



# IJRASET

International Journal For Research in  
Applied Science and Engineering Technology



---

# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

---

**Volume: 6      Issue: XII      Month of publication: December 2018**

**DOI:**

**[www.ijraset.com](http://www.ijraset.com)**

**Call:  08813907089**

**E-mail ID: [ijraset@gmail.com](mailto:ijraset@gmail.com)**

# Boundary Value Problem of First Order Random Differential Inclusions

Biradar S.B.<sup>1</sup>, D. S. Palimkar<sup>2</sup>

<sup>1</sup>Department of Mathematics, Shri Madhavrao Patil College, Murum Tq.: Omerga Dist.:Usmanabad

<sup>2</sup>Department of Mathematics, Vasanttrao Naik College, Nanded MS

**Abstract:** In this paper, investigate the nonlinear boundary value problem of first order random differential inclusions and prove the existence results through classical fixed point theorem.

**Keywords:** Multivalued map, random differential inclusions, Nonlinear boundary conditions, Condensing map, random fixed point, Upper and lower random solutions. 2000 Mathematics Subject Classification: 60H25, 47H10, 34A60.

## I. DESCRIPTION OF THE PROBLEM

Consider the first order random differential inclusions with boundary conditions

$$y'(t, \omega) \in F(t, \omega, y(t, \omega)), \text{ for a.e. } t \in J = [0, T], \omega \in \Omega \quad (1.1)$$

$$L(y(0, \omega), y(T, \omega)) = 0$$

Where  $F : J \times \Omega \times R \rightarrow 2^R$  is a compact and convex valued multivalued map and  $L : R^2 \times \Omega \rightarrow R$  is a continuous single-valued map.

The method of upper and lower solutions has been successfully applied to study the existence of multiple solutions for initial and boundary value problems of first and second order differential inclusions. This method has been used in Bernfeld-Lakshmikantham [10], Heikkila-Lakshmikantham [8], Carl-Heikkila-Kumpulainen [5], Cabada [4], Frigon [7], Lakshmikantham-Leela [17], Nkashama [13].

The problem(1.1)is new to the differential inclusions. In this paper, we establish an existence result for (1.1) through the existence of upper and lower random solutions and random fixed point theorem for condensing maps of Martelli[19].

## II. AUXILIARY RESULTS

We will need some basic definitions and Lemmas from multivalued analysis.

1) *Definition 2.1.* A multivalued map  $F : J \times R \times \Omega \rightarrow 2^R$  is said to be an  $L^1$ - random caratheodory , if

- a)  $t \rightarrow F(t, y, \omega)$  is measurable for each  $y \in R, \omega \in \Omega$ .
- b)  $y \rightarrow F(t, y, \omega)$  is upper semicontinuous for almost all  $t \in J, \omega \in \Omega$ .
- c) For each  $k > 0$ , there exists  $h_k \in L^1(J, R_+, \Omega)$  such that

$$\|F(t, y, \omega)\| = \sup\{|v| : v \in F(t, y, \omega)\} \leq h_k(t, \omega) \text{ for all } |y| \leq k \text{ and for almost all } \omega \in \Omega, t \in J.$$

So let us start by defining what we mean by a solution of problem (1.1).

2) *Definition 2.2.* A function  $y \in AC(J, R, \Omega)$  is said to be a solution of (1.1) if there exists a function  $v \in L^1(J, R, \Omega)$  such that  $v(t, \omega) \in F(t, y(t, \omega), \omega)$  a.e. on  $J, y'(t, \omega) = v(t, \omega)$  a.e. on  $J, \omega \in \Omega$  and  $L(y(0, \omega), y(T, \omega)) = 0$ .

The following concept of lower and upper solutions for (1.1) has been introduced by Halidias and Papageorgiou in [14] for second order multivalued boundary value problems.

3) *Definition 2.3.* A function  $\alpha \in AC(J, R, \Omega)$  is said to be a lower solution of (1.1) if there exists  $v_1 \in L^1(J, R, \Omega)$  such that  $v_1(t, \omega) \in F(t, \alpha(t, \omega), \omega)$  on  $J, \alpha'(t, \omega) \leq v_1(t, \omega)$  a.e. on  $J, \omega \in \Omega$  and  $L(\alpha(0, \omega), \alpha(T, \omega)) \leq 0$ .

