iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 6 **Issue:** IV **Month of publication:** April 2018

DOI: http://doi.org/10.22214/ijraset.2018.4186

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887
Volume 6 Issue IV, April 2018- Available at www.ijraset.com

Estimation of Population Growth with Mobility Rate and Expansion Rate by Cubic Spline

Vinay R Bhavsar[1], Mukesh Patel[2]
[1]Department of Mathematics, UTU, Bardoli.
[2] Department of Mathematics, UTU, Bardoli.

Abstract: This paper explores the method of estimating the population growth in various time intervals at different locations. The nature of growth of population over the various locations is constructed by both mobility rate of population-movement and expansion rate of migrated population. Where, mobility rate of population-movement controls the over expansion of population, mainly at initial locations by expansion rate of migrated population such that the distribution of population at all locations remains uniformly. For that, the Mathematical Model of Partial Differential Equation (PDE) is formed and its solution is derived by the numerical method; Cubic Spline Explicit and Implicit with different initial and boundary conditions, as a population growth prediction with respect to time and location.
Keywords: Population Growth, Mobility Rate of Movement, Migrated Population Expansion Rate, Cubic Spline Method-Explicit and Implicit, PDE

I. INTRODUCTION

Larry King took live interview of Prof. Stephan Hawking (Cambridge University), one of most intelligent persons in the world. He asked him a key question that “what worries you the most?” Hawking said, “My biggest worry is population growth, and if it continues at the current rate, we will be standing shoulder to shoulder in 2600. Something has to happen, and I don’t want it to be a disaster” [1]. Population problem is one of the major issues for every nation. There are many parameters affected on a growth of population viz. birth rate, death rate, fertility rate and migration rate. Nowadays, every government is taking some smart steps or making some policies for the solution of this unpredicted parameter. In which migration is one of the parameter which increases the population. In this modern arena, every individual is moving towards developing area for getting the better life style, health environment and working opportunities etc. This sudden migration of people towards the developing area creates ecological imbalance which converts residential area into a multi-hazard or overcrowded region and that increases the existing population [2]. So it is necessary for any government to make developmental plan giving due importance to bring down the population to a bearable level. Here we are considering not only the migration parameter but also the nature of movement of population for calculating the population growth.

II. POPULATION GROWTH – MOBILITY RATE AND EXPANSION RATE

It is assumed that the nature of advection-diffusion equation (Burger’s equation) and population growth equation are similar where the mobility rate of movement and migrated population expansion rate are following the nature of advection and diffusion respectively. The mobility rate of movement (advection) term is describing a change in population with respect to time and space when a person entering in the region and that depends on mobility rate of movement of people. Also it slightly increases the population at some of the locations but not at all. The migrated population expansion rate (diffusion) term describes a change in flow of population at various locations with the rate of migrated population expansion. The nature of migration is random; if the rate is increased then, the distance is decreased. If the area is increased then the population expansion is also increased. This generally creates overcrowding at initial locations as per its nature [3],[4]. If we follow only one of the terms describe above to fix population growth then it is found that population will not be distributed uniformly over the locations. Hence it is difficult to identify the actual nature of population growth and predict so. Thus, for the betterment we use both terms to calculate rather to predict growth of population at various locations in a particular region. For that, the one dimensional partial differential Burger’s equation is taken as follows which will be numerically solved [5].

$$\frac{\partial P}{\partial t} + a\frac{\partial P}{\partial x} = m\frac{\partial^2 P}{\partial x^2}\ ;\ m > 0 \qquad (1)$$

Here a = mobility rate of movement and m = migrated population expansion rate.

The equation (1) gives growth of population (P) in time t at place x. It will be determined by any analytical as well as numerical methods along with required initial and boundary conditions. Here we are following Cubic Spline Method for the solution of one dimensional partial differential equation. Also we are taking two different types of initial and boundary conditions for the prediction the population growth.

$$\text{Case-(i) } P(x,0) = p_0 + p_1 e^{-\alpha x}, \; P(0,t) = p_0 + p_1$$
$$\text{Case-(ii) } P(x,0) = p_0 + p_1 e^{-\alpha x}, \; P(0,t) = p_0 + p_1 e^{-\alpha t}$$

III.SPLINE METHOD [6],[7]

An equation which satisfies the own characteristic with any initial and/or boundary condition is called analytic solution. It is not necessary that every PDE has analytical solution therefor another approach is also required for solving PDE. There are numerous types of numerical approach exists to solve (partial differential equation) PDE. Here we will use cubic spline numerical method for the solution of one dimensional PDE. The method spline is nothing but it is piecewise connecting polynomials of any degree. There are many types of spline polynomials exists viz., linear spline, quadratic spline, cubic spline and quantic spline etc. Out of these, the cubic spline of degree three has been found the most popular approximation method. Hence, we are adopting cubic spline method and replacing all the PDE term by approximation terms. Also the method has two parts for calculating the PDE, one is Explicit Method of Cubic Spline and another is Implicit Method of Cubic Spline. In Explicit Method we use forward difference and central difference formulas for the LHS of eq. (1) and replacing RHS term of equation (1) by the second derivatives $S''_{i,j}$. In Implicit Method same formula will be followed which we used in Explicit Method but here the second order partial derivative term of equation is replacing by average of $S''_{i,j}, S''_{i,j+1}$.

A. *Explicit Method [6],[7]*

The partial differential equation form as population growth equation is solved by cubic spline explicit method is as below.

Consider equation (1) with the initial and boundary conditions as case (i) and (ii). Region x is divided into n-equal subintervals of width h i.e. the points of subintervals are $x_0, x_1, x_2, \dots, x_n$. The time interval $j\Delta t$ (j=0,1,…) are the mesh points at $x_0, x_1, x_2, \dots, x_n$. $P_{i,j}$ is represented as a value of P at the i^{th} mesh point in time $j\Delta t$.

