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An Analytical Investigation for Analysis of Fluid Flow within a Microchannel of Rectangular Cross Section

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Abstract: The present work considers fluid flow within a straight micro channel of rectangular cross section using Integral technique. The flow is represented by the Navier-Stokes equation subjected to slip boundary conditions and solved by Integral method considering both Ritz and Kantorovich profiles. Results obtained is validated and agreed with existing literature well. Thus, along with the velocity distribution, Poiseuille number and slip coefficient are also determined for both the Ritz and Kantorovich profiles. The results are presented in a comparative manner along with the existing literature to predict accuracy level of all the profiles. It is observed that the accuracy level for the velocity profile obtained in each profile depends on the aspect ratio where as for the prediction of Poiseuille number and slip coefficient, all the profile show less dependency on the aspect ratio.

Keywords: Rectangular microchannel, Slip-flow, Velocity distribution, Knudsen number, Poiseuille number, slip coefficient.

I. INTRODUCTION

In current days, fluid flow in microchannel is an emerging area of research due to the growing demand of microfluidic devices and systems. It plays an important role in various applications of medical and biomedical areas, computer chips, and chemical separations etc. In this connection, a new research area, namely Micro-electro-mechanical systems (MEMS) is opened up where non-continuum behavior exists [1], and the components like, micro-valves, micro-pumps and actuators are miniaturized, integrated and assembled. However, the fundamental understanding of flow characteristics such as velocity distribution, Poiseuille number and slip coefficient is vital for systematic design and process control of microfluidic applications [2]. It is noticed that the flow in the micro level is associated with the inclusion of slip velocity [3] and does not obey the classical continuum physics. Thus, such a flow is coupled with a non-zero fluid velocity at boundary walls. It occurs when the value of Knudsen number (*Kn*) ranges from 0.001 to 0.1 and corresponding flow is called as slip-flow [3]. In such a flow, the Navier–Stokes equation is combined with the slip-flow condition [4, 5]. In this connection, it is found that suitable analytical solutions for the flow through the microchannel are less developed.

The present work is focused accordingly to propose a suitable analytical technique for analysis of the fluid flow within a microchannel of rectangular cross section. In this context, some related research works are reviewed. Hooman [3] presented a superposition approach to investigate forced convection in microducts of arbitrary cross-section in slip-flow. It is found that applying an average slip velocity and temperature jump, the no-slip/no-jump with some minor modifications is applicable. Kundu *et al.* [6] established approximate analytical techniques to determine the velocity distribution considering laminar flow and no-slip boundary condition through straight rectangular channels. They adopted some exact and approximate analytical solutions for the prediction of velocity and temperature distributions. Chakraborty [7] considered flow problems within a straight microchannel of arbitrary cross-section using three general solution methods, namely complex function analysis, membrane vibration analogy and variational method. Kuddusi [8] demonstrated slip flow in a rectangular microchannel of heated walls using integral transform methods. The 2-*D* temperature field and Nusselt numbers as function of aspect ratio are also predicted by Morini [9] in case of fully developed thermal region of rectangular ducts at constant wall temperature considering a laminar fully developed velocity profile. Theofilis *et al.* [10] determined velocity distribution of a fluid flow through a rectangular channel solving the Navier-Stoke equation. They assumed a constant pressure gradient along length of the channel. Peng *et al.* [11] presented analytical solution for viscous flow in an equilateral triangular tube to irregular triangular tubes. The solution is examined and compared with numerical simulation. It is seen from the existing literature that the analytical techniques presented are lengthy, complex and laborious. Hence, in the present study, Integral technique considering both the Ritz and Kantorovich profiles is adopted for gaseous flow within a rectangular

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microchannel. The Navier–Stokes equation is solved assuming slip boundary conditions at walls to determine velocity distribution for the pressure driven gas flow within the microchannel.

II. DESCRIPTION OF THE PROBLEM

In the present study, a pressure driven fully developed laminar gas flow within a straight microchannel of rectangular cross section is considered as shown in Fig. 1. The centre of cross section of the channel is considered as origin of the Cartesian coordinate. The fluid flows only in the z-direction under steady state condition. A width of 2L parallel to the x-axis and a depth of 2l parallel to the y-axis are considered for the cross section. The flow is assumed as viscous incompressible with constant properties.

