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Integral Points on the Ternary Quadratic Diophantine Equation $y^2 = 33x^2 + 4^t$, $t \geq 0$

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Abstract: The binary quadratic equation $y^2 = 33x^2 + 4^t$ representing hyperbola is considered for finding its integer solutions. A few interesting properties among the solutions are presented. Also, we present infinitely many positive integer solutions in terms of Generalized Fibonacci sequences of numbers, Generalized Lucas sequences of numbers.
Keywords: Binary quadratic integral solutions, generalized Fibonacci Sequences of numbers, generalized Lucas Sequences of numbers.

AMS Mathematics Subject Classification: 11D09

Notations

$GF_n(k,s)$: Generalized Fibonacci Sequences of rank n.

$GL_n(k,s)$: Generalized Lucas Sequences of rank n.

$$t_{m,n} = n\left(1 + \frac{(n-1)(m-2)}{2}\right)$$

$$P_n^m = \frac{[n(n+1)((m-2)(n+(5-m))]}{6}$$

$$\Pr_n = n(n+1)$$

$$Ct_{m,n} = \frac{mn(n+1)}{2} + 1$$

$$S_n = 6n(n-1) + 1$$

I. INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$, where D is non-square positive integer has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values [1,2,4]. In [3] infinitely many Pythagorean triangles in each of which hypotenuse is four times the product of the generators added with unity are obtained by employing the non-integral solutions of binary quadratic equation $y^2 = 3x^2 + 1$. In [5] a, special Pythagorean triangle is obtained by employing the integral solutions of $y^2 = 182x^2 + 14$. In [6] different pattern of infinitely many Pythagorean triangles are obtained by employing the non-integral solutions of $y^2 = 14x^2 + 4$. In this context one may also refer [7,8]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation $y^2 = 33x^2 + 4^t$ representing a hyperbola. A few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration a few patterns of Pythagorean triangles are obtained.

II. METHODS OF ANALYSIS

Consider the binary quadratic equation

$$y^2 = 33x^2 + 4^t, \; t \geq 0 \qquad (1)$$

with least positive integer solutions is

$$x_0 = 4(2)^t, \qquad y_0 = 23(2)^t$$

To obtain the other solutions of (1),

consider the Pell equation

$$y^2 = 33x^2 + 1 \tag{2}$$

whose general solution $(\tilde{x}_n, \tilde{y}_n)$ is represented by

$$\tilde{x}_n = \frac{1}{2\sqrt{33}} g_n \,,\ \tilde{y}_n = \frac{1}{2} f_n$$

in which,

$$f_n = \left(23 + 4\sqrt{33}\right)^{n+1} + \left(23 - 4\sqrt{33}\right)^{n+1}$$

$$g_n = \left(23 + 4\sqrt{33}\right)^{n+1} - \left(23 - 4\sqrt{33}\right)^{n+1} \ ,\ n = -1, 0, 1, 2......$$

Applying Brahmagupta lemma between the solutions of $\left(x_0, y_0\right)$ and $\left(\tilde{x}_n, \tilde{y}_n\right)$, the general solutions of equation (1) are found to be

$$y_{n+1} = \frac{23(2)^t}{2} f_n + 2(2)^t \sqrt{33} g_n$$

$$x_{n+1} = 2(2)^t f_n + \frac{23(2)^t}{2\sqrt{33}} g_n$$

The recurrence relations satisfied by the values of x_{n+1} and y_{n+1} are respectively.

$$x_{n+3} - 46x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 46y_{n+2} + y_{n+1} = 0$$

A few numerical examples are presented in the table 1 below:

Table 1: Numerical examples

n	x_{n+1}	y_{n+1}
-1	$4(2)^t$	$23(2)^t$
0	$184(2)^t$	$1057(2)^t$
1	$8460(2)^t$	$48599(2)^t$
2	$388976(2)^t$	$2234497(2)^t$
3	$17884436(2)^t$	$102738263(2)^t$
4	$822295080(2)^t$	$4723725601(2)^t$

A. A Few Interesting Relations Between The Solutions Are Given Below

1) $x_{n+3} - 1057x_{n+1} - 184y_{n+1} = 0$
2) $23x_{n+2} - x_{n+1} - 4y_{n+2} = 0$
3) $23x_{n+2} - 23x_{n+1} - 184y_{n+2} = 0$
4) $1057x_{n+3} - x_{n+1} - 184y_{n+3} = 0$
5) $1057y_{n+1} + 6072x_{n+1} - y_{n+3} = 0$
6) $1057y_{n+2} + 132x_{n+1} - 23y_{n+3} = 0$
7) $x_{n+1} - 46x_{n+2} - x_{n+3} = 0$
8) $4y_{n+1} + 1057x_{n+2} - 23x_{n+3} = 0$