For approximating the function P by a cubic spline method let us consider $S(x)$ is a polynomial of degree k over each subinterval $[x_i, x_{i+1}]$. For finding the polynomial of $S(x_i)$ first we calculate $S''(x_i)$ by solving set of simultaneous equations for, i =0,1, 2, 3… n-1. We should note that values of P at x = x_0 and x = x_n are known by initial conditions. Here we take $S''(x_i) = S''_{i,j}$.

$$\frac{(P_{i,j+1}-P_{i,j})}{\Delta t} + a * \frac{(P_{i+1,j}-P_{i-1,j})}{2\Delta x} = mS''_{i,j} \tag{2}$$

Similarly we get the formula of $S''_{i-1,j}, S''_{i+1,j}$ from the above equation (2).Now substitute values of $S''_{i,j}$, $S''_{i-1,j}$ and $S''_{i+1,j}$ into below equation. We get,

$$S''_{i-1,j+1} + 4S''_{i,j+1} + S''_{i+1,j+1} = \frac{6}{h^2}\{P_{i-1,j+1} - 2P_{i,j+1} + P_{i+1,j+1}\} \tag{3}$$

$$\left[\frac{(P_{i-1,j+1}-P_{i-1,j})}{\Delta tm} + a * \frac{(P_{i,j}-P_{i-2,j})}{2\Delta xm}\right] + 4\left[\frac{(P_{i,j+1}-P_{i,j})}{\Delta tm} + a * \frac{(P_{i+1,j}-P_{i-1,j})}{2\Delta xm}\right] + \left[\frac{(P_{i+1,j+1}-P_{i+1,j})}{\Delta tm} + a * \frac{(P_{i+2,j}-P_{i,j})}{2\Delta xm}\right] = \frac{6}{h^2}(P_{i-1,j} - 2P_{i,j} + P_{i+1,j}) \tag{4}$$

$$(P_{i-1,j+1} - P_{i-1,j}) + 4(P_{i,j+1} - P_{i,j}) + (P_{i+1,j+1} - P_{i+1,j}) = \frac{6\Delta tm}{h^2}(P_{i-1,j} - 2P_{i,j} + P_{i+1,j}) - \frac{a\Delta tm}{2\Delta xm}[(P_{i,j} - P_{i-2,j}) + 4(P_{i+1,j} - P_{i-1,j}) + (P_{i+2,j} - P_{i,j})] \tag{5}$$

Here $r = \frac{\Delta tm}{h^2}, b = \frac{\Delta t}{\Delta x}$

$$(P_{i-1,j+1} - P_{i-1,j}) + 4(P_{i,j+1} - P_{i,j}) + (P_{i+1,j+1} - P_{i+1,j}) = (6rP_{i-1,j} - 12rP_{i,j} + 6rP_{i+1,j}) - \frac{ab}{2}[(P_{i,j} - P_{i-2,j}) + 4(P_{i+1,j} - P_{i-1,j}) + (P_{i+2,j} - P_{i,j})] \tag{6}$$

$$P_{i-1,j+1} + 4P_{i,j+1} + P_{i+1,j+1} = (1 + 6r)P_{i-1,j} + (4 - 12r)P_{i,j} + (1 + 6r)P_{i+1,j} - \frac{ab}{2}[(P_{i,j} - P_{i-2,j}) + 4(P_{i+1,j} - P_{i-1,j}) + (P_{i+2,j} - P_{i,j})] \tag{7}$$

Here,$P_{i,j}$'s (j=0 and i=1 to 10) are the initial values calculated from the given condition of equation (1).

Equation (7) recognized as cubic spline explicit formula for the solution of this set of simultaneous equations.

Now $P_{0,j+1}$ & $P_{10,j+1}$ are known values by the given initial conditions. The set of simultaneous equations found in explicit method contains (n – 1) unknowns. These (n – 1) equations from (7) with (n – 1) unknown can be solved by any standard method.

Furthermore, the values of P are known at (j+1)th step, we can calculate the next step (j + 2) by repeating the same process. At each step the set of simultaneous equations in (n – 1) unknowns give tri-diagonal matrix. It can be calculated by any standard method, thus the method can repeat by above steps. The value of r shows the convergence and stability of cubic spline explicit method.

B. Implicit Method [6],[7]

Now, the terms of equation (1) is replace by corresponding implicit method,

$$\frac{(P_{i,j+1}-P_{i,j})}{\Delta t}+a*\frac{(P_{i+1,j}-P_{i-1,j})}{2\Delta x}=m\frac{(S''_{i,j}+S''_{i,j+1})}{2} \quad (8)$$

Here $S''_{i,j}$, $S''_{i,j+1}$ denote second derivatives at $x = x_i$ at the time j and j + 1 respectively. However, second derivatives at (j+1)th step cannot be calculated as the values of P are unknown. We use the relationship of the equation $S''_{i,j}$ at jth and j+1th steps and rewrite it in the following forms.

$$S''_{i-1,j}+4S''_{i,j}+S''_{i+1,j}=\frac{6}{h^2}\{P_{i-1,j}-2P_{i,j}+P_{i+1,j}\} \quad (9)$$

$$S''_{i-1,j+1}+4S''_{i,j+1}+S''_{i+1,j+1}=\frac{6}{h^2}\{P_{i-1,j+1}-2P_{i,j+1}+P_{i+1,j+1}\} \quad (10)$$

$i = 1,2,3, \ldots$ as $P_{i,j=0}$ will be obtain from initial condition of equation (1).