Similarly, a function  $\beta \in AC(J, R, \Omega)$  is said to be upper solution of (1.1) if there exists  $v_2 \in L^1(J, R, \Omega)$  such that  $v_2(t, \omega) \in F(t, \beta(t, \omega), \omega)$  a.e. on  $J$ ,  $\beta'(t, \omega) \geq v_2(t, \omega)$  a.e. on  $J$ ,  $\omega \in \Omega$  and  $L(\beta(0, \omega), \beta(T, \omega)) \geq 0$ .

For the multivalued map  $F$  and for each  $y \in C(J, R, \Omega)$  we define  $S \frac{1}{F, y}$  by

$$S \frac{1}{F, y} = \{v \in L^1(J, R, \Omega) : v(t, \omega) \in F(t, y(t, \omega), \omega) \text{ for a.e. } t \in J, \omega \in \Omega\}.$$

Our main result is based on the following :

a) *Lemma 2.1.* [18]. Let  $I$  be a compact real interval and  $X$  be a Banach space. Let  $F : I \times X \rightarrow CC(X); (t, y) \rightarrow F(t, y)$  measurable with respect to  $t$  for any  $y \in X$  and u.s.c. with respect to  $y$  for almost each  $t \in I$  and  $S \frac{1}{F, y} \neq \emptyset$  for any

$y \in C(I, X)$  and let  $\Gamma$  be a linear continuous mapping from  $L^1(I, X)$  to  $C(I, X)$  then the operator

$$\Gamma \circ S \frac{1}{F} : C(I, X) \rightarrow CC(C(I, X)), y \mapsto (\Gamma \circ S \frac{1}{F})(y) := \Gamma \left( S \frac{1}{F, y} \right)$$

is a closed graph operator in  $C(I, X) \times C(I, X)$ .

b) *Lemma 2.2.* [19] Let  $G : X \rightarrow CC(X)$  be an u.s.c. condensing map. If the set

$$M := \{v \in X : \lambda v \in G(v) \text{ for some } \lambda > 1\}$$

is bounded, then  $G$  has a fixed point.

### III. MAIN RESULT

We are now in a position to state and prove our existence result for the problem (1.1).

#### A. Theorem 3.1

Suppose  $F : J \times R \times \Omega \rightarrow CC(R)$  is an  $L^1$ -random Caratheodory multivalued map.

And assume the following conditions.

there exist  $\alpha$  and  $\beta$  in  $W^{1,1}(J, R, \Omega)$  lower and upper random solutions respectively for the problem (1.1) such that,

$$\alpha \leq \beta,$$

$L$  is a continuous single-valued map in  $(x, y) \in [a(0), \beta(0)] \times [a(T), \beta(T)]$  and nonincreasing in  $y \in [\alpha(T), \beta(T)]$

are satisfied. Then the problem (1.1) has at least one random solution  $y \in W^{1,1}(J, R, \Omega)$

such that

$$a(t, \omega) \leq y(t, \omega) \leq \beta(t, \omega) \text{ for all } t \in J, \omega \in \Omega.$$

Proof. Transform the problem into a fixed point problem. Consider the following modified problem

$$y'(t, \omega) + y(t, \omega) \in F_1(t, y(t, \omega), \omega), \text{ a.e. } t \in J. \tag{3.1}$$

$$y(0, \omega) = \tau((0, y(0, \omega), \omega) - L(\bar{y}(0, \omega), \bar{y}(T, \omega))) \tag{3.2}$$

Where  $\omega \in \Omega$ ,  $F_1(t, y, \omega) = F(t, \omega, \tau(t, y, \omega)) + \tau(t, y, \omega) = \max\{a(t, \omega), \min\{y, \beta(t, \omega)\}\}$  and

$$\bar{y}(t, \omega) = \tau(t, \omega, y(t, \omega)).$$

Remark(i) Notice that  $F_1$  is an  $L^1$ -random Caratheodory multivalued map with compact convex values and there exists  $\emptyset \in L^1(J, R_+, \Omega)$  such that

$$\| F_1(t, y(t, \omega), \omega) \| \leq \theta(t, \omega) + \max(\sup_{t \in J} |a(t, \omega)|, \sup_{t \in J} |\beta(t, \omega)|) \text{ and all } y \in C(J, R, \Omega).$$

By the definition of  $\tau$  it is clear that  $\alpha(0, \omega) \leq y(0, \omega) \leq \beta(0, \omega)$ .

