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L-R Intuitionistic Fuzzy Numbers in Decision Making

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Abstract: Ranking Intuitionistic fuzzy numbers is an important aspect of decision making in a fuzzy environment. In fuzzy decision-making problems, Intuitionistic fuzzy numbers must be ranked before an action is taken by a decision maker. This article is about ranking Intuitionistic Fuzzy numbers and describes a ranking Method for ordering LR Intuitionistic fuzzy numbers based on the area of fuzzy numbers. This method is Simple in evaluation and can rank various types of LR Intuitionistic fuzzy numbers and also crisp Numbers which are considered to be a special class of Intuitionistic fuzzy numbers.
Keywords: Fuzzy number, LR Fuzzy number, Intuitionistic Fuzzy number, Ranking function, Triangular Fuzzy Number.

I. INTRODUCTION

The theory of fuzzy set introduced by L.A.Zadeh [1965] has achieved successful application in various fields. The fuzzy set was extended to develop intuitionistic fuzzy set by adding an additional non-membership degree which may express more abundant and flexible information as compared with the fuzzy set. The concept of intuitionistic fuzzy set proposed by Krassmir.Atanassov in 1986 is found to be highly useful to deal with vagueness. The intuitionistic fuzzy set theory has been applied in different fields. For example decision making problems, logic programming etc. Ranking fuzzy numbers were first proposed by Jain [10] for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. Since then, various procedures to rank fuzzy quantities are proposed by various researchers. Bortolan and Degani [6] reviewed some of the ranking methods[3-15] for ranking fuzzy subsets.Yuvan [16] presented a criterion for evaluating fuzzy ranking method. Ranking fuzzy numbers are an important tool in decision making. In fuzzy decision analysis, fuzzy quantities are used to describe the performance of lternatives in modeling real - world problems. The development in ranking fuzzy number can also found in[1-2] In order to rank fuzzy quantities, each quantity is converted into a real number and compared by defining a ranking function from the set of fuzzy numbers to a set of real numbers which assign a real number to each fuzzy number where a natural order exists. This articles is ranking of LR-intuitionistic fuzzy numbers is introduced. This method is simple in evaluation can rank various types of intuitionstic fuzzy numbers, fuzzy numbers and also crisp numbers. In this section some basic definitions and notations of Fuzzy Sets, Intuitionistic Fuzzy Sets and LR-Fuzzy numbers are discussed with example.

II. PRELIMINARIES

Fuzzy Sets

Let R be the real line , then a fuzzy set A in R is defined to be a set of ordered pairs, $A = \{x, \mu_A(x)/x \in R\}$, where $\mu_A(x)$ is called the member function for the fuzzy set. The member function maps each element of R to be a membership value between 0 and 1.

Fuzzy number

A fuzzy number $\bar{A}$ is a fuzzy set on a real line that satisfies the condition

1. $\bar{A}$ must be a normal fuzzy set
2. $\alpha_{\bar{A}}$ must be closed interval for every $\alpha \in [0, 1]$.
3. The support of $\bar{A}$, $0 + \bar{A}$must be bounded

LR fuzzy numbers

A fuzzy number Ā on R is said to be LR-fuzzy number. If there exist real numbers and s, t ≥ 0 such that

$$= \begin{cases} L\left(\frac{m - x}{s}\right), & x \leq m \\ R\left(\frac{x - m}{t}\right), & x \geq m \end{cases}$$

$\mu_A(x)$

In which L(x) and R(x) are continuous and non decreasing functions on the real line .L(1)=R(1)=0.L is the left, R is the right reference functions, m is the mean value, s and t are called left and right spreads of membership functions.