9) $4y_{n+3} + x_{n+2} - 23x_{n+3} = 0$
10) $23x_{n+1} - x_{n+2} + 4y_{n+1} = 0$
11) $23y_{n+2} - 132x_{n+2} + y_{n+1} = 0$
12) $x_{n+3} - 23x_{n+2} - 4y_{n+2} = 0$
13) $y_{n+3} - 132x_{n+2} - 23y_{n+2} = 0$
14) $23x_{n+1} - 1057x_{n+2} + 4y_{n+3} = 0$
15) $y_{n+1} + 132x_{n+2} - 23y_{n+3} = 0$
16) $23y_{n+1} + 132x_{n+3} - 1057y_{n+2} = 0$
17) $23y_{n+3} - 132x_{n+3} - y_{n+2} = 0$
18) $y_{n+1} + 6072x_{n+3} - 1057y_{n+3} = 0$
19) $132x_{n+1} + 23y_{n+1} - y_{n+2} = 0$
20) $y_{n+3} + y_{n+1} - 46y_{n+2} = 0$

B. Each Of The Following Expression Represents A Nasty Number

1) $\frac{1}{4(2)^t}\left(276x_{2n+3} - 12684x_{2n+2} + 48(2)^t\right)$
2) $\frac{1}{184(2)^t}\left(276x_{2n+4} - 583188x_{2n+2} + 2208(2)^t\right)$
3) $\frac{1}{2^t}\left(276y_{2n+2} - 1584x_{2n+2} + 12(2)^t\right)$
4) $\frac{1}{23(2)^t}\left(276y_{2n+3} - 72864x_{2n+2} + 276(2)^t\right)$
5) $\frac{1}{1057(2)^t}\left(276y_{2n+4} - 3350160x_{2n+2} + 12684(2)^t\right)$
6) $\frac{1}{4(2)^t}\left(12684x_{2n+4} - 583188x_{2n+3} + 48(2)^t\right)$
7) $\frac{1}{23(2)^t}\left(12684y_{2n+2} - 1584x_{2n+3} + 276(2)^t\right)$
8) $\frac{1}{2^t}\left(12684y_{2n+3} - 72864x_{2n+3} + 12(2)^t\right)$
9) $\frac{1}{23(2)^t}\left(12684y_{2n+4} - 3350160x_{2n+3} + 276(2)^t\right)$
10) $\frac{1}{1057(2)^t}\left(583188y_{2n+2} - 1584x_{2n+4} + 12684(2)^t\right)$

11) $\frac{1}{23(2)^t}\left(583188y_{2n+3}-72864x_{2n+4}+276(2)^t\right)$

12) $\frac{1}{2^t}\left(583188y_{2n+4}-3350160x_{2n+4}+12(2)^t\right)$

13) $\frac{1}{4(2)^t}\left(2208y_{2n+2}-48y_{2n+3}+48(2)^t\right)$

14) $\frac{1}{184(2)^t}\left(101520y_{2n+2}-48y_{2n+4}+2208(2)^t\right)$

15) $\frac{1}{4(2)^t}\left(101520y_{2n+3}-2208y_{2n+4}+48(2)^t\right)$

C. *Each Of The Following Expression Represents A Cubical Integer*

1) $\frac{1}{2(2)^t}\left[69x_{n+2}-3171x_{n+1}+23x_{3n+4}-1057x_{3n+3}\right]$

2) $\frac{1}{4(2)^t}\left[3x_{n+3}-6339x_{n+1}+x_{3n+5}-2113x_{3n+3}\right]$

3) $\frac{1}{2^t}\left[138y_{n+1}-792x_{n+1}+46y_{3n+3}-264x_{3n+3}\right]$

4) $\frac{1}{2^t}\left[6y_{n+2}-1584x_{n+1}+2y_{3n+4}-528x_{3n+3}\right]$

5) $\frac{1}{1057(2)^t}\left[138y_{n+3}-1675080x_{n+1}+46y_{3n+5}-558360x_{3n+3}\right]$

6) $\frac{1}{2(2)^t}\left[3171x_{n+3}-145797x_{n+2}+1057x_{3n+5}-48599x_{3n+4}\right]$

7) $\frac{1}{23(2)^t}\left[6342y_{n+1}-792x_{n+2}+2114y_{3n+3}-264x_{3n+4}\right]$

8) $\frac{1}{2^t}\left[6342y_{n+2}-36432x_{n+2}+2114y_{3n+4}-12144x_{3n+4}\right]$

9) $\frac{1}{23(2)^t}\left[6342y_{n+3}-1675080x_{n+2}+2114y_{3n+5}-558360x_{3n+4}\right]$

10) $\frac{1}{1057(2)^t}\left[291594y_{n+1}-792x_{n+3}+97198y_{3n+3}-264x_{3n+5}\right]$

11) $\frac{1}{2^t}\left[12678y_{n+2}-1584x_{n+3}+4226y_{3n+4}-528x_{3n+5}\right]$

12) $\frac{1}{2^t}\left[291594y_{n+3}-1675080x_{n+3}+97198y_{3n+5}-1558360x_{3n+5}\right]$