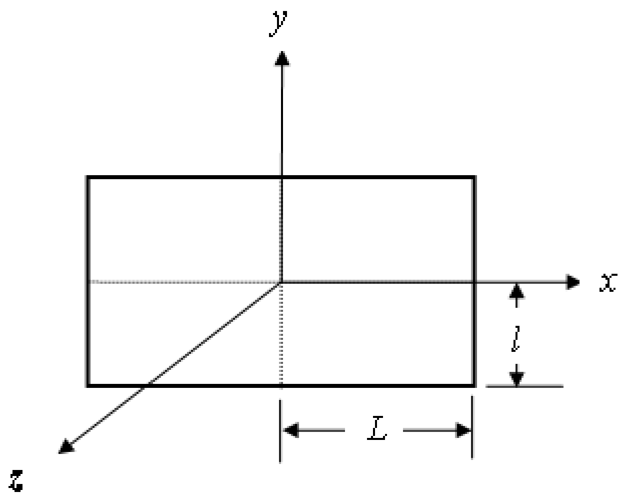

Fig. 1 A schematic of the channel cross section

III.MATHEMATICAL MODELLING

The present work considers a viscous incompressible flow within the straight microchannel of rectangular cross section. Considering a hydro-dynamically developed flow, the conservation of momentum equation along the axial direction (*z*-axis) is written as

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{dp}{dz} \tag{1}$$

Also $\frac{dp}{dz}$ is the pressure gradient in the flow direction and assumed as a constant in the present study.

Equation (1) is subjected to the following boundary conditions

$$u = u_s \text{ at } x = \pm L \text{ and } u = u_s \text{ at } y = \pm l \tag{2a}$$

$$\frac{\partial u}{\partial x} = 0 \text{ at } x = 0 \text{ and } \frac{\partial u}{\partial y} = 0 \text{ at } y = 0 \tag{2b}$$

where u_s is slip velocity at the solid walls of the channel.

The average velocity (u_m) of the flow is determined as

$$u_m = \frac{\int_0^L \int_0^l u dx dy}{\int_0^L \int_0^l dx dy} \tag{3a}$$

The slip velocity (u_s) at the walls is expressed [3, 8] as

$$u_s = \frac{F-2}{F} Kn\, D_h \left.\frac{\partial u}{\partial n}\right|_{wall} \tag{3b}$$

In equation (2a), the boundary conditions are non-homogeneous. In order to eliminate the non-homogeneity of the boundary conditions and present the results as free of units, the governing equation (1) is represented as a non-dimensional equation by introducing following non-dimensional variables as [3]

$$U = -\frac{\mu}{L_c^{\,2}(dp/dz)}(u - u_s)\,,\; X = \frac{x}{L}\,,\; Y = \frac{y}{l} \;\text{ and }\; A = \frac{l}{L}$$

Substituting these non-dimensional variables in the equation (1), the governing momentum equation becomes

$$\frac{\partial^2 U}{\partial X^2} + \frac{1}{A^2}\frac{\partial^2 U}{\partial Y^2} = -1 \tag{4}$$

The non-dimensional form of the boundary conditions becomes

$$U = 0 \;\text{ at }\; X = \pm 1 \;\text{ and }\; Y = \pm 1 \tag{5a}$$

$$\frac{\partial U}{\partial X} = 0 \;\text{ and }\; \frac{\partial U}{\partial Y} = 0 \;\text{at}\; X = 0\,, Y = 0 \tag{5b}$$

The equation (4) is the non-dimensional form of Navier-Stokes equation for a fully developed flow subjected to no-slip boundary condition. Accordingly, the flow velocity U is considered as the velocity for no-slip condition and expressed as U_{ns} . However, in the present microchannel flow, a non-zero velocity is present at the boundary walls. Assuming a constant slip velocity at the boundary, a normalized velocity ($\overline{U}$) of flow within the microchannel is expressed as [3]

$$\overline{U}(\mathrm{X},\mathrm{Y}) = B\overline{U}_{ns} + 1 - B \tag{6}$$

where $B = \dfrac{1}{1 + \left(\dfrac{2-F}{F\;U_{ns,m}}\,Kn\left(\dfrac{2A}{1+A}\right)^2\right)}$ and the normalized no-slip velocity $\overline{U}_{ns} = \dfrac{U_{ns}}{U_{ns,m}}$. The mean velocity for no-slip condition is determined as $U_{ns,m} = \dfrac{\int_0^1\int_0^1 U_{ns}\,dXdY}{\int_0^1\int_0^1 dXdY}$.