Also from equation (8) we take another similar term for $S''_{i-1,j+1}$ & $S''_{i+1,j+1}$

$$\left.\begin{aligned} S''_{i-1,j+1}&=\frac{2(P_{i-1,j+1}-P_{i-1,j})}{\Delta tm}+a*\frac{(P_{i,j}-P_{i-2,j})}{\Delta xm}-S''_{i-1,j}\\ S''_{i,j+1}&=\frac{2(P_{i,j+1}-P_{i,j})}{\Delta tm}+a*\frac{(P_{i+1,j}-P_{i-1,j})}{\Delta xm}-S''_{i,j}\\ S''_{i+1,j+1}&=\frac{2(P_{i+1,j+1}-P_{i+1,j})}{\Delta tm}+a*\frac{(P_{i+2,j}-P_{i,j})}{\Delta xm}-S''_{i+1,j}\end{aligned}\right\} \quad (11)$$

Substituting the values of (11) into equation (10). We get,

$$\left[\frac{2(P_{i-1,j+1}-P_{i-1,j})}{\Delta tm}+a*\frac{(P_{i,j}-P_{i-2,j})}{\Delta xm}\right]-S''_{i-1,j}+4\left[\frac{2(P_{i,j+1}-P_{i,j})}{\Delta tm}+a*\frac{(P_{i+1,j}-P_{i-1,j})}{\Delta xm}-S''_{i,j}\right]+\left[\frac{2(P_{i+1,j+1}-P_{i+1,j})}{\Delta tm}+a*\frac{(P_{i+2,j}-P_{i,j})}{\Delta xm}\right]-S''_{i+1,j}=\frac{6}{h^2}\{P_{i-1,j+1}-2P_{i,j+1}+P_{i+1,j+1}\} \quad (12)$$

$$\left[\frac{2(P_{i-1,j+1}-P_{i-1,j})}{\Delta tm}\right]+4\left[\frac{2(P_{i,j+1}-P_{i,j})}{\Delta tm}\right]+\left[\frac{2(P_{i+1,j+1}-P_{i+1,j})}{\Delta tm}\right]=\frac{6}{h^2}\{P_{i-1,j+1}-2P_{i,j+1}+P_{i+1,j+1}\}-\left[a*\frac{(P_{i,j}-P_{i-2,j})}{\Delta xm}+4a*\frac{(P_{i+1,j}-P_{i-1,j})}{\Delta xm}+a*\frac{(P_{i+2,j}-P_{i,j})}{\Delta xm}\right]+S''_{i-1,j}+4S''_{i,j}+S''_{i+1,j} \quad (13)$$

From (9) we have;

$$S''_{i-1,j}+4S''_{i,j}+S''_{i+1,j}=\frac{6}{h^2}\{P_{i-1,j}-2P_{i,j}+P_{i+1,j}\}$$

Substitute above equation in (13), we get

$$\left[\frac{2(P_{i-1,j+1}-P_{i-1,j})}{\Delta tm}\right]+4\left[\frac{2(P_{i,j+1}-P_{i,j})}{\Delta tm}\right]+\left[\frac{2(P_{i+1,j+1}-P_{i+1,j})}{\Delta tm}\right]=\frac{6}{h^2}\{P_{i-1,j+1}-2P_{i,j+1}+P_{i+1,j+1}\}-\left[a*\frac{(P_{i,j}-P_{i-2,j})}{\Delta xm}+4a*\frac{(P_{i+1,j}-P_{i-1,j})}{\Delta xm}+a*\frac{(P_{i+2,j}-P_{i,j})}{\Delta xm}\right]+\frac{6}{h^2}\{P_{i-1,j}-2P_{i,j}+P_{i+1,j}\} \quad (14)$$

Here $r=\frac{\Delta tm}{h^2}$, $b=\frac{\Delta t}{\Delta x}$ and simplify the above terms we get,

$$(1-3r)P_{i-1,j+1}+(4+6r)P_{i,j+1}+(1-3r)P_{i+1,j+1}=(1+3r)P_{i-1,j}+(4-6r)P_{i,j}+(1+3r)P_{i+1,j}-\frac{ab}{2}\left[(P_{i,j}-P_{i-2,j})+4(P_{i+1,j}-P_{i-1,j})+(P_{i+2,j}-P_{i,j})\right] \quad (15)$$

Here, Equation (15) is recognized as cubic spline implicit method for finding the solution equation (1).

Again $P_{0,j+1}$ & $P_{10,j+1}$ are known due to the given initial conditions. Here, the bunch of simultaneous equations obtained by implicit method contains (n–1) unknowns. We can solve (n-1) equation with (n-1) unknowns by any standard method and the calculated values from (j+1) step we can derived the further values for j+2, j+3,… In this case, again the value of r depends on the convergence and stability of the method as per explicit method.

IV.EXPERIMENTAL RESULT

For testing the applicability of Cubic Spline explicit and implicit method, we are assuming the PDE of equation (1) with some constant values, initial and boundary conditions.

A. Explicit Method Case-(i)

Let us consider the equation (1) with two different Cases-(i) & (ii).

$$\frac{\partial P}{\partial t} + a\frac{\partial P}{\partial x} = m\frac{\partial^2 P}{\partial x^2}\ ;\ m > 0\,,\ 0 < x < 10,\ t > 0$$

For Case-(i), Let the population at various location be distributed as $P(x,t)$. The initial condition is $P(x,0) = p_0 + p_1 e^{-\alpha x}$, where p_0 , p_1 and α are constants and let us assumes the value of $p_0 = 500$, $p_1 = 100$ and $\alpha = 0.4$. here 'a' is mobility rate which will depend on movement term therefor the value of a = 0.1. Furthermore the boundary condition at particular time is $P(0,t) = p_0 + p_1 = 500 + 100 = 600$. Let migrated population expansion rate 'm' be taken as 0.15. Now, we shall find the solution of the equation (1) which satisfying above conditions.

