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On New Functions in Soft Topological Space

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Abstract: Some newly functions like contra almost soft feebly regular irresolute, almost soft F.reg. irresolute are introduced and also further works are soft feebly regular set-connected with separation axioms in it.

Keywords : soft feebly open, soft feebly closed, soft feebly regular open, soft feebly regular closed, soft feebly regular T_1,weakly soft feebly Hausdrouff, ultra soft feebly regular Hausdroff space, soft feebly regular T_2.

I. PRELIMINARIES

1) *Definition 1.1 [5]:* Let X be an initial universe set and let E be the set of all possible parameters with respect to X. Let P(X) denote the power set of X. Let A be a nonempty subset of E. A pair (F,A) is called soft set over X, where F is a mapping given by F:A→P(X). A soft set (F,A) on the universe X is defined by the set of ordered pairs (F,A)={(x,f_A(x)):x$\tilde{\in}$E,f_A(x)$\tilde{\in}$P(X)} where f_A: E→P(X) such that f_A(x)= φ if x∉A. Here f_A is called an approximate function of the soft set (F,A). The collection of soft set (F,A) over a universe X and the parameter set A is a family of soft sets denoted by $SS(x)_A$.
2) *Definition 1.2[4]:* A set set (F,A) over X is said to be null soft set denoted by φ if for all e$\tilde{\in}$A, F(e) = φ. A soft set (F,A) over X is said to be an absolute soft set denoted by A if all e$\tilde{\in}$A, F(e)=X.
3) *Definition 1.3[6]:* Let Y be a nonempty subset of X, then Y denotes the soft set (Y,E) over X for which Y(e)=Y, for all e$\tilde{\in}$E. In particular, (X,E) will be denoted by X.
4) *Definition 1.4 [6]:* Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if (i)φ,X$\tilde{\in}$τ(ii)If (F,E),(G,E)$\tilde{\in}$τ then (F,E)$\tilde{\cap}$(G,E)$\tilde{\in}$τ(iii) If { $(F_i,E)\}_{i\in I}$$\tilde{\in}$τ then $\tilde{\bigcup}_{i\in I}$$(F_i,E)$ $\tilde{\in}$τ. The pair (X,τ,E) is called a soft topological space. Every member of τ is called a soft open set. A soft set (F,E) is called soft closed in X if $(F,E)^c$$\tilde{\in}$τ.
5) *Definition 1.5:* Let (X,τ,E) be a soft topological space over X and let (A,E) be a soft set over X (i) the soft interior[8] *of* (A,E) is the soft set $\widetilde{int}$(A,E)=$\tilde{\cup}${(O,E):(O,E) which is soft open and(O,E)$\tilde{\subseteq}$(A,E)} (ii) the soft closure[6] *of* (A,E) is the soft set $\widetilde{cl}$(A,E) =$\tilde{\cap}${ (F,E) : (F,E) which is soft closed and (A,E) $\tilde{\subseteq}$ (F,E)}. Clearly $\widetilde{cl}$(A,E) is the smallest soft closed set over X which contains (A,E) and $\widetilde{int}$(A,E) is the largest soft open set over X which is contained in (A,E).
6) *Definition 1.6 [3]:* In a soft topological space (X,τ,E), a soft set
a) (A,E) is said to be soft feebly-open set if (A,E) $\tilde{\subseteq}$ s $\widetilde{cl}(\widetilde{int}$(A,E)).
b) (A,E) is said to be soft feebly-closed set if s $\widetilde{int}(\widetilde{cl}$(A,E))$\tilde{\subseteq}$(A,E).

It is said to be soft feebly-clopen if it is both soft feebly-open and soft feebly-closed.