The equation (4) is solved to determine the velocity profile considering no-slip condition and hence the velocity profile for the slip flow is determined by the equation (6). In the present work, Integral for both Ritz and Kantorovich profiles are considered in order to determine the velocity distribution. In flow through microchannel, slip coefficient β_s which measures the velocity slip at the boundary is expressed as $\beta_s = \dfrac{u_s}{u_m}$ and the Poiseuille number as $Po = f\,\mathrm{Re}$. Both the parameters are important in predicting fluid flow characteristics within the microchannel and expressed as [3]

$$\beta_s = 1 - B \tag{7a}$$

$$Po = B\left(\frac{2}{U_{ns,m}}\right)\left(\frac{2A}{1+A}\right)^2 \tag{7b}$$

Finally, equations (1-7) are solved using the three solution techniques to predict the velocity distribution within the microchannel.

A. *Solution method by Chakraborty [7]*

In this section, equation (4) is solved in finite series form using the analytical technique proposed by Chakraborty [7] and finally the slip velocity is determined as

$$\overline{U} = \frac{\pi B}{2B_1}\left[\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3}\left\{1 - \frac{\cosh\left(2n+1\right)\pi\, A\, Y/2}{\cosh\left(2n+1\right)\pi\, A/2}\cos\frac{(2n+1)\pi\;X}{2}\right\}\right] + 1 - B \tag{8}$$

where $B = \left[1 + \dfrac{\pi^4(2-F)}{32\,F\,B_1}Kn\left(\dfrac{2A}{1+A}\right)^2\right]^{-1}$ and $B_1 = \displaystyle\sum_{n=0}^{\infty}\frac{1}{(2n+1)^4}\left\{1 - \frac{\tanh\left(2n+1\right)\pi\, A/2}{(2n+1)\,\pi\, A/2}\right\}$.

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The Poiseuille number (Po) is expressed as

$$Po = \frac{\pi^4}{16}\left[\sum_{n=0}^{\infty}\frac{1}{(2n+1)^4}\left\{1-\frac{\tanh(2n+1)\pi A/2}{(2n+1)\pi A/2}\right\}+\frac{\pi^4(2-F)}{32F}Kn\left(\frac{2A}{1+A}\right)^2\right]^{-1}\left(\frac{2A}{1+A}\right)^2 \quad (9)$$

B. Integral method

In this section an approximate analytical solution using Integral method is presented. The governing equation (4) has been solved for Ritz profile and Kantorovich profile.

1) Ritz profile

Here, the equation (4) is solved by the Integral method using second-order approximation of the Ritz profile. Accordingly, a truncated series for the velocity profile is assumed as

$$U_{ns} = (1-X^2)(1-y^2)(a_0 + a_1 X^2) \quad (10)$$

where a_0 and a_1 are two unknowns require two conditions as

$$\int_0^1\int_0^1 \left(\partial^2 U/\partial X^2 + \partial^2 U/A^2\partial Y^2 + 1\right)\delta X dX dY = 0 \quad (11)$$

$$\left.\frac{\partial^2 U}{\partial X^2}\right|_{\substack{X=0\\Y=0}} + \frac{1}{A^2}\left.\frac{\partial^2 U}{\partial Y^2}\right|_{\substack{X=0\\Y=0}} + 1 = 0 \quad (12)$$

Both the equations (11) and (12) are solved to determine a_0 and a_1 and hence, equation (10) is used to obtain the normalized no-slip velocity $U_{ns}(X,Y)$. Substituting $U_{ns}(X,Y)$ in equation (6), the velocity profile is determined as

$$\overline{U} = \frac{9B}{4}(1-X^2)(1-Y^2)\left(\frac{1+A^2}{3+26A^2}\right)\left(\frac{2+25A^2}{1+A^2}+5X^2\right)+1-B \quad (13)$$

where $B = \left[1+\frac{9(2-F)}{F}\left\{\frac{(1+A^2)(1+10A^2)}{3+26A^2}\right\}Kn\left(\frac{2}{1+A}\right)^2\right]^{-1}$