Let the length of the area (x) is divided into 10 equal subintervals x_0 to x_{10} and the length of each subinterval is $h = \Delta x = 1$. Also let time interval is $\Delta t = 1$ and $m = 0.15$ so we get values of r & b are,

$$r = \frac{\Delta t m}{h^2} = \frac{1 * 0.15}{(1)^2} = 0.15\,,\ b = \frac{\Delta t}{\Delta x} = \frac{1}{1} = 1$$

$$\therefore 1 + 6r = 1.9 \text{ and } 4 - 12r = 2.2$$

Now substituting the values of 1 + 6r and 4 – 12r in equation (7). For $j = 0$ and $i = 1,2,3,\ldots$ the equation are,

$$i = 1\,, 4P_{1,1} + P_{2,1} = (1.9)P_{0,0} + (2.2)P_{1,0} + (1.9)P_{2,0} - \frac{0.1}{2}\left[\left(P_{1,0} - P_{-1,0}\right) + 4\left(P_{2,0} - P_{0,0}\right) + \left(P_{3,0} - P_{1,0}\right)\right] - P_{0,1}$$

$$i = 2\,, P_{1,1} + 4P_{2,1} + P_{3,1} = (1.9)P_{1,0} + (2.2)P_{2,0} + (1.9)P_{3,0} - \frac{0.1}{2}\left[\left(P_{2,0} - P_{0,0}\right) + 4\left(P_{3,0} - P_{1,0}\right) + \left(P_{4,0} - P_{2,0}\right)\right]$$

$$i = 3\,, P_{2,1} + 4P_{3,1} + P_{4,1} = (1.9)P_{2,0} + (2.2)P_{3,0} + (1.9)P_{4,0} - \frac{0.1}{2}\left[\left(P_{3,0} - P_{1,0}\right) + 4\left(P_{4,0} - P_{2,0}\right) + \left(P_{5,0} - P_{3,0}\right)\right]$$

…

$$i = 9\,, P_{8,1} + 4P_{9,1} = (1.9)P_{8,0} + (2.2)P_{9,0} + (1.9)P_{10,0} - \frac{0.1}{2}\left[\left(P_{9,0} - P_{7,0}\right) + 4\left(P_{10,0} - P_{8,0}\right) + \left(P_{11,0} - P_{9,0}\right)\right] - P_{10,1} \quad (16)$$

Similarly, for $i = 4$ to 9 we can find out other equations. Here the term $P_{-1,0} = 0$ and $P_{11,0} = 0$ because it is out of table value. Here we get 9 algebraic equations in 9 unknowns with tri-diagonal matrix. Similarly for j = 1, 2,..we get another 9 algebraic equations. Proceeding in this way, the results calculated by explicit method are shown in Table I and plotted in Fig. 1.

Table I: Population Growth: PDE by Cubic Spline-Explicit Method-Case-(i)

Location (x)	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	$t = 7$	$t = 8$	$t = 9$	$t = 10$
0	600.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0
1	567.0	565.2	566.8	566.9	568.2	568.4	569.5	569.7	570.5	570.7	571.5
2	544.9	550.0	550.3	553.3	553.7	556.0	556.5	558.4	558.9	560.5	560.9
3	530.1	531.8	536.0	536.8	540.0	540.9	543.5	544.3	546.5	547.3	549.3
4	520.2	521.8	522.7	525.6	526.7	529.2	530.3	532.6	533.6	535.7	536.6
5	513.5	514.5	515.6	516.6	518.0	519.2	520.8	522.0	523.7	524.8	526.7
6	509.1	509.6	511.0	510.7	512.0	512.5	513.8	514.7	516.0	517.3	518.4
7	506.1	507.0	505.6	507.8	507.4	509.2	509.5	511.0	511.7	512.9	514.0
8	504.1	502.6	506.1	506.0	508.5	508.7	510.5	510.9	512.2	512.9	513.9
9	502.7	509.7	510.7	513.2	513.6	515.2	515.5	516.6	516.9	517.8	518.2
10	501.8	501.8	501.8	501.8	501.8	501.8	501.8	501.8	501.8	501.8	501.8

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887
Volume 6 Issue IV, April 2018- Available at www.ijraset.com

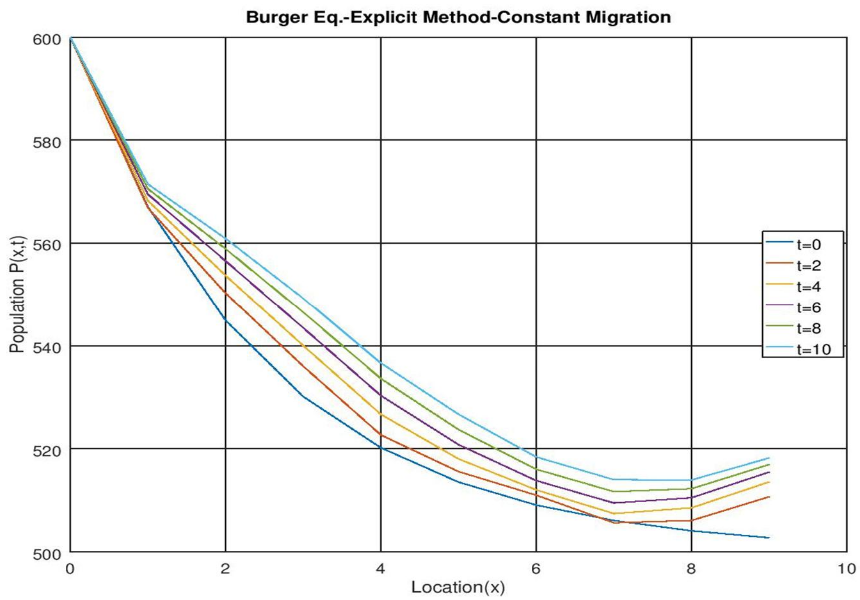


Fig. 1: Population Growth: PDE by Cubic Spline-Explicit Method-Case-(i)

Fig. 1 shows that, initially time t=0 the overall population was distributed in exponentially decreasing order form in the region. Here the mobility rate of movement 'a' controls the over expansion of population at initial locations and increase the movement of population towards the low populated far locations, so that there is an equal opportunity to all locations in increment of population. The Fig. 1 shows that at every time interval the population at initial location x=0 will be remain constant because of the boundary condition taken. Hence, the supposed growth in population at remaining locations x=1 to 9 at different time (t=1, t=3, t=5, t=10) will be seen above figure.