A LR – fuzzy number $\bar{A}$is denoted by three real numbers s, t and m as whose meaning are define in figure(1)

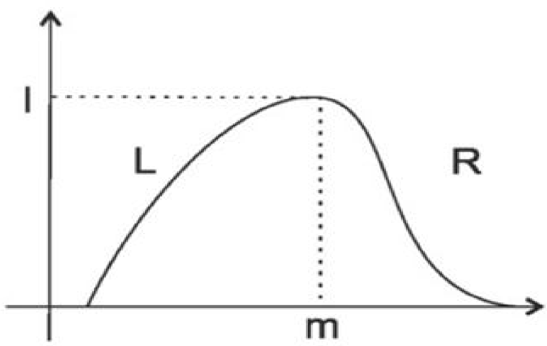

Figure 1:

Value of fuzzy number

$\bar{A}$ is the fuzzy number with α - cut representation $[L^{-1}(x), R^{-1}(x)]$ .S is a reducing function then the value of Āis defined by

$$Val(\bar{A}) = \int_0^1 f(\alpha)[L^{-1}(x)+R^{-1}(x)]d\alpha$$

Regular function

A function f: [0,1] → [0, 1] is a reducing function if it is increasing f(0)=0 and f(1)=1.We say that is a regular function then f (α)dα =1/2

Ambiguity of fuzzy number

If is a fuzzy number with -cut representation $[L^{-1}(x), R^{-1}(x)]$, S is a reducing function then the Ambiguity of Ā is defined by

$$Amb(\bar{A}) = \int_0^1 f(\alpha)[R^{-1}(x) - L^{-1}(x)]d\alpha$$

Intuitionistic fuzzy set

An Intuitionistic fuzzy set (IFS) A in E is defined as an object of the following form

$A = \{< x, \mu_A(x), \nu_A(x)> /x \in E\}$

Where the function is $\mu_A : E \to [0,1]$ and $: E \to [0,1]$ ν_A

Define the degree of membership and the degree of non-membership of the element x E , respectively. Each ordinary fuzzy set may be written as

$\{< x, \mu_A(x), 1 - \mu_A(x) > /x\in E\}$

Degree of non-Determinacy

The value of $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the degree of non-Determinacy (or uncertainty) of the element $x \in E$ to the Intuitionistic fuzzy set (IFS) A.

Triangular Intuitionistic fuzzy numbers

A TIFN $\bar{A}$ is a subset of Intuitionistic fuzzy set in R with following a membership and non-membership function as follows

$$\mu_{\bar{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{for } a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases} \qquad \nu_{\bar{A}}(x) = \begin{cases} \dfrac{a_1 - x}{a_2 - a'_1} & \text{for } a'_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a'_3 - a_2} & \text{for } a'_2 \leq x \leq a'_3 \\ 1 & \text{otherwise} \end{cases}$$

where $a'_1 \leq a_1 \leq a_2 \leq a_3 \leq a'_3$

Ranking function

An efficient for comparing the fuzzy numbers is by the use of ranking function $R:F(R) \rightarrow R$, where F(R) is a set of fuzzy numbers defined on set of real numbers which maps each fuzzy number into the real line where the natural order exist.

III. A NEW METHOD FOR RANKING OF INTUITIONISTIC FUZZY NUMBERS

Here we construct a new ranking system for LR intuitionistic Fuzzy numbers. The new algorithm for ranking LR intuitionistic Fuzzy numbers.

Definition

For any LR intuitionistic Fuzzy numbers $\bar{A} =< m_a; s_a; t_a; s'_a; t'_a >_{LR}$

$$\bar{A}_l = ma + \frac{1}{2} H_l \qquad \bar{A}_u = ma + \frac{1}{2} H_u$$

$$\bar{B}_u = mb + \frac{1}{2} H_u \qquad \bar{B}_l = mb + \frac{1}{2} H_l \qquad (3.1)$$

Where H_l and H_u are defined as follows

$$H_l = \frac{\int_0^1 L^{-1}(x) d\alpha}{\int_0^1 L^{-1}(x) d\alpha + \int_0^1 R^{-1}(x) d\alpha} \quad (3.2) \qquad H_u = \frac{\int_0^1 R^{-1}(x) d\alpha}{\int_0^1 L^{-1}(x) d\alpha + \int_0^1 R^{-1}(x) d\alpha} \quad (3.3)$$

Definition

Suppose that $\bar{A} =< m_a; s_a; t_a; s'_a; t'_a >_{LR}$ and $\bar{B} =< m_b; s_b; t_b; s'_b; t'_b >_{LR}$

be two LR intuitionistic Fuzzy numbers.

Let $\bar{R}(\bar{A}, \bar{B}) = \bar{A}_u - \bar{B}_u \qquad \underline{R}(\bar{A}, \bar{B}) = \bar{A}_l - \bar{B}_l \qquad (3.4)$

$\bar{A}_l, \bar{B}_l, \bar{A}_u, \bar{B}_u$

Where are defined

Definition

Let $\overline{A} =< m_a; s_a; t_a; s'_a; t'_a >_{LR}$ and $\overline{B} =< m_b; s_b; t_b; s'_b; t'_b >_{LR}$ be two LR intuitionistic Fuzzy numbers. Then we have

$$\overline{R}(\bar{A},\bar{B}) = -\overline{R}(\bar{A},\bar{B}) = \overline{R}(-\bar{A},-\bar{B})$$
$$\underline{R}(\bar{A},\bar{B}) = -\underline{R}(\bar{A},\bar{B}) = \underline{R}(-\bar{A},,-\bar{B}) \qquad (3.5)$$

Definition

Assume $\overline{A} =< m_a; s_a; t_a; s'_a; t'_a >_{LR}$ and $\overline{B} =< m_b; s_b; t_b; s'_b; t'_b >_{LR}$ be two LR intuitionistic fuzzy numbers and $\underline{R}(\overline{B},\overline{A}) \geq 0$

The relation < and ≈ on IF(R) are defined as follows

$$\overline{A} \approx \bar{B} \text{ iff } \underline{R}(\overline{A},\bar{B}) = \overline{R}(\overline{A},\bar{B}) \quad \overline{A} < \bar{B} \text{ iff } \underline{R}(\overline{A},\bar{B}) > \overline{R}(\overline{A},\bar{B}) \qquad (3.6)$$

Theorem 3.1

Suppose $\overline{A} \prec \overline{B}$ .Then it can be proved that $-\overline{A} > -\bar{B}$

Proof

By equation(3.5),we have $\overline{A} \leq \bar{B}$ iff $\underline{R}(\overline{A},\bar{B}) > \overline{R}(\overline{A},\bar{B})$

$$\underline{R}(\overline{A},\bar{B}) > \overline{R}(\overline{A},\bar{B})$$

using equation(3.5) , we have $\underline{R}(-\overline{A},-\bar{B}) > \overline{R}(-\overline{A},-\bar{B})$

$-\overline{A} > -\bar{B}$ Now from equation(3.6) we obtained

Theorem 3.2

Assume $\overline{A}, \overline{B}$ and $\overline{C}$ be three intuitionistic Fuzzy numbers.Then the following relations hold

1. $\overline{A} \approx \overline{A}$, for every $\overline{A}$ (Reflexivity)
2. $\overline{A} \approx \bar{B}$, then $\bar{B} \approx \overline{A}$.(symmetry)
3. if $\bar{A} \approx \bar{B}$, and $\bar{B} \approx \overline{C}$ then $\bar{A} \approx \overline{C}$ (Transitivity)

Proof

First part is obvious,because

$$\overline{A} = \overline{A} \Leftrightarrow \underline{R}(\overline{A},\overline{A}) = \overline{R}(\overline{A},\overline{A}) \Leftrightarrow \overline{A}_l - \overline{A}_l = \overline{A}_u - \overline{A}_u$$

Now for symmetry property, assume that $\overline{A} = \bar{B}$

then $\overline{A} = \bar{B} \Leftrightarrow \underline{R}(\bar{B},\overline{A}) = \overline{R}(\overline{A},\bar{B}) \Leftrightarrow \bar{B}_l - \overline{A}_l = \overline{A}_u - \bar{B}_u$

since we can write

$$\bar{B}_l - \bar{A}_l = \bar{A}_u - \bar{B}_u \text{ as a } \bar{A}_l - \bar{B}_l = \bar{B}_u - \bar{A}_u$$

$$\bar{A} = \bar{B} \Leftrightarrow \bar{A}_l - \bar{B}_l = \bar{B}_u - \bar{A}_u = \underline{R}(\bar{A}, \bar{B}) = \overline{R}(\bar{B}, \bar{A}) \Leftrightarrow \bar{B} = \bar{A}$$

For transitivity property, assume that

$\bar{A} = \bar{B}$ *and* $\bar{B} = \bar{C}$. Hence from $\bar{A} = \bar{B}$. We have

$$\underline{R}(\bar{B}, \bar{A}) = \overline{R}(\bar{A}, \bar{B}) \text{ or } \bar{B}_l - \bar{A}_l = \bar{A}_u - \bar{B}_u$$

Also from $\bar{B} = \bar{C}$. We have

$$\underline{R}(\bar{C}, \bar{B}) = \overline{R}(\bar{B}, \bar{C}) \text{ or } \bar{C}_l - \bar{B}_l = \bar{B}_u - \bar{C}_u$$

Then $\underline{R}(\bar{B}, \bar{A}) = \overline{R}(\bar{A}, \bar{B})$ and $\underline{R}(\bar{C}, \bar{B}) = \overline{R}(\bar{B}, \bar{C})$ *yield*

$$\bar{C}_l - \bar{A}_l = \bar{A}_u - \bar{C}_u$$

Thus $\underline{R}(\bar{C}, \bar{A}) = \overline{R}(\bar{A}, \bar{C})$ *or* equivalently we have $\bar{A} = \bar{C}$

The above lemma shows that the relation is an equivalence relation on IF(R).

Theorem 3.3

If $\overline{A} \le \overline{B}$ and $\overline{C} \le \overline{D}$ *then* $\overline{A} + \overline{C} \le \overline{B} + \overline{D}$

Proof

Since $\overline{A} \le \overline{B}$ and $\overline{C} \le \overline{D}$, the following results are holds

$$\underline{R}(\overline{B}, \overline{A}) \le \overline{R}(\overline{A}, \overline{B}) \Rightarrow \overline{B}_l - \overline{A}_l \ge \overline{A}_u - \overline{B}_u \qquad (2.7)$$

$$\underline{R}(\overline{D}, \overline{C}) \le R(\overline{C}, \overline{D}) \Rightarrow \overline{D}_l - \overline{C}_l \ge \overline{C}_u - \overline{D}_u \qquad (2.8)$$

From (2.7) and (2.8), we obtained,

$$(\overline{B}_l + \overline{D}_l) - (\overline{A}_l + \overline{C}_l) \ge (\overline{A}_u + \overline{C}_u) - (\overline{B}_u + \overline{D}_u) \Rightarrow \underline{R}(\overline{B} + \overline{D}, \overline{A} + \overline{C}) \ge \overline{R}(\overline{A} + \overline{C} < \overline{B} + \overline{D}) \qquad (2.9)$$

It follows that

$$\overline{A} + \bar{C} \le \bar{B} + \bar{D}$$

IV. NUMERICAL EXAMPLES

Algorithm

For two intuitionistic fuzzy number $\overline{A}$ and $\overline{B}$, assume that $\overline{B} \ge \overline{A}$ .compute

Example

$$\underline{R}(\overline{B}, \overline{A}) = \overline{B}_l - \overline{A}_l \text{ and } \overline{R}(\overline{A}, \overline{B}) = \overline{A}_u - \overline{B}_u$$

$$Let\ \ d = \underline{R}(B, A) - \overline{R}(A, B) \quad \text{Then}$$

i) $d = 0, then\ \overline{A} = \overline{B}$

ii) $d > 0, then\ \overline{A} < \overline{B}\ else\ \overline{A} > \overline{B}$

Here we present example to illustrate our method

Now for simplicity, consider the following triangular intuitionistic fuzzy numbers.

$\overline{A}$ = <0.5;0.1,0.5,0.2,0.6 > $\overline{B}$ = < 0.7;0.3,0.3,0.4,0.5> , $\overline{C}$ =<0.9;0.5,0.1,0.6,0.2

$$\mu_{\bar{A}}(x) = \begin{cases} L\left(\frac{m-x}{s}\right), & x \le m \\ R\left(\frac{x-m}{t}\right), & x \ge m \end{cases}$$

$$L(x) = L\left(\frac{x-m}{s}\right)$$

$$\alpha = \frac{m-x}{s}$$

$$x = m - s\alpha$$

$$L^{-1}(x) = m - s\alpha$$

$$R(x) = R\left(\frac{x-m}{t}\right)$$

$$\alpha = \frac{x-m}{t}$$

$$x = m + \alpha t$$

$$R^{-1}(x) = m + \alpha t$$

$$H_l = \frac{\int_0^1 (m - s\alpha)\, d\alpha}{\int_0^1 (m - s\alpha)\, d\alpha + \int_0^1 (m + \alpha t)\, d\alpha}$$

$$H_u = \frac{\int_0^1 (m - \alpha t)\, d\alpha}{\int_0^1 (m - s\alpha)\, d\alpha + \int_0^1 (m + \alpha t)\, d\alpha}$$

$$H_l = \frac{m - 0.5s}{2m - 0.5s + 0.5t}$$

$$H_u = \frac{m + 0.5t}{2m - 0.5s + 0.5t}$$

	H_l	H_u	L	u
$\overline{A}$	0.375	0.625	0.6875	0.8125
$\overline{B}$	0.4183	0.5416	0.92915	0.92708

$$v_{\bar{A}}(x) = \begin{cases} L\left(\dfrac{s'-m+x}{s'}\right), & x \le m \\ R\left(\dfrac{t'-x+m}{t'}\right), & x \ge m \end{cases}$$

$$H_l = \frac{m - 0.5s'}{2m + 0.5t' - 0.5s'}$$

$$H_u = \frac{m - 0.5t'}{2m + 0.5t' + 0.5s'}$$

	H_l	H_u	L	u
$\overline{A}$	0.333	0.666	0.665	0.833
$\overline{B}$	0.345	0.6552	0.8725	1.028
$\overline{C}$	0.375	0.675	1.0875	1.2125

	$\mu_{\bar{A}}(x)$	$v_{\bar{A}}(x)$		
$\underline{R}(\overline{B},\overline{A})$	0.2416	0.206	$\underline{R}(\overline{B},\overline{A}) > \overline{R}(\overline{A},\overline{B})$	$\overline{A} < \overline{B}$
$\overline{R}(\overline{A},\overline{B})$	−0.1145	−0.195		
$\underline{R}(\overline{C},\overline{B})$	0.1789	0.215	$\underline{R}(\overline{C},\overline{B}) > \overline{R}(\overline{B},\overline{C})$	$\overline{B} < \overline{C}$
$\overline{R}(\overline{B},\overline{C})$	−0.2646	−0.215		

$\underline{R}(\overline{C},\overline{A})$	0.4205	0.421	$\underline{R}(\overline{C},\overline{A}) > \overline{R}(\overline{A},\overline{C})$	$\overline{A} < \overline{C}$
$\overline{R}(\overline{A},\overline{C})$	−0.379	−0.3795		

$\bar{A} < \bar{B} < \bar{C}$

V. CONCLUSION

In this work a method is introduced that ranks LR Intuitionistic fuzzy number using a simple and naive manner. This method ranks LR intuitionistic fuzzy numbers as well as triangular and trapezoidal intuitionistic fuzzy numbers. This method will be very helpful for decision makers who are dealing with ranking procedures. This method which is simple in calculation not only gives satisfactory results to well defined problems, but also gives a correct ranking order to the problem. This method takes very less interaction to obtain a solution.

If we use fuzzy set we deal only the membership function, while we use intuitionistic fuzzy set we concentrate the membership function as well as the non-membership function. So we get more accurate value for the function.

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