The Poiseuille number (Po) is determined as

$$Po = 18\left[\frac{(3+26A^2)}{(1+10A^2)(1+A^2)}+\left\{\frac{9(2-F)}{F}Kn\left(\frac{2}{1+A}\right)^2\right\}\right]^{-1}\left(\frac{2}{1+A}\right)^2 \quad (14)$$

2) Kantorovich profile

The solution of equation (4) is determined by the Integral method using Kantorovich profile as

$$U = (1-Y^2)[F_1(X)+F_2(X)] \quad (15)$$

Equations (11) and (12) are used to determine the unknowns F_1 and F_2, which are the functions of X. The successive equations are

$$(A^4D^4 - 27A^2D^2 + 60)F_1 = 30A^2 \quad (16)$$

$$(A^4D^4 - 27A^2D^2 + 60)F_2 = 0 \quad (17)$$

Hence, F_1 and F_2 are determined considering the boundary conditions $F_1(1)=0$ and $F_2(1)=0$. Substituting the values of F_1 and F_1 in equation (15), the normalized no-slip velocity, $U_{ns}(X,Y)$, is obtained and finally, the slip velocity is by substituting $U_{ns}(X,Y)$ in equation (6) found as

$$\overline{U} = \frac{3B(1-Y^2)}{2B_2}\left[\begin{array}{l}\left(\frac{2-\beta}{\beta-\alpha}\right)\frac{\cosh X\sqrt{\alpha}/A}{\cosh\sqrt{\alpha}/A}+\left(\frac{2-\alpha}{\alpha-\beta}\right)\frac{\cosh X\sqrt{\beta}/A}{\cosh\sqrt{\beta}/A}+\\ \frac{(2-\alpha)(2-\beta)}{(\beta-\alpha)}\left(\frac{\cosh X\sqrt{\alpha}/A}{\cosh\sqrt{\alpha}/A}-\frac{\cosh X\sqrt{\beta}/A}{\cosh\sqrt{\beta}/A}\right)Y^2+1\end{array}\right]+1-B \quad (18)$$

where $B=\left[1+\frac{3(2-F)}{FB_2}Kn\left(\frac{2}{1+A}\right)^2\right]^{-1}$ and $B_2=\frac{(2-\alpha)(7-\beta)}{5(\beta-\alpha)}\left(\frac{\tanh\sqrt{\alpha}/A}{\sqrt{\alpha}/A}\right)+\frac{(2-\alpha)(7-\beta)}{5(\alpha-\beta)}\left(\frac{\tanh\sqrt{\beta}/A}{\sqrt{\beta}/A}\right)+1$.

The Poiseuille number (Po) is determined as

$$Po=6\left[\begin{array}{l}\frac{(2-\alpha)(7-\beta)}{5(\beta-\alpha)}\left(\frac{\tanh\sqrt{\alpha}/A}{\sqrt{\alpha}/A}\right)+\frac{(2-\alpha)(7-\beta)}{5(\alpha-\beta)}\left(\frac{\tanh\sqrt{\beta}/A}{\sqrt{\beta}/A}\right)+1\\ +\frac{3(2-F)}{F}Kn\left(\frac{2}{1+A}\right)^2\end{array}\right]^{-1}\left(\frac{2}{1+A}\right)^2 \quad (19)$$

where $\alpha=\left(27/2-\sqrt{489}/2\right)^{1/2}$ and $\beta=\left(27/2+\sqrt{489}/2\right)^{1/2}$

IV. RESULTS AND DISCUSSION

The Navier-Stokes equation subjected to slip boundary conditions is solved using various analytical techniques to determine the velocity distribution, and corresponding slip coefficient (β_s) and Poiseuille number (Po). The Ritz and Kantorovich profiles of Integral technique are adopted. The results obtained are compared with numerical solution and an existing work by Chakraborty (2008) for the validation purpose.

In Fig. 2 and 3, velocity distribution obtained is presented in y-direction at X = 0 for A = 1.0 and A = 0.5, respectively, based on the Ritz and Kantorovich profiles and compared with the solution of the existing method (Chakraborty [7]). A good agreement of the present solutions is observed with the existing method. The deviation is slightly higher in case of A = 1.0 than A = 0.5. It is also observed that the deviation is more in the flow towards the centre of the channel.

