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Bernoulli's Differential Equation for Linear and Non-Linear Equation with Fluid Flow

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Abstract: The Bernoulli equation is an approximate relation between pressure, velocity and elevation. Bernoulli's principle can be applied to various types of fluid flow, resulting in various forms of Bernoulli equation. The simple form of Bernoulli's equation is valid for incompressible flows. "Bernoulli's differential equation for linear and non-linear equation with fluid flow", in this paper we discussed about how the fluid flow will be in case of both linear and non-linear equation. For that, we are solving first order non-linear Bernoulli differential equation to the linear form, and also solving second order differential equation. From that we are concluded the fluid flow.
Index Terms: Bernoulli's differential equation, change of variable, integrating factor, steady flow, turbulence flow.

I. INTRODUCTION

Fluid dynamics is "the branch of applied science that is concerned with the movement of liquids and gases". Fluid dynamics is one of two branches of fluid mechanics, which is the study of fluids and how forces affect them. Bernoulli's principle can be derived from the principle of conservation of energy. This states that, in a steady flow, the sum of all forms of energy in a fluid along a streamline is the same at all points on that streamline. This requires that the sum of kinetic energy, potential energy and internal energy remains constant. Bernoulli's principle can also be derived directly from Isaac Newton's Second Law of Motion. If a small volume of fluid is flowing horizontally from a region of high pressure to a region of low pressure, then there is more pressure behind than in front. This gives a net force on the volume, accelerating it along the streamline. Hear we are using differential equation to solve Bernoulli's linear and non-linear equation to determine the flow type.

A. Bernoulli Differential Equation

The Bernoulli differential equation is an equation of the form $y' + p(x)y = q(x)y^{\alpha}$. Where p(x) and q(x) are continuous functions on the interval and α is a real number. This is a first order non-linear differential equation .The Bernoulli equation was one of the first differential equations to be solved, and is still one of very few non-linear differential equations that can be solved explicitly. Most other such equations either have no solutions, or solutions that cannot be written in a closed form, but the Bernoulli equation is an exception

B. Derivation Of First Order And Second Order Linear And Non-Linear Differential Equation

A generalized form of the linear equation that can be solved, is the Bernoulli differential equation

$$y' + p(x)y = q(x)y^{\alpha}$$

1) When α =0, the Bernoulli equation becomes $y' + p(x)y = q(x)$ which is linear.
2) When α=1, the Bernoulli equation becomes $y' + p(x)y = q(x)y$ that is $y' + [p(x) - q(x)y] = 0$, which is both separable and linear.
3) When $\alpha \neq$0 and $\alpha \neq$1, the Bernoulli equation is non-linear.

a) *Case:1*

The Bernoulli equation $y' + p(x)y = q(x)y^{\alpha}$ can be solved by the change of variable v=$y^{1-\alpha}$

Proof

When α =0 and α =1, the given equation becomes linear and it is easy to solve.

v=$y^{1-\alpha}$

$v' = (1-\alpha)y^{-\alpha}\, y'$

=(1-α)$y^{-\alpha}[-p(x)y + q(x)\, y^{\alpha}]$

=(1-α)(-p$y^{1-\alpha}$ +q)
=(1-α)[-p(x)v+q(x)]
The given equation turns to be

$$v' + (1-\alpha)p(x)v = (1-\alpha)q(x) \quad \text{-------(1)}$$

Which is linear.
Introducing integrating factor,

$$I(x) = e^{(1-\alpha)\int p(x)dx}$$

Multiplying the given equation by the integrating factor I(x), we get

$$I(x)v' + I(x)(1-\alpha)p(x)v = I(x)(1-\alpha)q(x)$$

We write,

$$Iv' + I(1-\alpha)pv = I(1-\alpha)q$$
$$Iv' = I(1-\alpha)q$$
$$Iv = (1-\alpha)\int q(I)dx$$

Since,

$$V=\frac{1-\alpha}{I}\int q(I)dx$$
$$=(1-\alpha)e^{-(1-\alpha)\int p(x)dx}\int(qe^{(1-\alpha)\int p(x)dx})dx$$

