established by optimizing return expectancy and risks.

# A. Return

Return is an amount of income or profit obtained from investment. It consists of two types: Actual Return is calculated according to historical data, and Expected Return which will be obtained by investors in the future. Individual Stock Return can be calculated as follows:  $R_{it} = \frac{D_{it}}{P_{it}-1} + \frac{P_{it}-P_{it-1}}{P_{it-1}}$ .

Portfolio Stock Return can be calculated as follows:  $R_p = \sum_{i=1}^n (W_i R_i)$ . Expected Return the average individual stock return; expected return on individual stocks can be calculated with the formula as follows:  $E(R_i) = \frac{\sum_{i=1}^n R_{it}}{N}$ . Expected Return on portfolio stock can be calculated with the formula as follows:  $E(R_p) = \sum_{i=1}^n (W_i E(R_i))$ .

# B. Risk

A risk constitutes the amount of deviation between the level of expected return and the level of actual return. Variance or standard deviation was used as the statistical measurement of its distribution. The higher the value of standard deviation the bigger the deviation and the higher the risk. The risk of individual stock can be calculated with the formula as follows:



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$$\sigma_i^2 = \frac{\sum_{t=1}^n (R_{it} - E(R_i))^2}{N} \quad or \quad \sigma_i = \sqrt{\frac{\sum_{t=1}^n (R_{it} - E(R_i))^2}{N}}$$

# C. Beta Risk Coefficient

Beta is used to measure the sensitivity of stocks toward market movement. A stock with high (low) beta is considered as high (low) risk security. Beta risk coefficient was calculated by using equation as follows:  $\beta_i = \frac{cov (R_i R_M)}{\sigma_{-}^2}$ .

#### D. Single Index Model

Single Index Model is based on observation that the price of a security is fluctuating in the same direction as market price index. Specifically, it can be observed that most stocks tend to undergo the increase in price when stock price index increases. Calculation to determine optimal portfolio will be made much easier if it is merely based on a figure which can determine whether a security can be included into the portfolio. This figure is the ratio excess return and beta. This ratio is  $R_i = \alpha_i + \beta_i \cdot R_M + e_i$ . Optimal portfolio will contain assets which have high ERB risk value. Therefore, a cut-off point which determines the limit of ERB value considered high is needed.

The next step is to determine the number of proportions of each security in optimal portfolio. The number of proportions for the isscurity is  $W_i = \frac{Z_i}{\sum_{j=1}^k Z_j}$ , with the Zi value of  $Z_i = \frac{\beta_i}{\sigma_e^2} (EB_i - C_i)$ . After the proportion of each stock was obtained, expected return of a portfolio was calculated. The risk of a portfolio was calculated as follows:  $\sigma_p^2 = \beta_p^2 \cdot \sigma_M^2 + (\sum_{i=1}^n W_i \cdot \sigma_e)^2$ .

#### E. Cluster Analysis

Cluster analysis is a technique used to identify the same object or individual by considering some criteria [7,10]. The cluster analysis is an analysis which is aimed to agglomerate some elements which bear a likeness to become different and mutually exclusive clusters as a research object [10]. Cluster analysis is a technical class used to classify an object or a case into a relatively homogenous cluster. The object of each cluster tends to be similar to one another and is far different from (not similar to) the object of the other clusters. Agglomerating was done according to the similarity among the objects. Similarity was obtained by minimizing the distance among the objects (within-cluster) and maximizing the distance among the objects (between-cluster)

#### F. Data Envelopment Analysis (DEA)

Data Envelopment Analysis (DEA) is functioned to evaluate the efficiency of a Decision Making Unit (DMU) that is responsible for using a number of inputs in order to obtain a targeted input. It is a non-parametric method which is basically developed from Linear Programming (LP) in the operation and economic research which is aimed to estimate the limit of production. It is used empirically to measure the efficiency of Decision Making Unit (DMU) production. Simply stated, the measurement of DMU is done by ratio - Output/Input. DEA-CCR model is used to find the value of CRS technical efficiency (CRS-TE) which is obtained by completing the equation as follows:

$$\max h_o(u, v) = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$
$$u_r \ge 0, r = 1, 2, \dots, s; v_i \ge 0, i = 1, 2, \dots, m$$

 $x_{io}$  is the number of i-inputs used by DMU,  $y_{ro}$  is the number of *r*-outputs used by DMU, *s* is the number of outputs, and m is the number of inputs. The variable of u is the weight of *r*-output while  $v_i$  is the weight of *i*-input, under the following conditions:

$$\max h_o(u, v) = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \le 1$$
  
 $j = 1, \dots, n \text{ (amount of DMU)}; u_r v_r \ge 0$ 

DEA-BBC model is similar to DEA-CCR. The difference between them is the addition of one inhibiting function in DEA-BCC model which is used to find the value of TE-VRS which is obtained by completing the equation as follows:  $\sum_{r=1}^{s} u_r y_{io} + u = 1$  (or  $\sum_{i=1}^{m} v_i x_{io} + v = 1$ ).

In general, the performance of a company is measured by using ratios such as Earning per Share (EPS), Net Profit (NP), Return on Asset (ROA), and Return on Equity (ROE). Besides income, other components to determine stock price is Rate of Return/ Price



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Earning Ratio (P/E Ratio). Besides PER, there is also Debt Equity Ratio (DER) which indicates how big a company is financed by a creditor in obtaining profit.