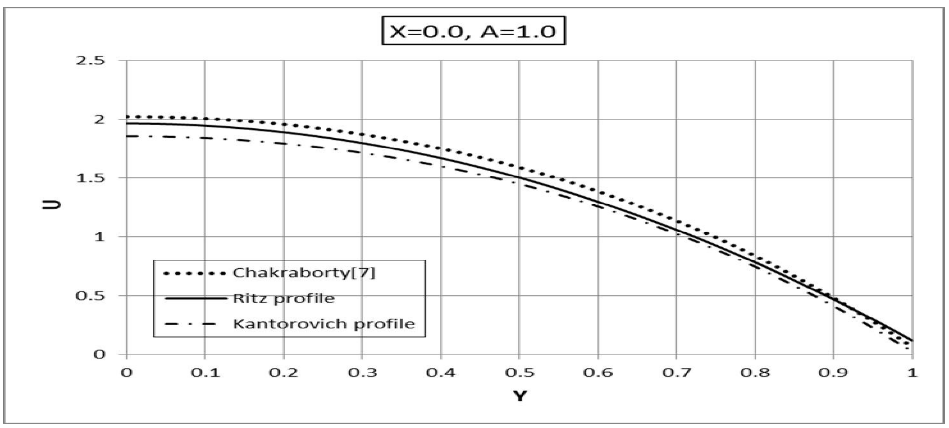

Fig. 2 Comparison of the velocity profile (U) obtained by the Ritz and Kantorovich profiles ($F=1$, $Kn=0.01$) with the profile by Chakraborty[7] for $A=1$

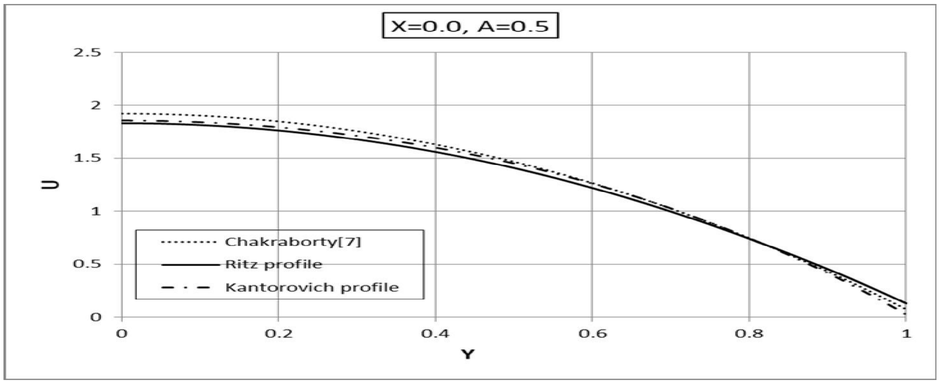

Fig. 3 Comparison of the velocity profile (U) obtained by the Ritz and Kantorovich profiles ($F=1$, $Kn=0.01$) with the profile by Chakraborty[7] for $A=0.5$.

Further, it is found in the literature that the Poiseuille number is of particular interest while analyzing the behaviour of flow within a microchannel. The Poiseuille number depends on the Knudsen number (*Kn*) which normally varies from 0.001 to 0.1 for the gaseous flow through microchannel. Hence, the Poiseuille number as a function of Knudsen number is evaluated and presented in Fig. 4 and 5 for *A* = 1.0 and *A* = 0.5 based on the Ritz and Kantorovich profiles. The variation is compared with the existing method. It is noted that the present prediction matches exactly with the existing literature. Hence, the Integral Ritz and Integral Kantorovich methods also approximate the fluid field well through the microchannel with less error.

