iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH
IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 6 **Issue:** V **Month of publication:** May 2018

DOI: http://doi.org/10.22214/ijraset.2018.5198

Super Efficiency in Data Envelopment Analysis and Stochastic Frontier Analysis

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Abstract : Super efficiency as a ranking methodology to distinguish the performance of DMU extreme efficiently. This model can lead to non-instability and instability when some of the inputs will be close to zero. DEA can solve the limitations of partial ratio analysis or multiple regression analysis. DEA is a specially designed procedure to determine the efficiency of a DMU that uses multiple inputs and outputs. The SFA method can add production limits with an error term with two components that allow for technical efficiency and other things that allow for random events that may affect individual producers. Deterministic equivalent to a stochastic super efficiency model can be converted to quadratic program. Finally, the limitation of the data allowed to evaluate the DMU then the proposed model sensitivity analysis can be used.
Keywords: Super efficiency, Decision Making Unit (DMU), Data Envelopment Analysis (DEA), Stochastic Frontier Analysis (SFA).

I. PRELIMINARY

Efficiency is the level of comparison between input (input) with the result (output) reflected in the ratio orratio between the two. If the output is greater than the input it can be said to be efficient and vice versa if the input is greater than the output then it is said to be inefficient. So the high low efficient is determined by the size of the resulting ratio .

Super efficiency [9] is a model developed by Andersen and Petersen (1993). Andersen and Petersen introduced a super efficiency for ranking methods in differentiating the performance of the extreme Decision Making Unit (DMU) efficiently. The uncertainty of the super efficiency model occurs when the efficiently evaluated DMUs can not reach the limits established by other DMUs by increasing input or decreasing output.

Mehrabian et.al [6] used heavy constraints on input and output weights on AP models solving some problems such as instability. Li et.al [5] modified the Mehrabian et.al model simultaneously by increasing output and reducing inputs of the same size called infeasibility. General weights are another method developed by Hosseinzadeh Lotfi et.al [4], for a unit of rank model through which units are evaluated and ranked multi-objectively [9].

Data Envelopment Analysis (DEA) was introduced by Charnes et.al [2]. DEA is basically the development of Linear Programming (LP). DEA method is used to evaluate the performance of an activity within the entity unit (organization). The working principle of DEA is to compare input and output data from DMU with other input and output data on homogeneous DMU. This comparison is done to obtain an efficiency value.

The above statement can be deduced DEA is requiring weights or scales for input and output DMU. The weights are negative and universal in nature, meaning that each DMU should be able to use the same set of weights to evaluate the ratio (weighted output / total weighted input) and the ratio should be no more than one (total weighted output / total weighted input ≤ 1) because if the dual result is more or less than one then the DMU is deemed not to be in relative efficiency or inefficiency, otherwise if its dual value is equal to one, or its efficiency value is 100% then DMU is considered efficiency.

Stochastic Frontier Analysis (SFA) was introduced by Aigner et.al (1977), and Meeusen and Van denk Broeck (1977). SFA is considered a parametric approach because it requires the creation of functions to build borders. D labeled as "stochastic" because this method considers randomness in calculating efficiency. This form of analysis is similar to regression analysis. However, where regression analysis will involve the average relationship between output level and input level, SFA involves the maximum relationship between these variables.

The use of SFA requires using one output and / or result in calculating efficiency. The use of multiple outputs and / or results will require assigning weights to various options, which can be complicated and often require too many judgments and assumptions. SFA is sensitive to low-performing outliers, as it lowers the overall border.

Based on its performance assessment, super-efficient as the basis for extreme DMU performance efficiently. The DEA method can evaluate homogeneous DMU performance. The SFA method is able to estimate the best technical efficiency of the DMU.

II. STOCHASTIC INPUT MODEL SUPER EFFICIENCY

Stochastic models of supplementation proposed in the correlation analysis which allows for possible stochastic variation in the input-output data $\tilde{x}_j = (\tilde{x}_{1j},........,\tilde{x}_{mj})^t$, $\tilde{y}_j = (\tilde{y}_{1j},........,\tilde{y}_{sj})^t$ becomes the random input and output associated with DMU_j (j = 1,....,n). $x_j = (x_{1j},........, x_{mj})^t$, $y_j = (y_{1j},........, y_{sj})^t$ indicates that it has the expected values of DMU_j input and output.