7) *Definition 1.7 [3]:* Let (X,τ,E) be a soft topological spaces and let (A,E) be a soft set over X.
a) Soft feebly-closure of a soft set (A,E) in X is denoted by $f\widetilde{cl}$(A,E) = $\tilde{\cap}${(F,E): (F,E) which is a soft feebly-closed set and (A,E)$\tilde{\subseteq}$(F,E)}.
b) Soft feebly-interior of a soft set (A,E) in X is denoted by $f\widetilde{int}$(A,E) = $\tilde{\cup}${(O,E) : (O,E) which is a soft feebly-open set and (O,E)$\tilde{\subseteq}$(A,E)}. Clearly $f\widetilde{cl}$(A,E) is the smallest soft feebly-closed set over X which contains (A,E) and $f\widetilde{int}$(A,E) is the largest soft feebly-open set over X which is contained in (A,E).
8) *Definition 1.8 ([5],[6],[1],[7]):* For a soft (F,E) over the universe U, the relative complement of (F,E) is denoted by (F,E)′ and is defined by (F,E)′ = (F′,E), where (F′,E), where F′ : E→P(U) is a mapping defined by F′(e) = U – F(e) for all e$\tilde{\in}$ E.
9) *Definition 1.9 ([2]):* A subset (A,E) of soft topological space (X,τ, E) is said to be soft feebly regular open (briefly soft F.reg.open) if (A,E) = $f\widetilde{int}(f\widetilde{cl}$(A,E)) where soft feebly interior and soft feebly closure are denoted by $f\widetilde{int}$ and $f\widetilde{cl}$. Here, always soft feebly regular open set is analyzed in the way if (A,E) is both soft feebly open and soft feebly closed.
10) *Definition 1.10 ([2]):* A subset (A,E) of soft topological space (X,τ, E) is said to be soft feebly regularly closed if (A,E) = $f\widetilde{cl}(f\widetilde{int}$(A,E)) (briefly soft F.reg.closed).

11) *Definition 1.11 ([2]):* A subset (A,E) of soft topological space (X,τ, E) is said to be soft feebly regular clopen if (A,E)= $f\widetilde{int}(f\widetilde{cl}(f\widetilde{int}$ (A,E))). On the other hand, if and only if (A,E) is soft F.reg.open and soft F.reg.closed.
12) *Definition 1.12 ([2]):* Let (A,E) be subset of soft topological space (X,τ, E). The soft feebly regular closure of (A,E) (briefly soft F.reg.$\widetilde{cl}$(A,E)) is the intersection of all soft feebly regular closed set containing (A,E) and the soft feebly regular interior of (A,E) (briefly soft F.reg. $\widetilde{int}$(A,E)) is the union of all soft feebly regular open sets contained in (A,E). The complement of soft feebly regular open set is soft feebly regular closed.
13) *Definition 1.13 ([2]):*. A function f: (X,τ, E) →(Y,τ, E) is said to be soft F.reg.open (resp. soft F.reg.closed) if the image of every soft open set (soft closed set) in X is soft F.reg.open (soft F.reg.closed) in Y.

II. SOFT FEEBLY REGULAR SET-CONNECTED FUNCTION

1) *Definition 2.1:* A function f: (X,τ, E) →(Y,τ, E) is said to be soft set- connected if f^{-1}(V,E) is soft clopen in X for every (V,E) is soft clopen subset of Y.
2) *Definition 2.2:* A function f: (X,τ, E) →(Y,τ, E) is said to be soft feebly regular set- connected (abbr. soft F.reg.set-connected) if f^{-1}(V,E) is soft clopen in X for every (V,E) is soft F.reg.clopen(Y).
3) *Theorem 2.3:* Let (X,τ,E) and (Y,τ,E) be soft topological space f: (X,τ, E) →(Y,τ, E)
 a) f is soft F.reg.set-connected
 b) $f^{-1}(f\widetilde{int}(f\widetilde{cl}$(G,E))) is soft clopen for every soft F.reg.open subset (G,E) of Y.
 i) *Proof:*
 ii) (i)⇒(ii) Let (G,E) be any soft F.reg.open subset of Y. Since $f\widetilde{int}(f\widetilde{cl}$(G,E)) is soft F.reg.open,by (i) it follows that $f^{-1}(f\widetilde{int}(f\widetilde{cl}$(G,E))) is soft clopen.

(ii)⇒(i) Let (V,E) be soft F.reg.open in Y. By (ii) $f^{-1}(f\widetilde{int}(f\widetilde{cl}$(V,E))) is soft clopen in Xand hence f is soft F.reg.set-connected.