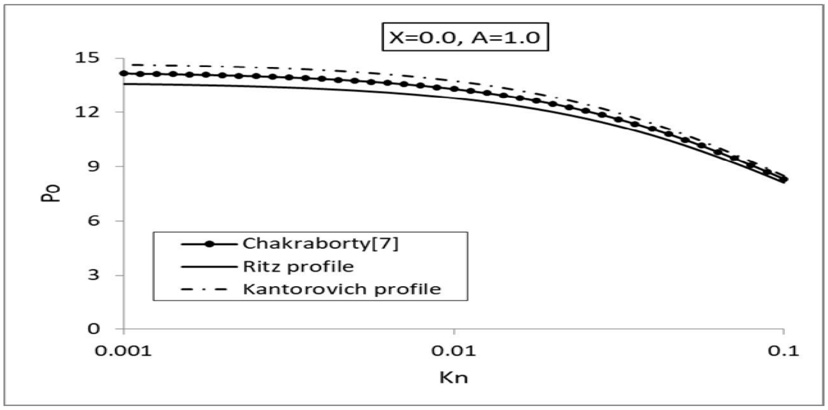

Fig. 4 Variation of Poiseuille number (Po) with Knudsen number (Kn) as determined by Ritz and Kantorovich profiles ($F = 1$) for $A = 1$.

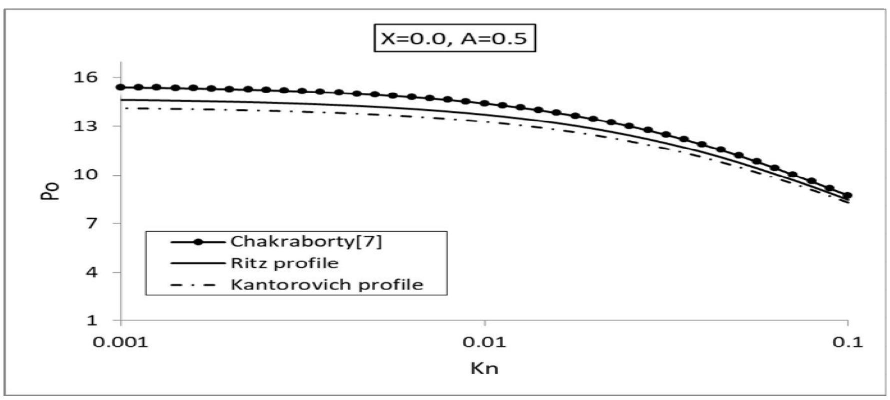

Fig. 5 Variation of Poiseuille number (Po) with Knudsen number (Kn) as determined by Ritz and Kantorovich profiles ($F = 1$) for $A = 0.5$.

In addition to the methods for solving flow field through the microchannel, it is stated that the slip coefficient which measures the velocity slip at the boundary and is of particular interest. The work accordingly predicts the slip coefficient evaluated at different Knudsen numbers for different analytical methods. The respective slip coefficients are summarized in Table 1 for *A* = 1.0 and in Table 2 for *A* = 0.5.

Table 1 A comparison of the slip coefficients for Ritz and Kantorovich profiles at A = 1.0

Solution methods	Slip coefficient (β)				
	Kn = 1.00E-03	Kn = 0.003162	Kn = 1.00E-02	Kn = 0.031623	Kn = 1.00E-01
Chakraborty[7]	0.017241	0.052562	0.149254	0.356824	0.636943
SOV	0.017241	0.052562	0.149254	0.356824	0.636943
Integral Ritz	0.016393	0.050066	0.142857	0.345141	0.625000
Integral Kantorovich	0.016949	0.051703	0.147059	0.352843	0.632911

Table 2 A comparison of the slip coefficient for Ritz and Kantorovich profiles at A = 0.5

Solution methods	Slip coefficient (β)				
	Kn = 1.00E-03	Kn = 0.003162	Kn = 1.00E-02	Kn = 0.031623	Kn = 1.00E-01
Chakraborty[7]	0.007737	0.024064	0.072333	0.1978	0.438116
SOV	0.007737	0.024064	0.072333	0.1978	0.438116
Integral Ritz	0.007353	0.022888	0.068966	0.189787	0.425532
Integral Kantorovich	0.007605	0.023659	0.071174	0.195054	0.433839

V. CONCLUSIONS

The present work considers a steady viscous flow within a straight microchannel of rectangular cross section. Ritz and Kantorovich profile of Integral technique are considered to obtain the flow profiles. The reduced form of the Navier-Stokes equation subjected to slip boundary conditions is solved using the analytical methods. The predictions obtained by the offered analytical techniques for $A = 1.0$ and $A = 0.5$ are compared with the existing work in literature for validation purpose. Subsequently, two important parameters of the flow through microchannel, namely Poiseuille number and slip coefficient are determined at different Knudsen numbers for the various analytical methods. It is observed for velocity distribution, the Integral Ritz and Integral Kantorovich methods provide closer solution at A=0.5 compared to A=1.0. For the prediction of Poiseuille number, almost all the profiles predict closer results with the existing literature.