The input and output components are equally distributed normally in stochastic input models constrained by inequality limits, where all slack variables are excluded from the destination function.

$$\begin{aligned} & \text{Max } \phi_o \\ & \text{S.t } P\{\textstyle\sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} - s_{i2}^{+} \le \tilde{x}_{io}\} \ge 1-\propto, \ i = 1,....,m \\ & P\{\textstyle\sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \ge \phi_o \tilde{y}_{ro}\} \ge 1-\propto, \ i = 1,....,s \\ & 1 = \textstyle\sum_{j=1}^{n} \lambda_j \\ & s_{i2}^{+}, \lambda_j \ge 0 \qquad (1) \end{aligned}$$

Where α is a predetermined value for the level of significance that is between 0 and 1. Solutions with $\phi_o = 1$, $\lambda_o = 1$, $\lambda_j = 0$ $(j \neq o)$, the optimal value ≥ 1. Stochastic efficiency with input model can be defined as follows.

A. Definition 1. (Stochastic Efficiency for Input Relaxation Models). DMU_o for stochastic efficiency if and only if the two thigs below are met:

(i) $\phi_o^* = 1$

(ii) the slack variable is 0 in all optimal solution alternatives.

DMU_o are called stochastic inefficient if the requirements of Definition 1 do not meet. Where, if optimal solution $\phi_o^* > 1$ or some slack variable is not zero, and DMUo is stochastic inefficient. If $\phi_o^* = 1$, then all results to evaluate DMU can be increased to $\phi_o^* \, y_{ro}$ (r = 1, ..., s) using other DMU convex combinations at the level of significance α.

The stochastic model corresponding to the input relaxation model is

$$\begin{aligned} & \text{Max } \phi_o + \varepsilon \left(\textstyle\sum_{i=1}^{m} s_{i1}^{-} + \sum_{r=1}^{s} s_{r1}^{+} - \sum_{i=1}^{m} s_{i2}^{+} \right) \\ & \text{S.t } P\{\textstyle\sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} + s_{i1}^{-} \le \tilde{x}_{io} + s_{i2}^{+}\} = 1-\propto, \ i = 1,....,m \\ & \quad P\{\textstyle\sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} - \phi_o \tilde{y}_{ro} \ge s_r^{+}\} = 1-\propto, \ r = 1,....,s \\ & 1 = \textstyle\sum_{j=1}^{n} \lambda_j \\ & s_{i1}^{-}, s_{i2}^{+}, \lambda_j, s_r^{+} \ge 0 \qquad (2) \end{aligned}$$

B. Definition 2. DMU_o is called stochastic relaxation efficient model input at a significance level α if the following conditions are met.

(i) $\emptyset_o^* = 1$

(ii) $s_{i1}^{-*} = s_{i2}^{+*} = s_r^{+*} = 0$ □i, □r

Based on the preceding assumption, the stochastic model of the proposed super efficiency model can be defined as below.

$$\begin{aligned} & \text{Max } \phi_0^s \\ & \text{S.t } P\left\{ \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j \tilde{x}_{ij} + s_{i1}^{-} \le \tilde{x}_{io} + s_{i2}^{+} \right\} = 1-\propto, \ i = 1,....,m \\ & \quad P\left\{ \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j \tilde{y}_{rj} - \phi_0^s \tilde{y}_{ro} \ge s_r^{+} \right\} = 1-\propto, \ r = 1,....,s \\ & 1 = \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j \\ & s_{i1}^{-}, s_{i2}^{+}, \lambda_j, s_r^{+} \ge 0 \qquad (3) \end{aligned}$$

Where α is a predetermined value between 0 and 1 for the level of significance and P represents the probability. DMU_o is superficially stochastic at significance α if the objective function has an optimal value of less than one. Therefore, if $\phi_0^{s*} < 1$ means DMU_o can reduce the result to be ϕ_0^{s*} percent of the output is still efficient, then the lower ϕ_0^{s*} the better the DMU.