4) *Remark 2.4*
 a) A subset (A,E) of soft topological space (X,τ, E) is said to be soft clopenif (A,E) is soft open and soft closed and .
 b) A soft topological space X is said to be soft clopenT_1 space if for any pair of distinct points x and y, there exists the soft clopen sets (G,E) and (H,E) such that x$\widetilde{\in}$ (G,E), y∉(G,E) and x∉(H,E), y$\widetilde{\in}$(H,E).
5) *Theorem 2.5:* If f: (X,τ, E) →(Y,τ, E) is soft F.reg.set-connected function and (A,E) is any subset of X, then the restriction f/(A,E) : (A,E)→ (Y,τ, E) is soft F.reg.set-connected function.
 a) *Proof:* Let (V,E) be a soft F.reg.open set in Y. By hypothesis f^{-1}(V,E) is soft clopen in X. We have f^{-1}(V,E)$\widetilde{\cap}$ (A,E)=$(f/(A,E))^{-1}$(V,E) is soft clopen in (A,E). Hence f/(A,E) is soft F.reg.set-connected function.
6) *Theorem 2.6:* Let f: (X,τ, E) →(Y,τ, E) be soft set-connected and g : (Y, τ,E)→(Z, τ,E) be soft F.reg.set- connected . Then g ∘f: (X, τ,E)→(Z,τ,E) is soft F.reg.set-connected function.
 a) *Proof:* Let(V,E) be soft F.reg.open in Z. Since g is soft F.reg.set-connected, g^{-1}(V,E) is soft clopen in Y. Since f is soft set-connected, $f^{-1}(g^{-1}$(V,E)) is soft clopen in X.Hence g ∘f is soft F.reg.set- connected.
7) *Theorem 2.7:* If f: (X,τ, E) →(Y,τ, E) is a surjective soft F.reg.open and soft F.reg.closed function and g:(Y,τ, E) →(Z,τ, E) is a function such that g ∘f: (X, τ,E)→(Z, τ,E) is soft F.reg.set-connected, then g is soft F.reg.set- connected.
 a) *Proof:* Let (V,E) be soft F.reg.open in Z.$(g∘f)^{-1}$(V,E) is soft clopen in X. That is $f^{-1}(g^{-1}$(V,E)) is soft clopen in X. Since f is surjective soft F.reg.open and soft F.reg.closed, $f(f^{-1}(g^{-1}$(V,E)))=g^{-1}(V,E) is soft clopen.Therefore g is soft F.reg.set-connected.
8) *Definition 2.8:* A function f: (X,τ, E) →(Y,τ, E) is said to be
 a) Soft F.reg.irresolute if the inverse image of every soft F.reg.open set in Y is soft F.reg.open in X.
 b) Contrasoft F.reg.irresolute if the inverse image of every soft F.reg.open set in Y is soft F.reg.closed in X.
 c) Almostsoft F.reg.irresolute if the inverse image of soft F.reg.open in Y is soft feebly open set in X.
 d) Contra almost soft F.reg.irresolute continuous if the inverse image soft F.reg.open in Y is soft feebly closed in X.
9) *Theorem 2.9:* Every contra soft F.reg.irresolute is contra almost soft F.reg.irresolute function.
 a) *Proof:* Suppose f: (X,τ, E) →(Y,τ, E) is a contra soft F.reg.irresolute function and (A,E) be any soft F.reg.open set in Y then f^{-1}(A,E) is soft F.reg.closed in X. Thus the inverse image of each soft F.reg.open set in Y is soft feebly closed in X. Therefore f is contra almost soft F.reg. irresolute function.

10) *Theorem 2.10:* The followings are equivalent for a function f: (X,τ, E) →(Y,τ, E)

a) F is contra almost soft F.reg.irresolute for every soft F.reg.closed set (F,E) of Y, f^{-1}(F,E) is soft feebly open set of X.
b) For each x∈X and each soft F.reg.closed set (F,E) of Y containing f(x),there exists soft feebly open set (U,E) containing x such that f(U,E) $\widetilde{\subseteq}$(F,E).
c) For each x∈X and each soft F.reg.open set (V,E) of Y not containing f(x), there exists feebly closed set K not containing x such that f^{-1}(V,E)$\widetilde{\subseteq}$ (K,E).

i) *Proof:* (1)⇒ (2): Let (F,E) be a soft F.reg.closed set in Y, then Y-(F,E) is a soft F.reg.open set in Y. By (1), f^{-1}(Y-(F,E)) = X – f^{-1}(F,E) is soft feebly closed set in X. This implies f^{-1}(F,E) is soft feebly open set in X.