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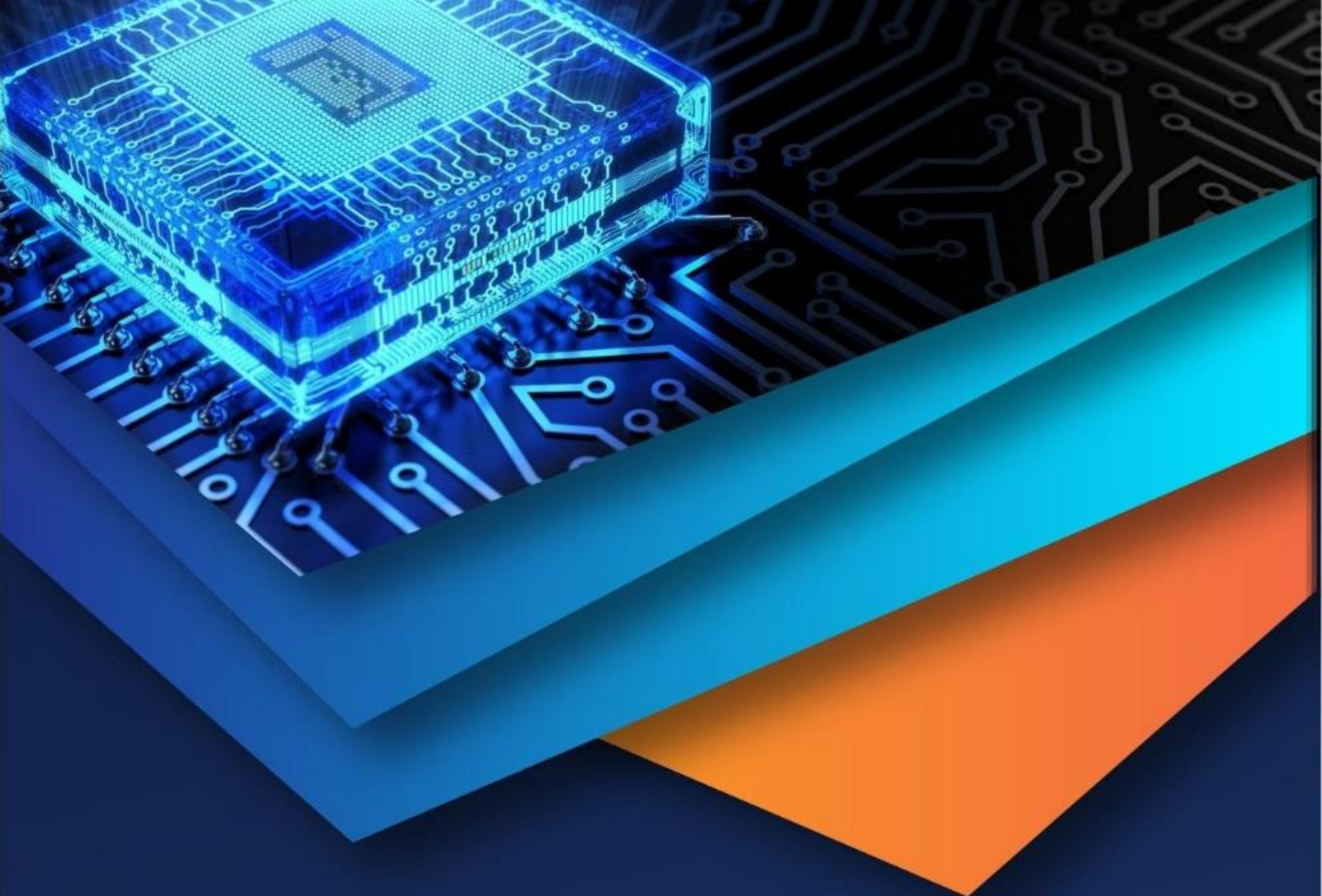

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