C. Equivalent Deterministic For Stochastic Super Efficiency Models

$$\begin{aligned} & \text{Max } \phi_o + \varepsilon \left(\textstyle\sum_{i=1}^{m} s_{i1}^{-} + \sum_{r=1}^{s} s_{r1}^{+} - \sum_{i=1}^{m} s_{i2}^{+} \right) \\ & \text{S.t } \textstyle\sum_{j=1}^{n} \lambda_j x_{ij} + s_{i1}^{-} - s_{i2}^{+} - \phi^{-1}(\propto)\sigma_i^{I}(\lambda) = x_{io}, \quad i = 1,....,m \\ & \phi_o y_{ro} - \textstyle\sum_{j=1}^{n} \lambda_j y_{rj} + s_r^{+} - \phi^{-1}(\propto)\sigma_r^{o}(\phi_o, \lambda) = 0, \quad r = 1,....,s \end{aligned}$$

$\sum_{j=1}^{n} \lambda_j = 1$

$s_{i1}^{-}, s_{i2}^{+}, \lambda_j, s_r^{+} \geq 0$ (4)

Where ϕ is the cumulative distribution function (cdf) of a random variable Normal standard and ϕ^{-1} is its inverse. It is considered that x_{ij} and y_{rj} produce input and output variables. Form σ_i^I (λ) and σ_r^o (ϕ_o, λ) is a nonlinear program. This nonlinear program can be converted into quadratic query problem. Let's say w_i^I and w_r^o is a nonnegative variable. Replace w_i^I and w_r^o with σ_i^I (λ) and σ_r^o (ϕ_o, λ) and add the following equation of equilibrium equation

$$(w_i^I)^2 = (\sigma_i^I(\lambda))^2$$

$$(w_r^o)^2 = (\sigma_r^o(\phi_o, \lambda))^2$$

Model (4) is transformed into a quadratic programming problem. Therefore, one can obtain the optimal value $\emptyset_o^* s_{i1}^{-*}, s_{i2}^{+*}$ and s_r^{+*} by solving the quadratic program. The three cases below should naturally occur in one case because the input evaluates DMU_o :

(i) Increase, in accordance with $s_{i2}^{+*} > 0$

(ii) Decrease, in accordance with $s_{i1}^{-*} > 0$

(iii) No change, in accordance with $s_{i1}^{-*} = s_{i2}^{+*}$

To ensure that at most one s_{i1}^{-*} and s_{i2}^{+*} model is positive, then add restrictions $s_{i1}^{-*} . s_{i2}^{+*} = 0$.

The deterministic input relaxation model of one of the above three cases occurs on the basic optimal solution. Therefore, the corresponding simplex and column methods are used s_{i1}^{-*}, s_{i2}^{+*} linear dependent. The following deterministic is an equivalent stochastic model (2).

$$\text{Max}\phi_o + \varepsilon \left(\sum_{i=1}^{m} s_{i1}^{-} + \sum_{r=1}^{s} s_{r1}^{+} - \sum_{i=1}^{m} s_{i2}^{+} \right)$$

$$\text{S.t } \sum_{j=1}^{n} \lambda_j x_{ij} + s_{i1}^{-} - s_{i2}^{+} - \phi^{-1}(\propto) w_i^I = x_{io}, \quad i = 1,....,m$$

$$\phi_o y_{ro} - \sum_{j=1}^{n} \lambda_j y_{rj} + s_r^{+} - \phi^{-1}(\propto) w_r^o = 0, \quad r = 1,....,s$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$(w_i^I)^2 = \sum_{j\neq 0}\sum_{k\neq 0} \lambda_j \lambda_k Cov(\tilde{x}_{ij}, \tilde{x}_{ik}) + 2(\lambda_0 - 1)\sum_{j\neq 0} \lambda_j Cov(\tilde{x}_{ij}, \tilde{x}_{i0}) + (\lambda_0 - 1)^2 \text{ var}(\tilde{x}_{i0})$$

$$(w_r^0)^2 = \sum_{k\neq 0}\sum_{j\neq 0} \lambda_k \lambda_j Cov(\tilde{y}_{rk}, \tilde{y}_{rj}) + 2(\lambda_0 - \phi_0)\sum_{k\neq 0} \lambda_k Cov(\tilde{y}_{rk}, \tilde{y}_{r0}) + (\lambda_0 - \phi_0)^2 \text{ var}(\tilde{y}_{r0})$$

$$s_{i1}^{-} . s_{i2}^{+} = 0$$

$$s_{i1}^{-}, s_{i2}^{+}, \lambda_j, s_r^{+}, w_i^I, w_r^o \geq 0 \quad (5)$$

Similarly, the following deterministic equations can be obtained with the super model stochastic efficiency, model (3).