B. Explicit Method Case-(ii)

Case-(ii), Let us assume the condition $P(x,0) = p_0 + p_1 e^{-\alpha x}$, with the boundary condition $P(0,t) = p_0 + p_1 e^{-\alpha t}$ and the values of $p_0 = 500$, $p_1 = 100$, $\alpha = 0.4$, migrated population expansion rate m = 0.15 and mobility rate of movement a = 0.1 are considered. We should follow the same process as per explicit method for Case-(i).

Here also we get 9 algebraic equations in 9 unknowns with tri-diagonal matrix. Similarly for j = 1, 2,..We get another 9 algebraic equations. Continuing in this way, the results are shown in Table II and plotted in Fig. 2.

Table II: Population Growth: PDE by Cubic Spline-Explicit Method-Case-(ii)

Location (x)	t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6	t = 7	t = 8	t = 9	t = 10
0	600.0	567.0	544.9	530.1	520.2	513.5	509.1	506.1	504.1	502.7	501.8
1	567.0	573.2	564.1	554.1	543.7	534.8	527.0	520.5	515.1	510.6	506.8
2	544.9	547.3	554.5	555.0	553.8	550.2	546.2	541.7	537.3	533.0	529.0
3	530.1	532.2	533.0	537.7	540.0	542.1	542.5	542.2	541.0	539.5	537.4
4	520.2	521.5	523.3	523.4	526.1	528.0	530.3	531.9	533.2	533.8	534.1
5	513.5	514.5	515.1	516.9	517.0	518.5	519.6	521.3	522.6	524.1	525.3
6	509.1	509.5	510.7	510.7	511.9	512.0	513.0	513.7	514.8	515.8	517.1
7	506.1	507.0	506.1	506.9	507.1	508.2	508.7	509.7	510.4	511.4	512.3
8	504.1	502.5	504.7	505.7	507.3	508.2	509.5	510.4	511.5	512.4	513.3
9	502.7	509.7	512.1	514.3	515.7	517.1	518.0	519.0	519.7	520.5	521.2
10	501.8	501.8	501.8	501.8	501.8	501.8	501.8	501.8	501.8	501.8	501.8

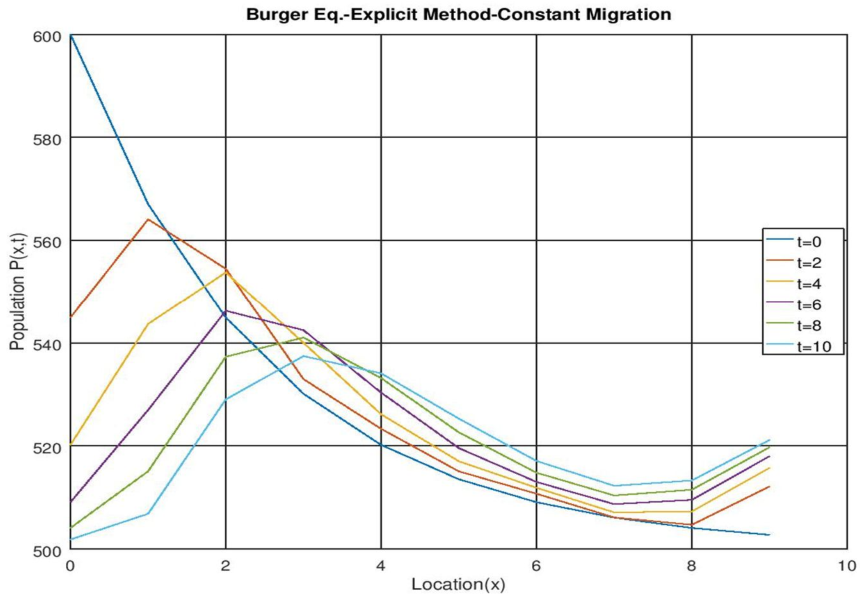


Fig. 2: Population Growth: PDE by Cubic Spline-Explicit Method-Case-(ii)

Fig. 2 indicates that, at time t = 0 the initial population is distributed at various locations in the form of exponentially decreasing order. At different time interval the resulting population at initial location (x=0) will be highly decrease because of the nature of boundary condition. As discussed earlier, due to mobility rate of movement ‘a’ and migrated population expansion rate ‘m’, the population is increased at far locations as compare to initials in order to maintain the uniformity in distribution.