# III. RESULT AND EXAMPLES OF IMPLEMENTATION

The samples used in this research were the stocks in BEI (Indonesia Stock Exchange). They were taken by using purposive sampling technique with the criteria, LQ45 companies in the period of January – December, 2013 consecutively. The data were secondary data which consisted of monthly stock price during the closing price, dividend of each stock, IHSG (Composite Stock Price Index) in the period of January – December, 2013, the Interest Rate of SBI (Bank Indonesia Certificate), and the data of Annual Financial Report) which had been audited by December 31, 2013.

## A. Stock Clustering

The process of agglomerating, using Ward method was done by using an SPSS software program at the stage of agglomerating. After the range among the variables was measured with Euclidean range, it was followed by agglomeration which was done gradually. The process of agglomerating could be illustrated in the form of dendogram which was read from left to right which vertical line indicated combined clusters, while the line in the scale indicated the range of combined clusters. The established dendogram indicated the separation between cluster 1 and cluster 2 with the Cut off point in the case no. 10 and no. 28. Each of the cluster members was as follows at figure 1:



.Figure 1. Dendogram with Ward Method

# B. Calculating Efficient Stocks with Data Envelopment Analysis (DEA)

In the Table 1 of the output of DEAP software version 2.1, DEA-CCR (TE-CSR) model, DMU which indicated stocks with efficient performance had 7 stocks: AALI, BSDE, INTP, LSIP, MNCN, PGAS, and UNVR. Meanwhile, in DEA-BCC (TE-VRS) model, DMU which indicated stocks with efficient performance had 8 stocks: AALI, BSDE, INTP, LSIP, MNCN, PGAS, TLKM, and UNVR. DMU which had work optimally had 10 DMUs: AALI, BSDE, INTP, LPKR, LSIP, MNCN, PGAS, TLKM, SMGR, and UNVR.



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DMU No.	DMU	TE-CRS	TE-VRS	$\mathbf{SE}$
1	AALI	1.000	1.000	1.000
2	ASRI	0.993	0.993	0.999
3	BMTR	0.822	0.870	0.999
4	BSDE	1.000	1.000	1.000
5	CTRA	0.727	0.797	0.912
6	INTP	1.000	1.000	1.000
7	LPKR	0.872	0.872	1.000
8	LSIP	1.000	1.000	1.000
9	MNCN	1.000	1.000	1.000
10	PGAS	1.000	1.000	1.000
11	PTBA	0.897	0.898	0.999
12	SMGR	0.992	0.993	1.000
13	SMRA	0.747	0.793	0.942
14	TLKM	0.938	1.000	1.000
15	UNVR	1.000	1.000	1.000

Table 1. Value DEA CRS and VRS, Output Results with Program DEAP 2.1.

## C. Optimizing Portfolio with Single Index Model

In the Single Index Model, the first step which should be done was calculating the value of Excess Return to Beta (ERB). Data of the level of return asset which was free from the risk was needed to fulfill the calculation of ERB; in this research, the average interest rate of monthly SBI during the period of the research was used, and the result of the average return was 0.65%. The following Table showed the ERB value of the 9<sup>th</sup> stock which had been put in the right order from the biggest ERB to the smallest one.

Based on Table 2, it was found there were 3 stocks with positive ERB values and 6 stocks with negative ERB values. A stock with negative ERB value indicated that the stock had the level of stock return which was still below the stock return with free risk. Optimal portfolio would consist of stocks which had high ERB values. The value of Cut off point ( $C^*$ ) would be used as the limitation of a stock entering portfolio, and the amount of Cut off point was the highest  $C_1$  value.

DMU No.	DMU	ERB	$C_i$
1	UNVR	0.027087	0.000054
2	PGAS	0.001972	0.000045
3	MNCN	0.002791	0.000006
4	BSDE	-0.000321	-0.000035
5	SMGR	-0.000959	-0.000454
6	LPKR	-0.001003	-0.000569
7	INTP	-0.001020	-0.000569
8	AALI	-0.001185	-0.001596
9	LSIP	-0.001003	-0.000569

**Table 2**. Value of ERB and Cut-off Point ( $C_i$ )

Cut off point (C\*) which was the highest  $C_1$  value was on the figure of 0.000054 or in the UNVR stock (Unilever Indonesia, Tbk). From the above table, it was found that there were three stocks which fulfilled the criteria for entering the establishment of optimal portfolio. The three stocks were UNVR, PGAS, and MNCN.

After the three stocks chosen to enter the establishment of optimal portfolio were known, the proportion (*wi*) invested in each stock in the portfolio was invested. The proportions of fund invested in each stock in the portfolio were as follows:

- 1) UNVR (Unilever Indonesia Tbk) was 0.650929 (65.9%);
- 2) PGAS (Perusahaan Gas Negara (Persero) Tbk) was 0.291388 (29.14%);
- 3) MNCN (Media Nusantara Citra Tbk) was 0.048709 (4.87%).

Portfolio established in these three stocks provided the level of expected return of 0.031213 per month with the standard deviation of 0.308259. This result was promising since the expected return portfolio was higher than the market expected return of 0.026621 which was still above the level of free risk return of 0.0065 per month.



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# IV. CONCLUSION

The proportion of each stock was different from one to another because the differences in stock price, the level of profit, and individual risk accompanying the stocks. Low risk would obtain low profit, and high risk would obtain high profit. The result of the research showed that the risk was more minimal than the calculation of optimal portfolio. Investors could choose their preference of which stocks which would be used to do the investment.

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