(2)⇒ (1) : Let (G,E) be a soft F.reg.open set of Y, then Y-(G,E) is a soft F.reg.closed set in Y. By (2), f^{-1}(Y-(G,E)) is soft feebly open set in X. This implies X-f^{-1}(G,E) is soft feebly open set in X, which implies f^{-1}(G,E) is feebly closed set in X. Therefore, (1) holds.

(2) ⇒ (3) : Let (F,E) be a soft F.reg.closed set in Y containing f(x), which impliesx$\widetilde{\in}$$f^{-1}$(F,E). By (2), f^{-1}(F,E) is soft feebly open in X containing x. Set (U,E) = f^{-1}(F,E), which implies (U,E) is feebly open in X containing x and f(U,E) = f(f^{-1}(F,E))$\widetilde{\subseteq}$ (F,E). Therefore (3) holds.

(3)⇒ (2) : Let (F,E) be a soft F.reg.closed set in Y containing f(x), which implies x$\widetilde{\in}$$f^{-1}$(F,E). From (3), there exists feebly open $(U,E)_x$ in X containing x such that f($(U,E)_x$)$\widetilde{\subseteq}$ (F,E).That is $(U,E)_x$$\widetilde{\subseteq}$$f^{-1}$(F,E). Thus f^{-1}(F,E) = $\widetilde{\cup}$ {$(U,E)_x$: x$\widetilde{\in}$$f^{-1}$(F,E)}, which is the union of soft feebly open sets. Therefore, f^{-1}(F,E) is soft feebly open set of X.

(3) ⇒ (4) : Let (V,E) be a soft F.reg.open set in Y not containing f(x). Then Y-(V,E) is a soft F.reg.closed set in Y containing f(x). From (3), there exists a feebly open set (U,E) in X containing x such that f(U,E)$\widetilde{\subseteq}$Y-(V,E). This implies (U,E)$\widetilde{\subseteq}$$f^{-1}$(Y-(V,E)) = X-$f^{-1}$(V,E).

(4)⇒ (3) : Let (F,E) be a soft F.reg.closed set in Y containing f(x). Then Y-(F,E) is a soft F.reg.open set in Y not containing f(x). From (4), there exists soft feebly closed set (K,E) in X not containing x such that f^{-1}(Y-(F,E))$\widetilde{\subseteq}$ (K,E).This implies X-f^{-1}(F,E)$\widetilde{\subseteq}$ (K,E). Hence, X-(K,E)$\widetilde{\subseteq}$$f^{-1}$(F,E), that is f(X-(K,E))$\widetilde{\subseteq}$ (F,E).Set (U,E) = X-(K,E), then (U,E) is soft feebly open set containing x in X such that f(U,E)⊂(F,E).

11) *Theorem 2.11:* The following are equivalent for a function f: (X,τ,E) →(Y,τ,E) f is contra almost soft F.reg.irresolute.

a) f^{-1}(soft F.reg.$\widetilde{int}$(softF.reg.$\widetilde{cl}$(G,E))) issoft feebly closed set in X for every soft F.reg.opensubset (G,E) of Y.
b) f^{-1}(soft F.reg.$\widetilde{cl}$(F.reg.$\widetilde{int}$(F,E))) is soft feebly open set in X for every soft F.reg.closedsubset (F,E) of Y.

i) *Proof:* (1)⇒ (2) : Let (G,E) be a soft F.reg.open set in Y.Then soft F.reg.$\widetilde{int}$(soft F.reg.$\widetilde{cl}$(G,E)) issoft F.reg.open set in Y. By (1) f^{-1}(soft F.reg.$\widetilde{int}$(soft F.reg.$\widetilde{cl}$(G,E))) $belongs\ to$ soft feebly closed set of X.

(2) ⇒ (1) : Obvious.

(1) ⇒ (3) : Let (F,E) be a soft F.reg.closed in Y. Then soft F.reg.$\widetilde{cl}$(softF.reg$\widetilde{int}$(G,E)) is soft F.reg.closed set in Y. By (1), f^{-1}(soft F.reg.$\widetilde{cl}$(soft F.reg $\widetilde{int}$ (G,E))) $belongs\ to\ soft\ f$eebly open set of X.

(3) ⇒ (1) : Obvious.