C. Implicit Method Case-(i)

The solution of equation (1) by Cubic spline implicit method,

Here $r = 0.15$, $= 1$, $a = 0.1$

Now, Substitute the value of r in (15) we get,

$$1-3r = 0.55\,,\ 1+3r = 1.45,\ 4-6r = 3.1,\ 4+6r = 4.9$$

For $j = 0$ and $i = 1,2,\ldots$

$$i = 1,\ (4.9)P_{1,1} + (0.55)P_{2,1}$$
$$= (1.45)P_{0,0} + (3.1)P_{1,0} + (1.45)P_{2,0} - \frac{0.1}{2}\left[\left(P_{1,0}-P_{-1,0}\right) + 4\left(P_{2,0}-P_{0,0}\right) + \left(P_{3,0}-P_{1,0}\right)\right] - (0.55)P_{0,1}$$
$$i = 2,\ (0.55)P_{1,1} + (4.9)P_{2,1} + (0.55)P_{3,1}$$
$$= (1.45)P_{1,0} + (3.1)P_{2,0} + (1.45)P_{3,0} - \frac{0.1}{2}\left[\left(P_{2,0}-P_{0,0}\right) + 4\left(P_{3,0}-P_{1,0}\right) + \left(P_{4,0}-P_{2,0}\right)\right]$$
$$i = 3,\ (0.55)P_{2,1} + (4.9)P_{3,1} + (0.55)P_{4,1}$$
$$= (1.45)P_{2,0} + (3.1)P_{3,0} + (1.45)P_{4,0} - \frac{0.1}{2}\left[\left(P_{3,0}-P_{1,0}\right) + 4\left(P_{4,0}-P_{2,0}\right) + \left(P_{5,0}-P_{3,0}\right)\right]$$

…

$$i = 9\,,\ (0.55)P_{8,1} + (4.9)P_{9,1} = (1.45)P_{8,0} + (3.1)P_{9,0} + (1.45)P_{10,0} - \frac{0.1}{2}\left[\left(P_{9,0}-P_{7,0}\right) + 4\left(P_{10,0}-P_{8,0}\right) + \left(P_{11,0}-P_{9,0}\right)\right] - (0.55)P_{10,1} \quad (17)$$

Similarly, for $i = 4,5,\ldots$ we can find out other equations.

In this method again we take the value of $P_{-1,0}$ for the 1st equation and $P_{11,0}$ for the last equation as zero because of the outer value from the table. So we have 9 algebraic equations in 9 unknowns with tri-diagonal matrix. Similarly, applying above method, we get the solution for $j = 1,2\ldots$ .we get another 9 algebraic equations. This can be solved easily by above method. Continuing in this way, the results obtained by implicit method are shown in Table III and plotted in Fig. 3

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887
Volume 6 Issue IV, April 2018- Available at www.ijraset.com

Table III: Population Growth: PDE by Cubic Spline-Implicit Method-Case-(i)

Location (x)	t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6	t = 7	t = 8	t = 9	t = 10
0	600.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0
1	567.0	565.5	565.1	565.3	565.6	565.9	566.3	566.7	567.1	567.4	567.8
2	544.9	548.6	550.6	552.1	553.3	554.5	555.5	556.5	557.4	558.3	559.1
3	530.1	532.0	534.4	536.5	538.5	540.2	541.8	543.3	544.7	546.0	547.2
4	520.2	521.5	522.9	524.4	526.1	527.7	529.3	530.9	532.4	533.9	535.3
5	513.5	514.4	515.4	516.4	517.5	518.6	519.8	521.1	522.3	523.6	524.9
6	509.1	509.7	510.3	511.0	511.6	512.4	513.2	514.0	515.0	516.0	517.0
7	506.1	506.5	506.7	507.1	507.6	508.3	509.0	509.8	510.6	511.5	512.4
8	504.1	503.8	504.7	505.9	507.1	508.2	509.3	510.4	511.3	512.3	513.2
9	502.7	508.1	511.3	513.5	515.2	516.5	517.6	518.6	519.4	520.2	520.9
10	501.8	501.8	501.8	501.8	501.8	501.8	501.8	501.8	501.8	501.8	501.8

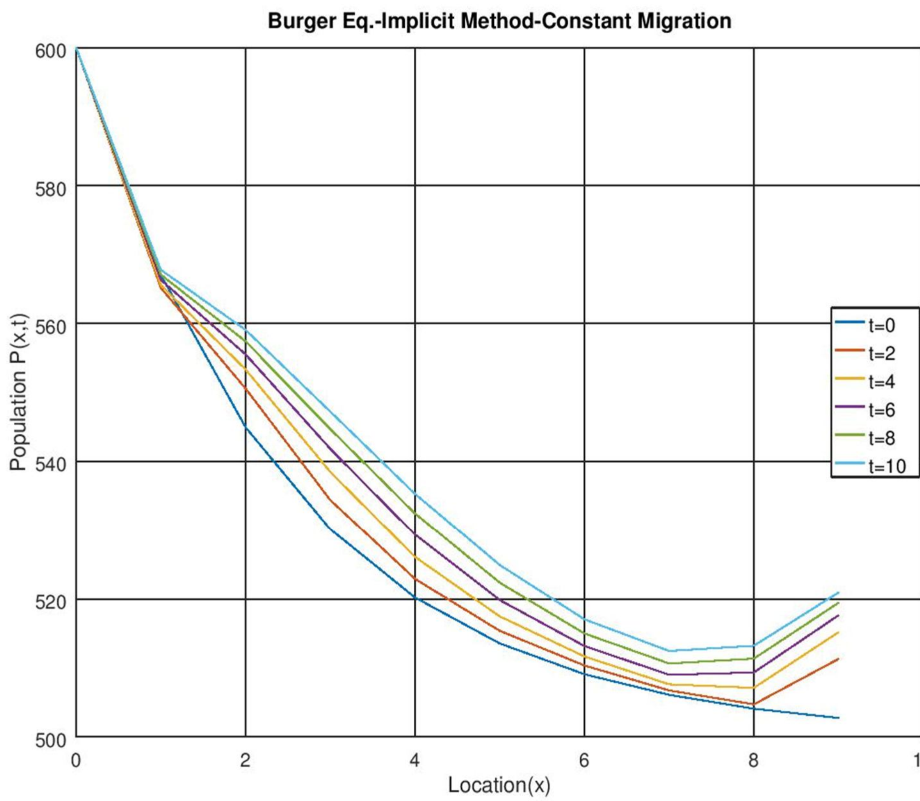


Fig. 3: Population Growth: PDE by Cubic Spline Implicit Method-Case-(i)

Here, Fig. 3 shows that, the nature in which the population is growing at various locations in different time period by implicit method is similar to the nature obtained by explicit method for Case-(i), discussed above.

