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# Remark on the Paper Entitled Lattice Points of a Cubic Diophantine Equation $11(x + y)^2 = 4(xy + 11z^3)$

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**Abstract:** In this paper, new sets of solutions to the cubic equation with three unknowns given by  $11(x + y)^2 = 4xy + 44z^3$  are presented.

**Keywords:** Ternary cubic, Integer solutions

## I. INTRODUCTION

When a search is made for cubic diophantine equations, the authors noticed a paper by Manju Somanath, J. Kannan, K. Raja [1] in which they have presented lattice points of the cubic diophantine equation  $11(x + y)^2 = 4xy + 44z^3$ . However, there are other interesting sets of solutions to the above equations that are exhibited in this paper.

## II. METHOD OF ANALYSIS

Consider the cubic equation with three unknowns given by

$$11(x + y)^2 = 4xy + 44z^3 \tag{1}$$

To start with, the substitution

$$y = (2k - 1)x \tag{2}$$

in (1) gives

$$(11k^2 - 2k + 1)x^2 = 11z^3$$

which is satisfied by

$$x = 121(11k^2 - 2k + 1)\alpha^3 \tag{3}$$

$$z = 11(11k^2 - 2k + 1)\alpha^2 \tag{4}$$

Note that (2) – (4) satisfies (1)

Again, the substitution

$$y = 2kx \tag{5}$$

in (1) leads to

$$(44k^2 + 36k + 11)x^2 = 44z^3$$

whose solutions are

$$x = 242(44k^2 + 36k + 11)\alpha^3 \tag{6}$$

$$z = 11(44k^2 + 36k + 11)\alpha^2 \tag{7}$$

Thus, (5)-(7) satisfy (1)

Further,

Introduction of the linear transformations

$$x = u + v, \quad y = u - v, \quad z = u \tag{8}$$

in (1) leads to

$$v^2 = u^2(11u - 10) \tag{9}$$

After performing some algebra, it is noted that (9) is satisfied by the following two choices of  $u$  and  $v$ :

$$1) \quad u = 11k^2 - 2k + 1, \quad v = (11k - 1)(11k^2 - 2k + 1)$$

$$2) \quad u = 11k^2 + 2k + 1, \quad v = (11k + 1)(11k^2 + 2k + 1)$$



In view of (8), the corresponding two sets of values to  $x, y, z$  satisfying (1) are represented below:

a) *Set 1:* Consider choice (i). The values of  $x, y, z$  are:

$$x = 11k(11k^2 - 2k + 1)$$

$$y = (2 - 11k)(11k^2 - 2k + 1)$$

$$z = 11k^2 - 2k + 1$$

b) *Set 2:* Consider choice (ii). The values of  $x, y, z$  are:

$$x = (11k + 2)(11k^2 + 2k + 1)$$

$$y = -11k(11k^2 + 2k + 1)$$

$$z = 11k^2 + 2k + 1$$

#### REFERENCE

- [1] Manju Somanath, J.Kannan, K.Raja, "Lattice Points Of A Cubic Diophantine Equation  $11(x + y)^2 = 4(xy + 11z^3)$ ", IJRASET, Volume 5, Issue 5, 1797-1800, 2017.



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