Clearly a solution to (3.1) – (3.2) is a fixed point of the operator  $N : C(J, R, \Omega) \rightarrow 2^{C(J, R, \Omega)}$  defined by

$$N(y) := \left\{ h \in C(J, R, \Omega) : h(t, \omega) = y(0, \omega) + \int_0^t [v(s, \omega) + \bar{y}(s, \omega) - y(s, \omega)] ds, v \in S_{F, y}^{-1} \right\}$$

Where

$$S_{F, \bar{y}}^{-1} = \{v \in S_{F, \bar{y}}^1 : v(t, \omega) \geq v_1(t, \omega) \text{ a.e. on } A_1 \text{ and } v(t, \omega) \leq v_2(t, \omega) \text{ a.e. on } A_2\}$$

$$S_{F, \bar{y}}^1 = \{v \in L^1(J, R, \Omega) : v(t, \omega) \in F(t, \bar{y}(t, \omega), \omega) \text{ for a.e. } t \in J\},$$

$$A_1 = \{t \in J : y(t, \omega) < \alpha(t, \omega) \leq \beta(t, \omega)\}, A_2 = \{t \in J : a(t, \omega) \leq \beta(t, \omega) < y(t, \omega)\}.$$

Since (i) For each  $y \in C(J, R, \Omega)$ . the set  $S_{F, y}^1$  is nonempty..

For each  $y \in C(J, R, \Omega)$ . the set  $S_{F, \bar{y}}^1$  is nonempty. Indeed, by (i) there exists  $v \in S_{F, y}^1$ . Set

$$w = v_1 \chi_{A_1} + v_2 \chi_{A_2} + v \chi_{A_3},$$

Where

$$A_3 = \{t \in J : \alpha(t, \omega) \leq \beta(t, \omega)\}$$

Then by decomposability  $w \in S_{F, \bar{y}}^1$

We shall show that  $N$  is a completely continuous multivalued map, u.s.c. with convex closed values. The proof will be given in several steps.

1) Step 1:  $N(y)$  is convex for each  $y \in C(J, R, \Omega)$ .

Indeed, if  $h, \bar{h}$  belong to  $N(y)$ , then there exist  $v \in S_{F, \bar{y}}^1$  and  $\bar{v} \in S_{F, \bar{y}}^1$  such that

$$h(t, \omega) = y(0, \omega) + \int_0^t [v(s, \omega) + \bar{y}(s, \omega) - y(s, \omega)] ds, \quad t \in J, \omega \in \Omega$$

And

$$\bar{h}(t, \omega) = y(0, \omega) + \int_0^t [\bar{v}(s, \omega) + \bar{y}(s, \omega) - y(s, \omega)] ds, \quad t \in J, \omega \in \Omega.$$

Let  $0 \leq k \leq 1$ . Then for each  $t \in J, \omega \in \Omega$ . we have

$$[kh + (1-k)\bar{h}](t, \omega) = y(0, \omega) + \int_0^t [kv(s, \omega) + (1-k)\bar{v}(s, \omega) + \bar{y}(s, \omega) - y(s, \omega)] ds.$$

Since  $S_{F, \bar{y}}^1$  is convex (because  $F$  has convex values) then

$$kh + (1-k)\bar{h} \in G(y).$$

2) Step 2:  $N$  sends bounded sets into bounded sets in  $C(J, R, \Omega)$

Let  $B_r := \{y \in C(J, R, \Omega) : \|y\|_\infty \leq r\}$ , ( $\|y\|_\infty := \sup\{|y(t, \omega)| : t \in J, \omega \in \Omega\}$ ) be a bounded set in

$C(J, R, \Omega)$  and  $y \in B_r$ , then for each  $h \in N(y)$  there exists  $v \in S_{F, \bar{y}}^1$  such that

$$h(t, \omega) = y(0, \omega) + \int_0^t [v(s, \omega) + \bar{y}(s, \omega) - y(s, \omega)] ds, t \in J, \omega \in \Omega.$$

Thus for each  $t \in J$  we get

$$\begin{aligned} |h(t, \omega)| &\leq |y(0, \omega)| + \int_0^t [|v(s, \omega)| + |\bar{y}(s, \omega)| + |y(s, \omega)|] ds \\ &\leq \max(\alpha(0, \omega), \beta(0, \omega)) + \|\theta_r\|_{L^1} + T \max(\gamma, \sup_{t \in J} |\alpha(t, \omega)| \sup_{t \in J} |\beta(t, \omega)|) + Tr. \end{aligned}$$

3) Step 3:  $N$  sends bounded sets in  $C(J, R, \Omega)$  into equicontinuous sets.