$$\text{Max } \phi_0^s$$

$$\text{S.t } \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j x_{ij} + s_{i1}^{-} - s_{i2}^{+} - \phi^{-1}(\propto)\sigma_i^I(\lambda) = x_{io}, \ i = 1,....,m$$

$$\phi_0^s y_{ro} - \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j y_{rj} + s_r^{+} - \phi^{-1}(\propto)\sigma_r^o(\phi_0^s, \lambda) = 0, \ r = 1,....,s$$

$$\sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j = 1$$

$$s_{i1}^{-}, s_{i2}^{+}, \lambda_j, s_r^{+} \geq 0 \quad (6)$$

So the following deterministic is obtained which is equivalent to stochastic input model super efficiency

$$\text{Max } \phi_0^s$$

$$\text{S.t } \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j x_{ij} + s_{i1}^{-} - s_{i2}^{+} - \phi^{-1}(\propto) w_i^I = x_{io}, \ i = 1,....,m$$

$$\phi_0^s y_{ro} - \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j y_{rj} + s_r^{+} - \phi^{-1}(\propto) w_r^o = 0, \ r = 1,....,s$$

$$\sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j = 1$$

$$(w_i^I)^2 = \sum_{j\neq 0}\sum_{k\neq 0} \lambda_j \lambda_k Cov(\tilde{x}_{ij}, \tilde{x}_{ik}) + 2\sum_{j\neq 0} \lambda_j Cov(\tilde{x}_{ij}, \tilde{x}_{i0}) + \text{var}(\tilde{x}_{i0})$$

$$(w_r^0)^2 = \sum_{k\neq 0}\sum_{j\neq 0} \lambda_k \lambda_j Cov(\tilde{y}_{rk}, \tilde{y}_{rj}) + 2\phi_0^s \sum_{k\neq 0} \lambda_k Cov(\tilde{y}_{rk}, \tilde{y}_{r0}) + (\phi_0^s)^2 \text{ var}(\tilde{y}_{r0})$$

$$s_{i1}^{-} * s_{i2}^{+} = 0$$

$$s_{i1}^{-}, s_{i2}^{+}, \lambda_j, s_r^{+}, w_i^I, w_r^o \geq 0 \quad (7)$$

D. Sensitivity Analysis

DEA allows all DMU data to vary simultaneously to change its status at least one DMU from efficient to inefficient, or otherwise. We will simplify the previous equation by assuming that only DMU_0, randomly varying the input and output, ie. $\sigma_{i0}^1 \neq 0, \sigma_{r0}^1 \neq 0, \sigma_{ij}^1 = 0,$ $and\ \sigma_{rj}^0 = 0\ (j \neq 0)$ for all i and r. Model can be written:

$$\text{Max } \phi_0^s$$

$$\text{S.t } x'_{i0} = \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j x'_{ij} + s_{i1}^- - s_{i2}^+, \qquad i = 1,....,m$$

$$0 = \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j y'_{rj} - \phi_0^s y'_{r0} - s_r^+, \qquad r = 1,....,s \qquad (8)$$

$$\sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j = 1$$

$$s_{i1}^-, s_{i2}^+, \lambda_j, s_r^+ \geq 0$$

where:

$$y'_{r0} = y_{r0} - \sigma_{r0}^0\ \phi^{-1}(\alpha), \qquad r = 1,......,s$$

$$y'_{rj} = y_{rj},\ j \neq 0, \quad r = 1, \ldots .. s \qquad (9)$$

$$x'_{i0} = x_{i0} + \sigma_{i0}^1\ \phi^{-1}(\alpha), \qquad i = 1,......,m$$

$$x'_{ij} = x_{ij},\ j \neq 0, \quad i = 1, \ldots .. m \qquad (10)$$

Therefore, model (8) is a deterministic equivalent stockhastik model (3) with the above assumption. This model is a super efficiency input model for DMU_0 with adjustable input and output values x'_{ij} , i = 1,....., m and y'_{rj}, r = 1,......,s as defined in models (9) and (10) .