III. SEPARATION AXIOMSIN SOFT TOPOLOGICAL SPACE

1) *Definition 3.1:* A soft topological space (X, τ,,E) is said to be soft feebly regular T_1 (brieflysoft F.reg.T_1) space if for any pair of distinct points x and y, there exists the soft F.reg.open sets (G,E) and (H,E) such that x$\widetilde{\in}$ (G,E), y$\notin$(G,E) and x$\notin$(H,E), y$\widetilde{\in}$(H,E).
2) *Definition 3.2:* A space X is said to be weakly soft feebly Hausdrouffif each elements of X is an intersection of soft F.reg.closed sets.
3) *Definition 3.3:* A soft topological space X is called ultra softF.reg.Hausdroff space, if for every pair of disjoint points x and y in X, there exist disjoint soft F.reg.clopen sets (U,E) and (V,E) in X containing x and y, respectively.
4) *Definition 3.4:* Asoft topological space X is said to be soft feebly regular T_2 (briefly soft F.reg.T_2) space if for any pair of disjoint points x and y, there exists disjoint soft F.reg.open sets (G,E) and (H,E) such that x$\widetilde{\in}$ (G,E) and y$\widetilde{\in}$ (H,E).
5) *Theorem 3.5:* If f: (X,τ, E) →(Y,τ, E) is a contra almost soft F.reg. irresolute injection and Y is weakly soft feebly Hausdorff then X is soft F.reg.T_1.

a) *Proof:* Suppose Y is weakly soft feebly Hausdorff. For any distinct points x and y in X, there exist soft F.reg.closed sets (V,E) and (W,E) in Y such that f(x)$\tilde{\in}$ (V,E), f(y)$\tilde{\notin}$(V,E), f(y)$\tilde{\in}$ (W,E) and f(x)$\tilde{\notin}$(W,E). Since f is contra almost soft F.reg.irresolute,f^{-1}(V,E) and f^{-1}(W,E) are soft F.reg.open subsets of X such that x$\tilde{\in}f^{-1}$(V,E), y$\tilde{\notin}f^{-1}$(V,E), y$\tilde{\in}f^{-1}$(W,E) and x$\tilde{\notin}f^{-1}$(W,E). This shows that X is soft F.reg.T_1.

6) *Theorem 3.6:* If f: (X,τ, E) →(Y,τ, E) is a soft F.reg.set-connected injection and Y is soft F.reg.T_1, then X is soft clopen T_1.

a) *Proof :* Since Y is soft F.reg.T_1 for any disjoint points x and y in X, there exist (V,E), (W,E) are soft F.reg.open(Y) such that f(x)$\tilde{\in}$(V,E), f(y)$\tilde{\notin}$(V,E), f(x)$\tilde{\notin}$(W,E), f(y)$\tilde{\in}$(W,E). Since f is soft F.reg.set-connected, f^{-1}(V,E) and f^{-1}(W,E) are soft clopen in X. Furthermore y$\tilde{\notin}f^{-1}$(V,E) and x$\tilde{\notin}f^{-1}$(W,E). This shows that X is soft clopen T_1.

7) *Theorem 3.8:* If f : (X, τ,E)→(Y, τ,E) and g: (X, τ,E)→(Y, τ,E) be soft F.reg. set-connected function and Y is soft F.reg. Hausdorff, then (F,E)={x∈X : f(x)=g(x)}is soft F.reg.closed in X}.

a) *Proof:* If x$\tilde{\in}$X-(F,E) then it follows that f(x)≠g(x). Since Y is soft F.reg.Hausdorff, there exist soft F.reg.open sets (V,E) and (W,E) such that f(x)$\tilde{\in}$(V,E), g(x)$\tilde{\in}$(W,E) and (V,E)$\tilde{\cap}$ (W,E)≠φ. Since f and g are soft F.reg.set-connected, $f^{-1}(f\,\widetilde{int}\,(f\,\widetilde{cl}(V,E)))$ and $g^{-1}(f\,\widetilde{int}\,(f\,\widetilde{cl}(W,E)))$ are soft clopen in X with x$\tilde{\in}f^{-1}(f\,\widetilde{int}(f\,\widetilde{cl}(V,E)))$ and x$\tilde{\in}g^{-1}(f\,\widetilde{int}\,(f\,\widetilde{cl}(W,E)))$.