D. Implicit Method: Case-(ii)

Now, consider equation (1) for Case-(ii). The same procedure we will follow as per the experimental results by implicit method. The values of $p_0 = 500$, $p_1 = 100$, $\alpha = 0.4$. and migration rate m=0.15 and a=0.1. The results Table IV for Case-(ii) by implicit method is shown below,

Table IV: Population Growth: PDE by Cubic Spline-Implicit Method-Case-(ii)

Location (x)	t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6	t = 7	t = 8	t = 9	t = 10
0	600.0	567.0	544.9	530.1	520.2	513.5	509.1	506.1	504.1	502.7	501.8
1	567.0	569.2	562.4	553.0	543.6	535.0	527.5	521.1	515.7	511.2	507.4
2	544.9	548.1	551.6	552.5	551.2	548.5	544.9	540.9	536.9	532.9	529.1
3	530.1	532.1	534.2	536.7	538.8	540.2	540.7	540.4	539.5	538.1	536.4
4	520.2	521.5	522.9	524.4	526.1	527.8	529.4	530.8	531.8	532.4	532.7
5	513.5	514.4	515.4	516.4	517.4	518.6	519.8	521.1	522.4	523.5	524.6
6	509.1	509.7	510.3	511.0	511.6	512.4	513.2	514.0	515.0	516.0	517.0
7	506.1	506.5	506.7	507.1	507.6	508.3	509.0	509.8	510.6	511.5	512.4
8	504.1	503.8	504.7	505.9	507.1	508.2	509.3	510.4	511.3	512.3	513.2
9	502.7	508.1	511.3	513.5	515.2	516.5	517.6	518.6	519.4	520.2	520.9
10	501.8	501.8	501.8	501.8	501.8	501.8	501.8	501.8	501.8	501.8	501.8

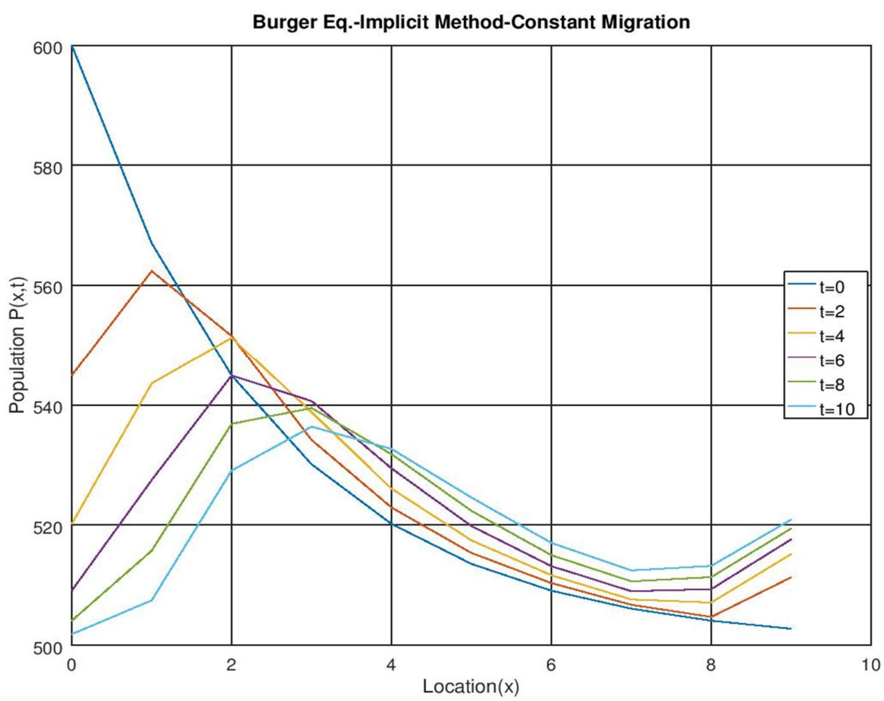


Fig. 4: Population Growth: PDE by Cubic Spline-Implicit Method-Case-(ii)

From the Fig. 4 the resulting nature in population increase by implicit method is as similar to the nature followed by explicit method for Case-(ii) which we have already seen in above case.

V. CONCLUSION

In the estimation of population growth by both Cubic spline - explicit and implicit numerical methods with the two major parameters time and space, having similar kind of results. Also the nature of initial and boundary conditions affect the pattern of growth of population. Furthermore, the key impact of the mobility rate of movement is, it controls the over expansion of population at some locations by migrated population expansion rate such that, the population distribution becomes equal in magnitude over the locations. If the initial and boundary conditions changed with the same numerical method which was discussed above, the population growth will be varied and can be projected respectively.

REFERENCE

[1] Prof. V.V. Singh1, Dr. Alka Mittal2, Dr. Neetish Sharma3, Prof. F. Smarandache, "Determinants of Population Growth in Rajasthan: An Analysis", 1 Dec 2010

[2] Harpreet Singh, "Increasing rural to urban migration in India: A challenge or an opportunity", International Journal of Applied Research 2016; 2(4): 447-450

[3] "Impact of Migration on Economic and Social Development: A review of evidence and emerging issues", Migration&Development-Ratha-GFMD_2010a

[4] Ronald Skeldon , "GLOBAL MIGRATION: DEMOGRAPHIC ASPECTS AND ITS RELEVANCE FOR DEVELOPMENT", Technical Paper No. 2013/6, United Nations ,New York, 2013

[5] Sharon Gane, "Solution of the advection equation using finite difference schemes and the method of characteristics", University of Glasgow, September 2000.