Let  $u_1, u_2 \in J, u_1 < u_2, B_r := \{y \in C(J, R, \Omega) : \|y\|_\infty \leq r\}$  be a bounded set in  $C(J, R, \Omega)$  and

$y \in B_r$ . For each  $h \in N(y)$  there exists  $v \in S_{F, \bar{y}}^1$  such that

$$h(t, \omega) = y(0, \omega) + \int_0^t [v(s, \omega) + \bar{y}(s, \omega) - y(s, \omega)] ds, t \in J, \omega \in \Omega.$$

We then have

$$\begin{aligned} |h(u_2) - h(u_1)| &\leq \int_{u_1}^{u_2} [|v(s, \omega) + \bar{y}(s, \omega)| + |y(s, \omega)|] ds \\ &\leq \int_{u_1}^{u_2} |\theta_r(s, \omega)| ds + (u_2 - u_1) \max(r, \sup_{t \in J} |\alpha(t, \omega)|, \sup_{t \in J} |\beta(t, \omega)|) + r(u_2 - u_1) \end{aligned}$$

As a consequence of Step2, Step 3 together with the Ascoli-Arzela theorem we can conclude that  $N : C(J, R, \Omega) \rightarrow 2^{C(J, R, \Omega)}$  is a compact multivalued map, and therefore, a condensing map.

4) Step.4:  $N$  has a closed graph.

Let  $y_n \rightarrow y_0, h_n \in N(y_n)$  and  $h_n \rightarrow h_0$ . We shall prove that  $h_0 \in N(y_0)$ .

$h_n \in N(y_n)$  means that there exists  $v_n \in S_{F, \bar{y}_n}^1$  such that

$$h_n(t, \omega) = y(0, \omega) + \int_0^t [v_n(s, \omega) + \bar{y}_n(s, \omega) - y_n(s, \omega)] ds, t \in J, \omega \in \Omega.$$

We must prove that there exists  $v_0 \in S_{F, \bar{y}_0}^1$  such that

$$h_0(t, \omega) = y(0, \omega) + \int_0^t [v_0(s, \omega) + \bar{y}_0(s, \omega) - y_0(s, \omega)] ds, t \in J, \omega \in \Omega.$$

Consider the linear continuous operator  $\Gamma : L^1(J, R, \Omega) \rightarrow C(J, R, \Omega)$  defined by

$$(\Gamma v)(t, \omega) = \int_0^t v(s, \omega) ds.$$

We have

$$\left\| \left( h_n - y(0, \omega) - \int_0^t [\bar{y}_n(s, \omega) - y_n(s, \omega)] ds \right) - \left( h_0 - y(0, \omega) + \int_0^t [\bar{y}_0(s, \omega) - y_0(s, \omega)] ds \right) \right\|_\infty \rightarrow 0.$$



From Lemma 2.1, it follows that  $\Gamma \circ S_F^{-1}$  is a closed graph operator.

Also from the definition of  $\Gamma$  we have that

$$\left( h_n(t, \omega) - y(0, \omega) - \int_0^t [\bar{y}_n(s, \omega) - y_0(s, \omega)] ds \right) \in \Gamma \left( S_{F, \bar{y}_n}^{-1} \right)$$

Since  $y_n \rightarrow y_0$  it follows from Lemma 2.1 that

$$h_0(t, \omega) = y(0, \omega) + \int_0^t [v_0(s, \omega) + \bar{y}_0(s, \omega) - y_0(s, \omega)] ds, t \in J, \omega \in \Omega.$$

For some  $v_0 \in S_{F, \bar{y}_0}^{-1}$

Next we shall show that  $N$  has a fixed point, by proving that

5) Step 5: The set  $M := \{v \in C(J, R, \Omega) : \lambda v \in N(v) \text{ for some } \omega \in \Omega, \lambda > 1\}$

is bounded. Let  $y \in M$  then  $\lambda y \in N(y)$  for some  $\lambda > 1$ . Thus there exists  $v \in S_{F, \bar{y}}^{-1}$  such that

$$y(t, \omega) = \lambda^{-1} y(0, \omega) + \lambda^{-1} \int_0^t [v(s, \omega) + \bar{y}(s, \omega) - y(s, \omega)] ds, t \in J.$$

Thus

$$|y(t, \omega)| \leq |y(0, \omega)| + \int_0^t |v(s, \omega) + \bar{y}(s, \omega) - y(s, \omega)| ds, \omega \in \Omega, t \in J.$$