III. CONCLUSION

Based on the study acyl menunju h m etode proposed kkan practically useful to rank the units efficiently obtained. In addition to developing stochastic versions of the super efficiency and deterministic models equivalent to the super efficiency models can be The conversion right in quadratic problem. The sensitivity analysis of the super efficiency model is used when it is possible to conduct research to evaluate the degree of use based on the parameters that determine the significance. Therefore, given the limitations of the data allowed to evaluate the DMU then the proposed model sensitivity analysis can be used.

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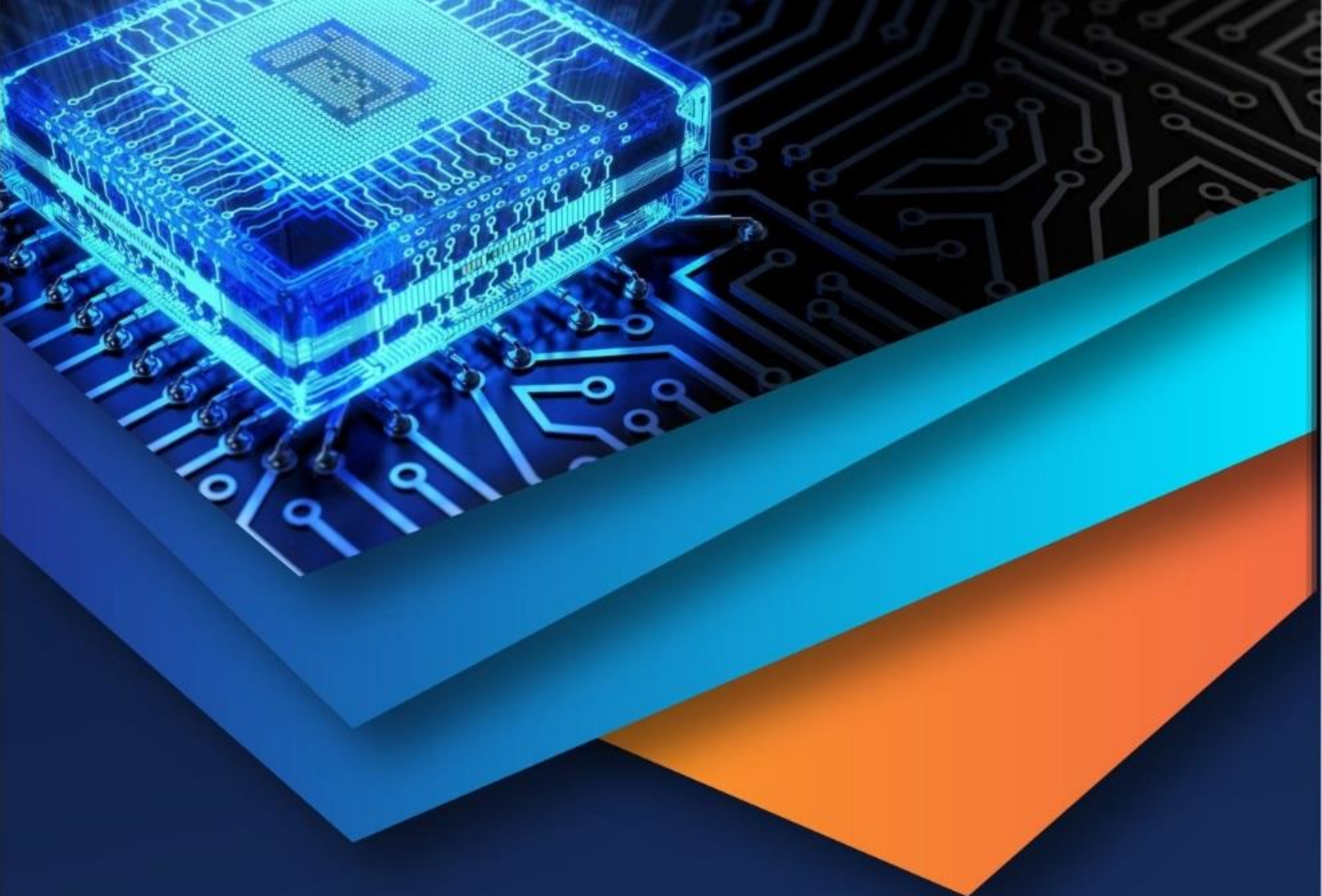

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