Express it in terms of y by using v=$y^{1-\alpha}$.
(ie) $y^{1-\alpha}$=(1-α) $e^{-(1-\alpha)\int p(x)dx}\int qe^{(1-\alpha)\int(p(x)dx}$dx
y={(1-α) $e^{-(1-\alpha)\int p(x)dx}\int(qe^{(1-\alpha)\int(p(x)dx})\,dx\}^{1\backslash 1-\alpha}$
From the solution of the linear equation is approximately have a same interval value in each substitution. Hence, from this the velocity is constant at any point in the tube or channel. Since the fluid is steady. One result of laminar flow is that the velocity of the fluid is constant at any point in the fluid. so, the flow is laminar also. In this case, the fluid is essentially incompressible and that the flow is steady.

b) Case:2

When $\alpha \neq 0$ and $\alpha \neq 1$, the Bernoulli equation is non-linear.

Proof:

$$y' + p(x)y = q(x)y^2 \ \ where\ \alpha = 2$$

Differentiate with respect to x,

$$y'' + py' + yp' = q(2y)y' + y^2q'$$
$$y'' + py' + yp' - 2qyy' - q'yy^2 = 0$$
$$y'' + (p-2qy)y' + (p'-q'y)y = 0$$
$$D^2 + (p-2qy)D + (p'-q'y)y = 0$$

Introduce e^{2-x} as a constant term of the above equation

Hence the equation becomes,

$$D^2 + (p-2qy)D + (p'-q'y)y = e^{2-x}$$
$$a = 1; \quad b = p-2qy; \quad c = (p'-q'y)y$$
$$y = \frac{(-b)\pm\sqrt{(b^2-4ac)}}{2a}$$
$$y = -(p-2qy) \pm \frac{\sqrt{(p-2qy)^{\,2} - 4(p'-q'y)y}}{2}$$

Squaring on both sides,

$$y^2 = \frac{(p-2qy)^2 \pm (p-2qy)^2 - 4(p'-q'y)y}{4}$$
$$y^2 = \frac{(p^2 + 4q^2y^2 - 4pqy) \pm (p^2 + 4q^2y^2 - 4pqy) - 4yp' + 4q'y^2}{4}$$

For the plus term we have,

$$y^2 = \frac{(p^2 + \sqrt{2}q^2y^2 - \sqrt{2}pqy - 2yp' + 2y^2q')}{2}$$

$$y = \pm\frac{\sqrt{p^2 + \sqrt{2}qy(qy - p) - 2y(p' + yq')}}{2}$$

For negative term we have,

$$y^2 = \frac{(-4yp' + 4y^2q')}{4}$$

$$y^2 = q'^{y^2} - yp'$$

$$y = \pm\sqrt{y(q'y - p')}$$

(For an easiest simplification take negative term value)

The roots are real and distinct,

C.F=$Ae^{\sqrt{y(q'y-p')}\,x} + Be^{\sqrt{y(q'y-p')}\,x}$

$$P.I = \frac{1}{D^2+(p-2qy)D+(p'-q'y)y}e^{2-x}$$

Substitute $D^2 = -a^2$ and differentiate with respect to x

$$= \frac{1}{-(2-x)^2 + (p' - 2qy' - 2yq' + p'y - q'y^2)}e^{2-x}$$

$$= \frac{e^{2-x}}{p'(1+y) - q'(2+y) - (x^2 - 4x + 2qy' + 4)}$$

simplify the equation we get,

$$= \frac{e^{2-x}}{(1+y)(p'+q') - (x-2)^2 - q' + 2qy'}$$

hence the solution is,

$$y = C.F + P.I$$

$$y = Ae^{\sqrt{y(q'y-p')}\,x} + Be^{\sqrt{y(q'y-p')}\,x} + \frac{e^{2-x}}{(1+y)(p'+q')-(x-2)^2-q'+2qy'}$$

From the solution of the non-linear equation is approximately have a different interval value in each substitution. Hence, the velocity is different for the fluid. From this fluid flow is turbulent. If velocity vectors components of fluid elements are not the functions of the time, the flow is called non-linear. Above a certain speed, the flow becomes turbulent.