From the definition of  $\tau$  there exists  $\theta \in L^1(J, R^+, \Omega^+)$  such that

$$\|F(t, \bar{y}(t, \omega), \omega)\| = \sup\{|v| : v \in F(t, \bar{y}(t, \omega), \omega)\} \leq \theta(t, \omega) \text{ for each } y \in C(J, R, \Omega).$$

$$|y(t, \omega)| \leq \max(a(0, \omega), \beta(0, \omega)) + \|\theta\|_{L^1} + T \max(\sup_{t \in J} |\alpha(t, \omega)|, \sup_{t \in J} |\beta(t, \omega)|) + \int_0^t |y(s, \omega)| ds.$$

Set

$$z_0 = \max(a(0, \omega), \beta(0, \omega)) + \|\theta\|_{L^1} + T \max(\sup_{t \in J} |\alpha(t, \omega)|, \sup_{t \in J} |\beta(t, \omega)|).$$

Using the Gronwall's Lemma [11], we get for each  $t \in J, \omega \in \Omega$ .

$$\begin{aligned} |y(t, \omega)| &\leq z_0 + z_0 \int_0^t e^{t-s} ds \\ &\leq z_0 + z_0(e^t - 1). \end{aligned}$$

Thus

This shows that  $M$  is bounded.

Hence, Lemma 2.2 applies and  $N$  has a random fixed point which is a solution to problem (3.1)- (3.2).

6) Step 6: We shall show that the solution  $y$  of (3.1)-(3.2) satisfies

$$\alpha(t, \omega) \leq y(t, \omega) \leq \beta(t, \omega) \text{ for all } t \in J, \omega \in \Omega.$$

Let  $y$  be a solution to (3.1) – (3.2). We prove that

$$\alpha(t, \omega) \leq y(t, \omega) \text{ for all } t \in J, \omega \in \Omega.$$

Suppose not. Then there exist  $t_1, t_2 \in J, t_1 < t_2$  such that  $\alpha(t_1) = y(t_1)$  and

$$\alpha(t, \omega) > y(t, \omega) \text{ for all } t \in (t_1, t_2).$$

In view of the definition of  $\tau$  one has

$$y'(t, \omega) + y(t, \omega) \in F(t, \alpha(t, \omega), \omega) + \alpha(t, \omega) \text{ a.e. on } (t_1, t_2).$$

Thus there exists  $v(t, \omega) \in F(t, \alpha(t, \omega), \omega)$  a.e. on  $J$  with  $v(t, \omega) \geq v_1(t, \omega)$  a.e. on  $(t_1, t_2)$  such that

$$y'(t, \omega) + y(t, \omega) = v(t, \omega) + \alpha(t, \omega) \text{ a.e. on } (t_1, t_2).$$

An integration on  $(t_1, t)$ , with  $t \in (t_1, t_2)$  yields

$$\begin{aligned} y(t, \omega) - y(t_1, \omega) &= \int_{t_1}^t [v(s, \omega) + (\alpha - y)(s, \omega)] ds \\ &> \int_{t_1}^t v(s, \omega) ds. \end{aligned}$$

Since  $\alpha$  is a lower solution to (1.1), then

$$\alpha(t, \omega) - \alpha(t_1, \omega) \leq \int_{t_1}^t v_1(s, \omega) ds, \quad t \in (t_1, t_2), \omega \in \Omega.$$

It follows from the facts  $y(t_1, \omega), v(t, \omega) \geq v_1(t, \omega)$  that

$$\alpha(t, \omega) < y(t, \omega) \text{ for all } t \in (t_1, t_2), \omega \in \Omega.$$

Which is a contradiction, since  $y(t, \omega) < \alpha(t, \omega)$  for all  $t \in (t_1, t_2)$ . Consequently

$$\alpha(t, \omega) \leq y(t, \omega) \text{ for all } \omega \in \Omega, t \in J.$$

Analogously, we can prove that

$$y(t, \omega) \leq \beta(t, \omega) \text{ for all } \omega \in \Omega, t \in J.$$

This shows that the problem (3.1) – (3.2) has a random solution in the interval  $[\alpha, \beta]$ .

Finally, we prove that every solution of (3.1) – (3.2) is also a solution to (1.1). We only need to show that

$$\alpha(0, \omega) \leq y(0, \omega) - L(\bar{y}(0, \omega), \bar{y}(T, \omega)) \leq \beta(0, \omega).$$

Notice first that we can prove that

$$\alpha(T, \omega) \leq y(T, \omega) \leq \beta(T, \omega).$$

Suppose now that  $y(0, \omega) - L(\bar{y}(0, \omega), \bar{y}(T, \omega)) < \alpha(0, \omega)$ . Then  $y(0, \omega) = \alpha(0, \omega)$  and

$$y(0, \omega) - L(\alpha(0, \omega), \bar{y}(T, \omega)) < \alpha(0, \omega).$$

Since  $L$  is nonincreasing in  $y$ , we have

$$\alpha(0, \omega) \leq \alpha(0, \omega) - L(\alpha(0, \omega), \alpha(T, \omega)) \leq \alpha(0, \omega) - L(\alpha(0, \omega), \bar{y}(T, \omega)) < \alpha(0, \omega).$$

Which is a contradiction.

Analogously we can prove that

$$y(0, \omega) - L(\tau(0, \omega), \tau(T, \omega)) \leq \beta(0, \omega).$$

Then  $y$  is a solution to (1.1).

#### IV. CONCLUSION

The fixed point theorems are useful for proving the existence theorems as well as for characterizing the solutions of different types of functional random differential equations on bounded or unbounded intervals of real line. The choice of the fixed point theorems depends upon the situations and the circumstances of the nonlinearities involved the problems. The selection of the fixed point theorems yields very powerful existence results as well as different characterizations of the nonlinear function differential equations. In this article, considered the nonlinear boundary value problem of first order random differential inclusions and prove the existence results through classical fixed point theorem.

## REFERENCES

- [1] J.Banas, D. K.Goebel, Measures of Noncompactness in Banach Spaces, MarcelDekker, New York, 1980.
- [2] M. Benchora, S.K. Ntouyas, On second order differential inclusions with periodicboundary conditions, Acta Math, Univ. Comen. LXIX (2000), 173–181,
- [3] A.T. Bharucha-Reid, Random Integral Equations, Academic Press, New York, 1972 .
- [4] A. Cabada, The monotone method for first order problems with linear and nonlinear boundary conditions, Appl. Math. Comp., 63 (1994), 163 – 186.
- [5] S.Carl, S.Heikila, M. Kumpulainen, On solvability of first order discontinuous scalar differential equations, Nonlinear Times Digest,2(1) (1995), 11-27.
- [6] B. C. Dhage, S. K. Ntouyas, D. S. Palimkar, Monotone Increasing Multi-valued condensing Random Operator and Random differential Inclusion, Electronic Journal of Qualitative Theory of Differential Equations,2006,Vol.15,1-20
- [7] M. Frigon,D.O'Regan, Existence results for some initial and boundary value problems without growth restriction, Proc. Amer. Math. Soc., 123(1)(1995), 207-216.
- [8] S. Heikila , V. Lakshmikantham, Monotone Iterative Techniques for Discontinuous Nonlinear Differential Equations, Marcel Dekker,New York, 1994.
- [9] N. Halidas,N.S. Papageorgious, Second order multivalued boundary value problems, Arch. Math. (Brno), 34 (1998), 267-284.
- [10] V.Lakshmikantham ,S. Leela, Remarks on first and second order periodic boundary value problems, Nonlinear Anal., 8 (3) (1984),281- 287.
- [11] A. Lasota, Z.Opial, An application of the Kakutani-Ky-Fan theorem in the theory of ordinary differential equations, Bull. Acad Pol. Sci. Ser. Sci.Math. Astronom. Phys., 13 (1965), 781-786.
- [12] M. Martelli, A Rothe's type theorem for non compact acyclic-valued maps, Boll.Un. Mat. Ital., 11 (1975), 70-76
- [13] M.N. Nkashama, A generalized upper and lower solutions method and multiplicity results for nonlinear first order ordinary differential equations, J.Math.Anal. Appl., 140 (1989), 381-395.
- [14] D.S.Palimkar, Existence of solution for third order random differential inclusion, Journal of Global Research in Mathematical Archive,Vol.1(4),2013,40-49.
- [15] D.S. Palimkar, Existence theory for random non-convex differential inclusion, Differential Equations and ControlProcesses,No.2,2013,1-11
- [16] D.S. Palimkar,Some Studies in Random Differential Inclusions,LAP LAMBERT Academic Publishing Co., Saarbrücken, Germany.





10.22214/IJRASET



45.98



IMPACT FACTOR:  
7.129



IMPACT FACTOR:  
7.429



# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089  (24\*7 Support on Whatsapp)