8) *Theorem 3.9:* Iff : (X, τ,E)→(Y, τ,E) is a contra almost soft F.reg. irresolute injective function and Y is ultra soft F.reg. Hausdroff space , then X is soft F.reg.T_2.

a) *Proof:* Let x and y be any two distinct points in X. Since f is injective f(x) ≠ f(y) and Y is ultra softF.reg.Hausdroff space, there exist disjoint soft F.reg.clopen sets (U,E) and (V,E) of Y containing f(x) and f(y), respectively. Then x∈f^{-1}(U,E) and y∈f^{-1}(V,E), where f^{-1}(U,E) and f^{-1}(V,E) are disjoint soft feebly open sets in X. Therefore X is soft F.reg.T_2.

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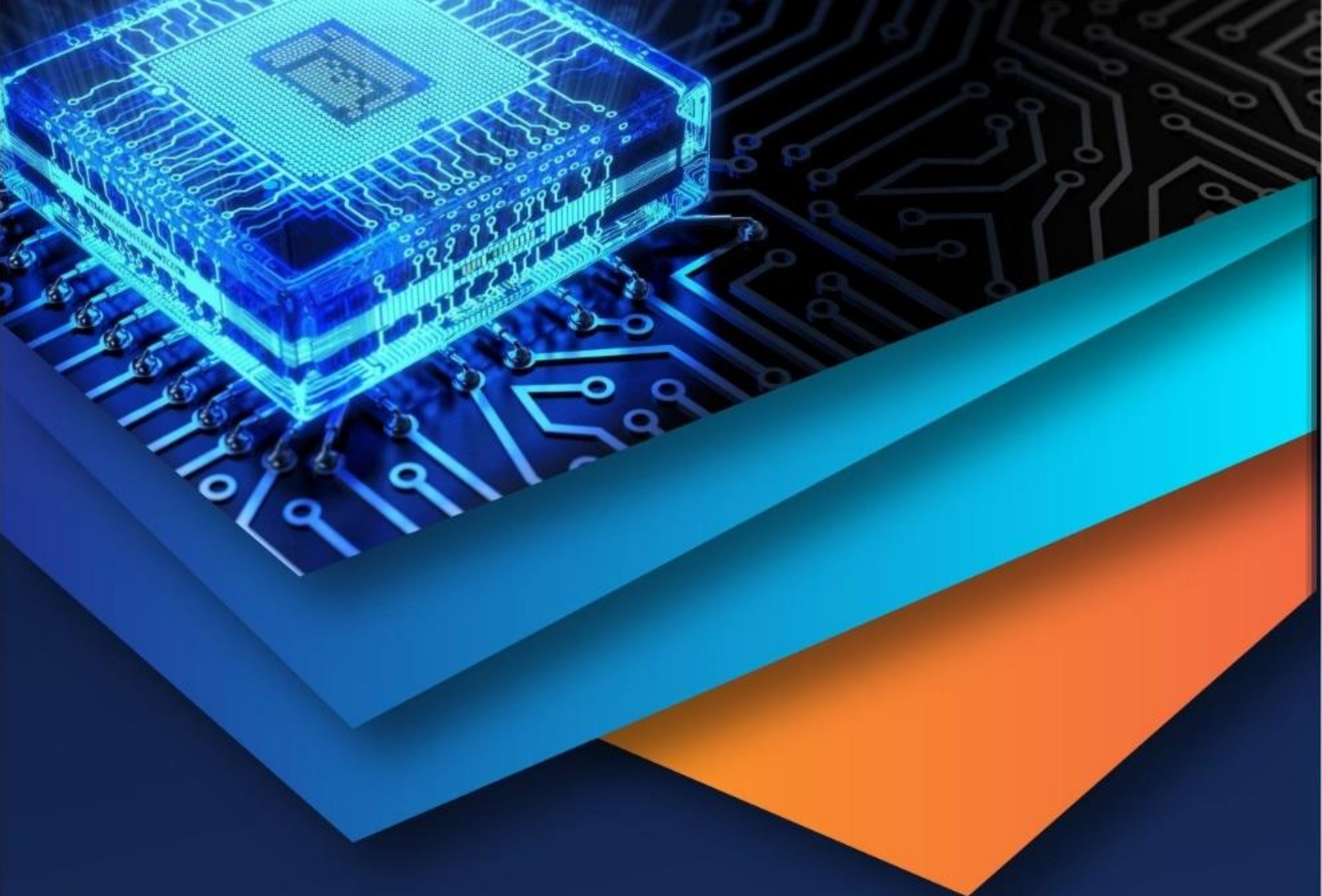

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