II. CONCLUSION

Thus in this paper we have discussed about the Bernoulli differential equation for the fluid flow in case of both linear and non-linear differential equation and we conclude that the flow is steady and laminar for linear and the flow is turbulent for non-linear.

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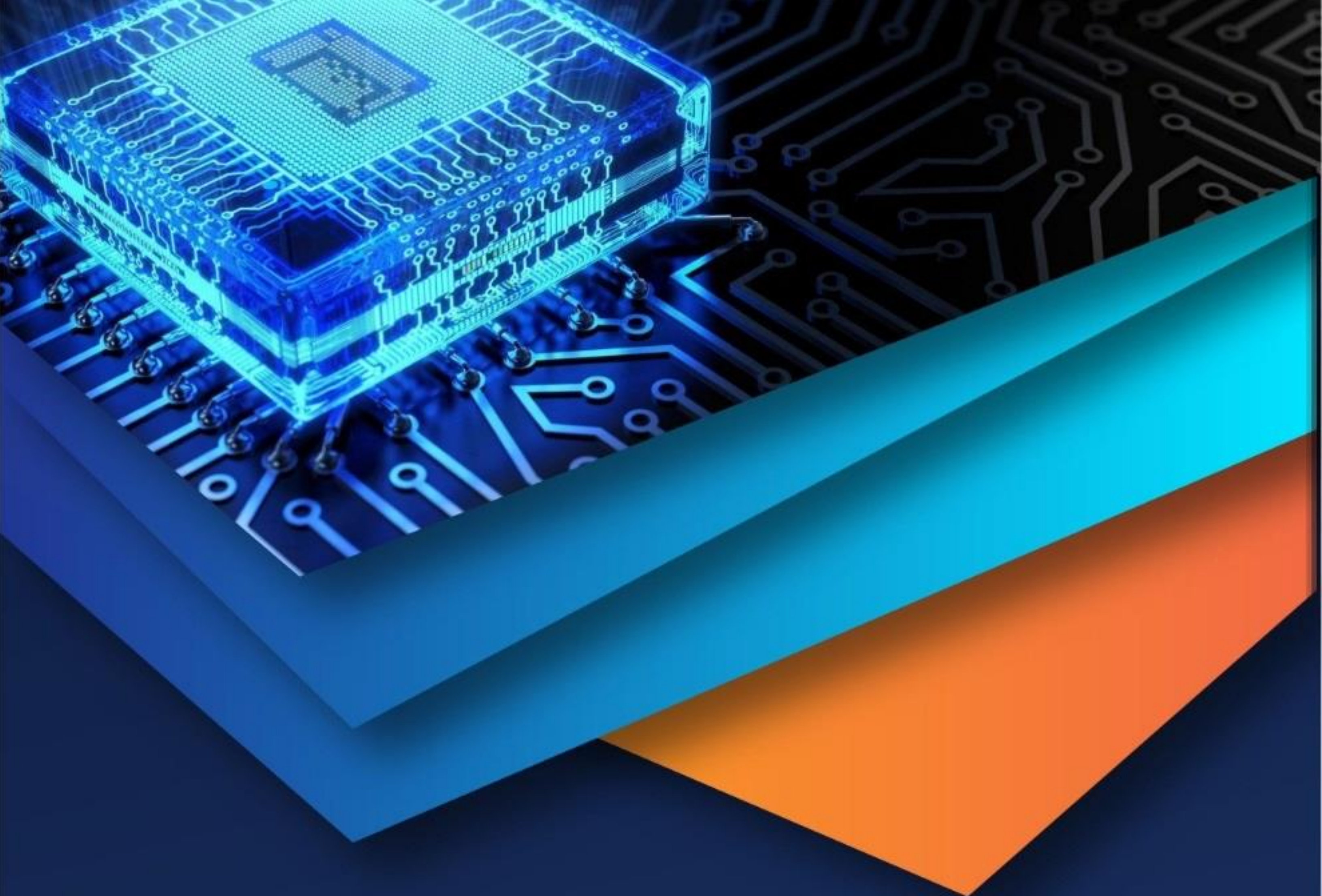

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