[6] William, F. A, "Numerical Methods for Partial Differential Equations". Academic Press Incorporated, London, 1992.

[7] O. A. Taiwo, O.S. Odetunde, "Application of Cubic Spline Collocation Method for the Numerical Solution of Parabolic Partial Differential Equations", International Journal of Engineering and Innovative Technology (IJEIT) Volume 1, Issue 3, March 2012, ISSN: 2277-3754

[8] L. R. Brown , Gary Gardner & Brian halwell, "Beyond Malthus: Sixteen Dimensions of the Population Problem", Linda Starke, Editor, ISBN 1-878071-45-9

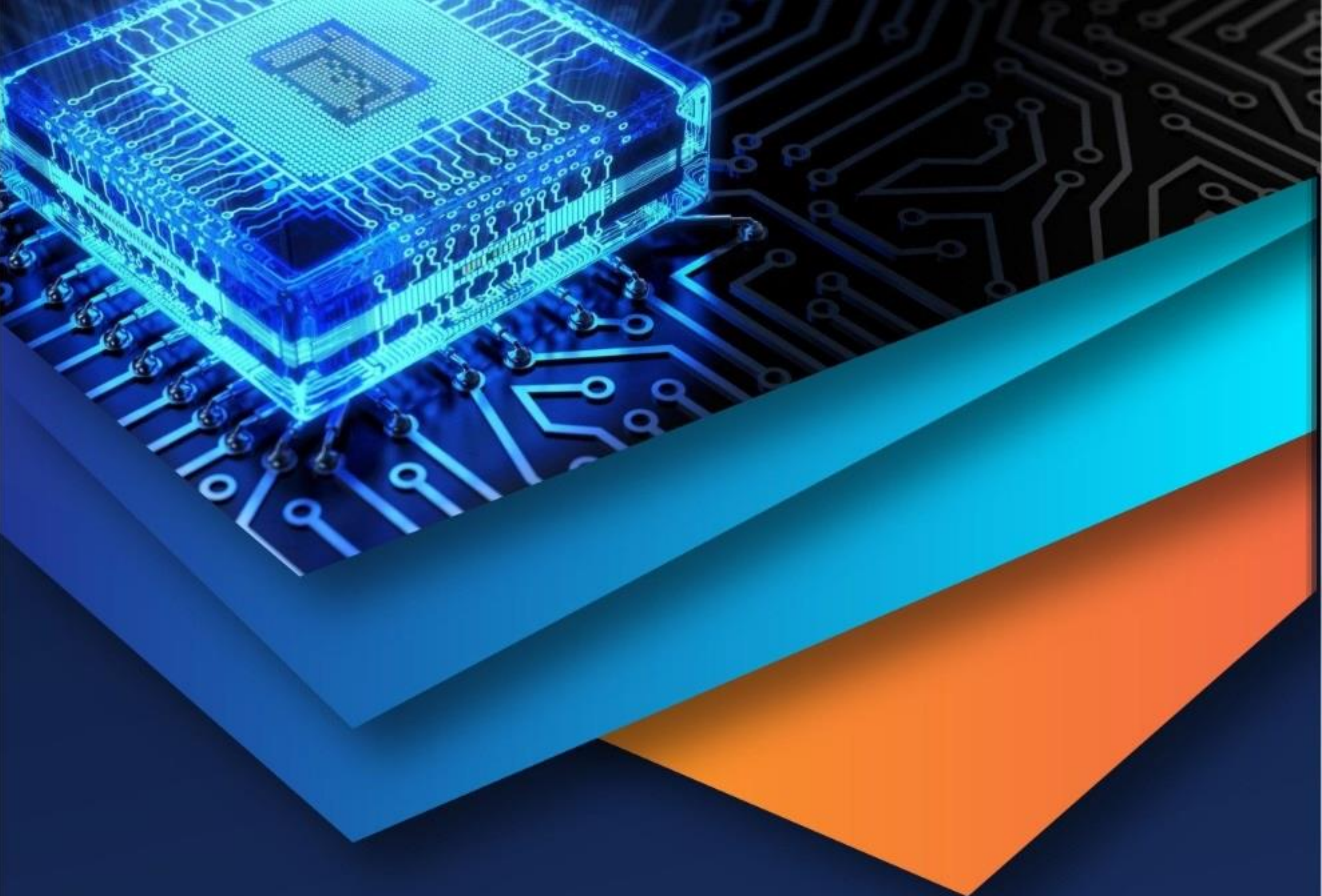
[9] Ahlberg, J. H., Nilson, E. N. and Walsh, J. H.: The Theory of splines and their applications, Academic Press, N. Y., 1967.

[10] Agrwal R.P.; Chow Y.M.; Wilson S.J. (Eds): Numerical Mathematics Singapore, 1988,I. S.N.M.vol. 86, Birkhauser verlage, 1988.

[11] Ahlberg, J. H. and Nilson, E. N.: Convergence properties of the spline fit, Notices Am. Math. Soc., 61T-219. 1961

[12] Ahlberg, J. H. and Nilson, E. N.: Convergence properties of the spline fit, J. Soc. Ind. Appl. Math-II, 95-104. 1962

[13] Albasiny E.L., Hoskins W.D.: Cubic spline solution of two point boundary value problems. Compute. J. 12(1969), 151-153.



10.22214/IJRASET

INDEX COPERNICUS 45.98

IMPACT FACTOR: 7.129

TOGETHER WE REACH THE GOAL

IMPACT FACTOR: 7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY