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# Study of Natural Frequency of a Power Transformer by Finite Element Method

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**Abstract:** Excessive vibration generated within power transformer winding generally triggers a mechanical defect. Mechanical defects can be buckling, bending, dislocation and breakage of winding. Calculation of natural frequency of a power transformer winding can provide knowledge of mode shapes, deflection and other parameters.

Some simplifications have been introduced to reduce the problem size and computing time for problem. In this paper various modeling procedure is described and outcomes of it has been discussed. In this paper attempt of finding natural frequency of the power transformer winding has been carried out by Comsol Multiphysics.

**Keywords –** Transformer, Natural frequency, Eigen frequency

## I. INTRODUCTION

Problem of determining Eigen frequencies of the power transformer has occupied engineers from many decades. Information about frequencies can help designer to build transformer which can withstand natural disaster such as earthquake, event of short circuit etc. In order to find out the natural frequency of the transformer various experiments has been conducted by engineers. The most important investigation on this field are those of Wagen, Blume and Boyajian, Bewley, Rudenberg, Pirenn and Vitins and Abetti. Experiments were conducted under different assumption. However, robust method for finding out natural frequency of the transformer is not available. Mathematical modeling is one way, but due to geometry of power transformer and boundary conditions procedure is numerically so complicated that no direct idea is obtained about the general distribution of the natural frequencies [1].

Vibration within the power transformer is caused by magnetic and magnetostrictive forces. Since the structure consisting of distributed mass and stiffness, it has infinite number of natural frequencies, one of which may coincide with one of the harmonic of above forces, resulting in failure of the transformer [2]. Natural frequency can be obtained through modal analysis while displacement can be obtained by type of vibration [3]. This approach comes with limitations as mathematical modeling become way complex to handle for vibration and displacement calculation.

In this paper various methods for calculation of natural frequency using commercial software is discussed and result of each outcome is discussed.

## II. GEOMETRY OF WINDING

Power transformer windings are generally made of metal coils, spacers, synthetic wood and glass fiber rod. Coils are generally made of copper while spacers are made of composite material or plastic. Agenda behind use of spacer to create spacer between two respective turns of coils.. Winding studied over here is disk type winding as shown in Figure.2.1. In order to simplify the design and increase computing time glass fiber and wood is ignored for frequency calculation.

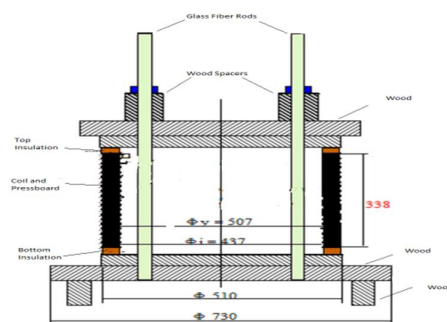


Figure Error! No text of specified style in document..1 Disc type winding

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### III. MATERIAL PROPERTIES

Most of the transformer industries are using either copper or aluminum for making coil of it. Operating temperature, speed of temperature rise during short circuit event, cost, and availability of raw materials play vital role for selection of the winding material. The transformer for what research is conducted has copper coil. Material property of the coil is shown in Table.3.1

Table 3.1 Material Properties of coils

Total length of the copper (L)	259.8 mm
Density ( $\rho$ )	8930 Kg/m <sup>3</sup>
Young modulus (E)	$12 \times 10^{10}$ (N/m <sup>2</sup> )
Mass of the coil (M)	114 Kg
Stiffness (K)	$2.30 \times 10^{10}$ (N/m)

With the help of loading and unloading curve for spacers in axial and normal direction young modulus in X,Y and Z directions has been found. The material property of spacer in Z direction is shown in Figure 3.1. Pressboard material property in axial direction is shown in Figure 3.2.

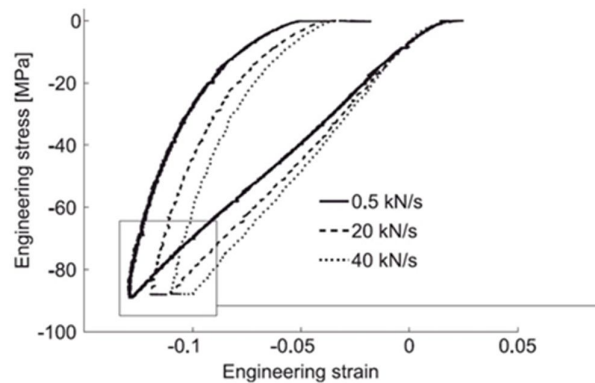


Figure3.1 Material property of spacer in Z direction

$$E_z = 0.8 \times 10^9 \text{ (Pa)}$$

$$E_x = 13.6 \times 10^9 \text{ (Pa)}$$

$$E_y = 11.4 \times 10^9 \text{ (Pa)}$$

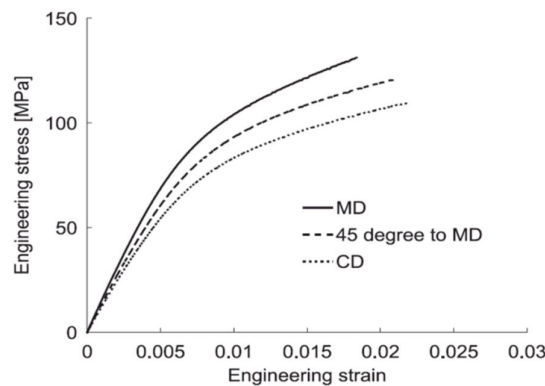


Figure 3.2 Pressboard material property in axial direction

### IV. MATERIAL PROPERTIES FOR 2-D AXIAL SYMMETRIC MODEL

We have material properties for the pressboard for 3D Model. In order to use available material property for 2D axisymmetric model, we have to convert them by using formula for changing Cartesian coordinate system to cylindrical coordinate system as below.

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$$\begin{bmatrix} E_r \\ E_{phi} \\ E_z \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

Where,  $\phi = (\tan^{-1}(E_y/E_x))$  and other values are taken from the previous step.

Solving above equation we get a material property for radial, axial and tangential direction. Material properties for 2D axisymmetric model is given in Table 4.1.

Table 4.1 Material properties for 2D axisymmetric model

$E_r$	$1.7525 \times 10^{10}$ Pa
$E_{phi}$	$-2.7855 \times 10^9$ Pa
$E_z$	$0.8 \times 10^9$ Pa

Where,  $E_r, E_{phi}$  and  $E_z$  are young moduls of the pressboard in radial, tangential and normal direction respectively.

From calculation, axial mode of the winding gives accurate result while in radial mode difference between experimental result and computed result is noticeable. By assuming, pressboard material properties also playing role in eigenfrequency calculation, we introduced orthotropic material for pressboard, which has different material property in different orthogonal directions along each axis. In other words, material properties depend on the direction in which they are measured. Huber’s equation has been used over here for finding Shear modulus for the orthotropic material. Orthotropic material property of pressboard is listed in Table 4.2.

Table 4.2. Orthotropic material property of pressboard

Young Modulus	Radial direction (Er)	Theta direction (Ephi)	Axial direction(Ez)
Shear Modulus (G)	$G_{RPHI} = \frac{\sqrt{E_r * E_{phi}}}{2 * (1 + \nu_p)}$	$G_{PHIZ} = \frac{\sqrt{E_{phi} * E_z}}{2 * (1 + \nu_p)}$	$G_{RZ} = \frac{\sqrt{E_r * E_z}}{2 * (1 + \nu_p)}$
	1.137e9 Pa	2.43e8 Pa	6.10e8 Pa

### V. EIGEN FREQUENCY

Finding eigen frequency of winding by mathematical model can be way complicated as winding is assembly of coils and spacers. In this case number of coil disk is 20 and spacers are 180. Eigen frequency mainly depends upon the geometry and material property of the system. Time and effectively utilization of resources is the key to success for industry. We have generated two dimensional and three dimensional model of winding and have compared our result with experimental result to justify the outcome.

### VI. MATHEMATICAL MODELLING

A natural frequency (Eigen frequency) is the frequency at which the structure would oscillate if it were disturbed from its rest position and then allowed to vibrate freely. To gain a better understanding of the system a single degree of freedom (SDOF) is studied over here. A SDOF system is defined as a system whose motion is defined just by a single independent co-ordinate (or function) e.g. ‘x’ which is function of time. As in this case, the SDOF systems are often used as a very crude approximation for a generally much more complex system. The representation of single degree of freedom is shown in Figure 6.1.

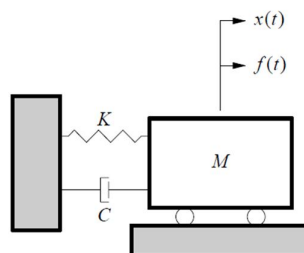


Figure 6.1 Single degree of freedom

Theoretically, it is expressed by using the equation of motion as below.

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$$[M]\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f(t) \tag{1}$$

Where, M is mass of the system (kg), C is co-efficient of viscous damping (Ns/m), K is a stiffness of the system (N/m), x (t) is displacement of the mass from its equilibrium (m) and f (t) is input force applied to the system (N). Assuming that damping is zero,

$$[M]\ddot{x}(t) + Kx(t) = f(t) \tag{2}$$

Taking Laplace transform of the equation 2.2 will result,

$$(s^2[M] + [K])X(s) = f(s) \tag{3}$$

For eigen frequency force function is generally considered as zero so assuming f (t) =0.

$$(s^2[M] + [K])X(s) = 0 \tag{4}$$

Future solving equation 2.4 and solving it would result as,

$$([K]^{-1}[M] + 1/s^2[I])X = 0 \tag{5}$$

Over here,

$$k/m = \omega_n^2 \text{ (Radian/sec)} \tag{6}$$

Where,  $\omega_n$  is angular frequency in Hz, to get eigen frequency conversation of angular frequency is required as below,

$$f = \omega_n/2\pi \text{ (Hz)} \tag{7}$$

Where f is the natural frequency of the system, Natural frequency is also known as an Eigen frequency of the system [3].

### VII. TWO DIMENSIONAL MODEL

To reduce the computing time and to verify the primary result 1D and 2D modeling and simulation of it are the most preferred way. As geometry of winding is circular 2D axisymmetric model were generated and computed.

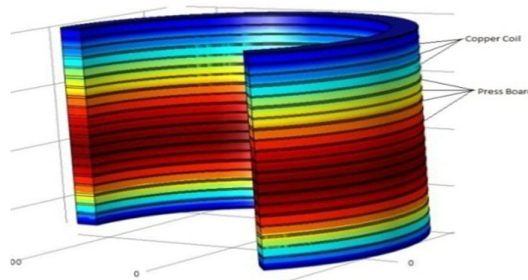


Figure 7.1 Two-dimensional axial

Various assumption and simplification were done on the model to reduce the computing time. As winding of power transformer is fixed on both end, boundary condition for both end are kept as fixed. In case of 2D axial symmetric model pressboard is considered as circular disk which tends to increase the stiffness of the problem. To overcome this problem young modulus of the spacers in X, Y and Z direction was reduced with respect to contact surface area of spacer with coil. Two-dimensional axial model is shown in figure 7.1

In reality due to electromagnetic reasons copper coils are round at edges. Each disc has 16 copper strands next to each other which are bounded with strong industrial glue. If during modeling of the coil above factor is considered than it tends to give precious result for eigenfrequency calculation. However, method tends to take much more computing time and result in poor distribution of mesh elements. Further simplifications are needed. Coil with round edges is shown in Figure 7.2.

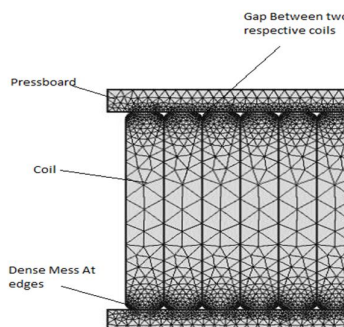


Figure 7.2 Coil with round edges

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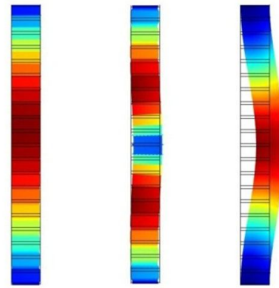


Figure 7.3 First and second axial and radial eigen mode shapes of winding

Inner diameter of the coil and height play most vital role in eigen frequency calculation. Keeping both as constant and changing outer diameter of coil can with respect to same amount of copper, which is used in one turn of coil rectangular cross-section area can be introduced. First and second axial and radial eigen mode shapes of winding is shown in Figure 7.3. Comparison of Experimental and Computed Eigen frequency is given in Table 7.1.

Table 7.1 Comparison of Experimental and Computed Eigen frequency

Mode	Eigen Frequency [Hz]	Experimental result [Hz]	Error [%]
1 <sup>st</sup> axial	344	380	10
2 <sup>nd</sup> axial	795	840	5
1 <sup>st</sup> radial	2347	2442	2

### VIII. THREE DIMENSIONAL MODEL

Two dimensional models have its own limitation as disturbance created due to geometry irregularity is not considered within the model. In case of three dimensional models no discretization of spacer material property is required. However, simplification used for the coil is still the same. As, winding is clamped from both end here we have assumed that end insulation also play vital role in frequency calculation. Moreover, boundary condition on the both ends are kept in such a way that it add constraints to displacement in direction perpendicular to the boundary but allows movement in tangential direction.

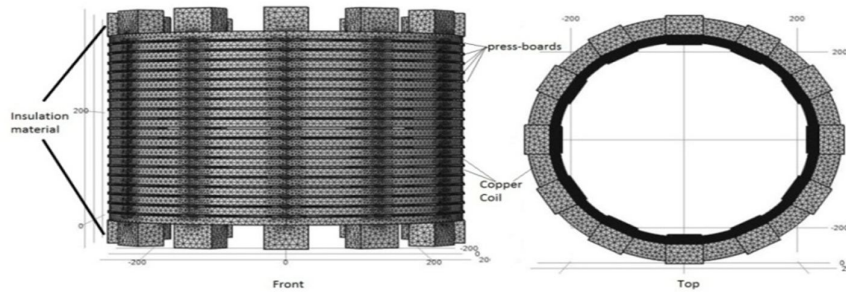


Figure 8.1 3D model of test winding

Above, shows the final meshed model for calculation. 160000 degree of freedom and 684958 mesh elements are introduced in computer model for more accurate results. However, computing time is way greater than two dimensional models. 3D model of test winding is shown in Figure 8.1. Table Error! No text of specified style in document.8.1 gives the Comparison of results.

Table Error! No text of specified style in document.8.1 Comparison of results

Mode	Computed	Experimental	Error in Percentage
1 <sup>st</sup> axial mode	383	380	0.7 %
2 <sup>nd</sup> axial mode	827	840	1.5 %
1 <sup>st</sup> radial mode	2445	2442	0.001 %

### IX. RESULTS AND DISCUSSION

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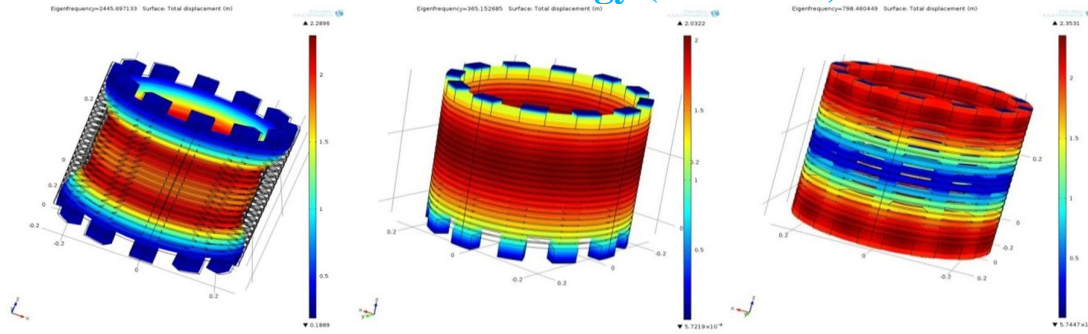


Figure 9.1 1st and 2nd axial and 1st radial mode of the winding

First three modes are the most important nodes in case of winding eigen frequency. Figure 9.1 shows 1st and 2nd axial and 1st radial mode of the winding. First mode gives the information about how much of a displacement a winding will have in case this mode is activated. Moreover, from the figure it is clear what first mode has maximum displacement in one direction only which suggest that this mode is asymmetric mode. But in case of 2<sup>nd</sup> mode displacement within the coil is symmetric along the axis, Maximum displacement is noticed at the end of the winding on both side. However, 3<sup>rd</sup> mode which is known as radial mode tends to gives more displacement in radial direction as shown in the figure. In order to find out the maximum displacement in the system for different eigen frequencies frequency sweep of the model with respect to displacement can be taken.

Comparisons of 2D and 3D computed model results with respect to experimental result. Table 8.1 shows the comparison of 2D and 3D models.

Table Error! No text of specified style in document.8.1 Comparison of 2D and 3D models

Mode	2D model	3D model	Experimental
1 <sup>st</sup> axial mode	344	383	380
2 <sup>nd</sup> axial mode	795	827	840
1 <sup>st</sup> radial mode	2347	2445	2442

By comparing the results obtained by 2D model and 3D model with respect to Experimental model it is easy to conclude that 3D model tends to give more accurate result as compare to 2D model. However, a price of high computing time has to be paid by user. It is suggested that 2D model should be used for checking the physic and boundary condition of the model. Once satisfied, 3D model approach can be considered for better results.

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