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13) $\frac{1}{2^t}\left[276y_{n+1} - 6y_{n+2} + 92y_{3n+3} - 2y_{3n+4}\right]$

14) $\frac{1}{23(2)^t}\left[6345y_{n+1} - 3y_{n+3} + 2115y_{3n+3} - y_{3n+5}\right]$

15) $\frac{1}{2^t}\left[12690y_{n+2} - 276y_{n+3} + 4230y_{3n+4} - 92y_{3n+5}\right]$

D. Each Of The Following Expression Represents A Bi-Quadratic Integer

1) $\frac{1}{2(2)^t}\left[92x_{2n+3} - 4228x_{2n+2} + 23x_{4n+5} - 1057x_{4n+4} + 12(2)^t\right]$

2) $\frac{1}{4(2)^t}\left[23x_{2n+4} - 8452x_{2n+2} + x_{4n+6} - 2113x_{4n+4} + 24(2)^t\right]$

3) $\frac{1}{2^t}\left[184y_{2n+2} - 1056x_{2n+2} + 46y_{4n+4} - 264x_{4n+4} + 6(2)^t\right]$

4) $\frac{1}{2^t}\left[8y_{2n+3} - 2112x_{2n+2} + 2y_{4n+5} - 528x_{4n+4} + 6(2)^t\right]$

5) $\frac{1}{1057(2)^t}\left[1184y_{2n+4} - 2233440x_{2n+2} + 46y_{4n+6} - 558360x_{4n+4} + 6342(2)^t\right]$

6) $\frac{1}{2(2)^t}\left[4228x_{2n+4} - 194396x_{2n+3} + 1057x_{4n+6} - 48599x_{4n+5} + 12(2)^t\right]$

7) $\frac{1}{23(2)^t}\left[8456y_{2n+2} - 1056x_{2n+3} + 2114y_{4n+4} - 264x_{4n+5} + 138(2)^t\right]$

8) $\frac{1}{2^t}\left[8456y_{2n+3} - 48576x_{2n+3} + 2114y_{4n+5} - 12144x_{4n+5} + 6(2)^t\right]$

9) $\frac{1}{23(2)^t}\left[8456y_{2n+4} - 2233440x_{2n+3} + 2114y_{4n+6} - 558360x_{4n+5} + 138(2)^t\right]$

10) $\frac{1}{1057(2)^t}\left[388792y_{2n+2} - 1056x_{2n+4} + 97198y_{4n+4} - 264x_{4n+6} + 6342(2)^t\right]$

11) $\frac{1}{2^t}\left[16904y_{2n+3} - 2112x_{2n+4} + 4226y_{4n+5} - 528x_{4n+6} + 6(2)^t\right]$

12) $\frac{1}{2^t}\left[388792y_{2n+4} - 2233440x_{2n+4} + 97198y_{4n+6} - 558360x_{4n+6} + 6(2)^t\right]$

13) $\frac{1}{2^t}\left[368y_{2n+2} - 8y_{2n+3} + 92y_{4n+4} - 2y_{4n+5} + 6(2)^t\right]$

14) $\frac{1}{23(2)^t}\left[8460y_{2n+2} - 4y_{2n+4} + 2115y_{4n+4} - y_{4n+6} + 138(2)^t\right]$

15) $\frac{1}{2^t}\left[16920y_{2n+3} - 368y_{2n+4} + 4230y_{4n+5} - 92y_{4n+6} + 6(2)^t\right]$

E. Each Of The Following Expression Represents A Quintic Integer

1) $\frac{1}{4(2)^t}\left[10x_{n+3}-21130x_{n+1}+5x_{3n+5}-10565x_{3n+3}+x_{5n+7}-2113x_{5n+5}\right]$

2) $\frac{1}{8(2)^t}\left[20x_{n+3}-42260x_{n+1}+10x_{3n+5}-21130x_{3n+3}+2x_{5n+7}-4226x_{5n+5}\right]$

3) $\frac{1}{2^t}\left[460y_{n+1}-2640x_{n+1}+230y_{3n+3}-1320x_{3n+3}+46y_{5n+5}-264x_{5n+5}\right]$

4) $\frac{1}{2^t}\left[20y_{n+2}-5280x_{n+1}+10y_{3n+4}-2640x_{3n+3}+2y_{5n+6}-528x_{5n+5}\right]$

5) $\frac{1}{1057(2)^t}\left[460y_{n+3}-5583600x_{n+1}+230y_{3n+5}-2791800x_{3n+3}+46y_{5n+7}-558360x_{5n+5}\right]$

6) $\frac{1}{2(2)^t}\left[10570x_{n+3}-485990x_{n+2}+5285x_{3n+5}-242995x_{3n+4}+1057x_{5n+7}-48599x_{5n+6}\right]$

7) $\frac{1}{23(2)^t}\left[21140y_{n+1}-2640x_{n+2}+10570y_{3n+3}-1320x_{3n+4}+2114y_{5n+5}-264x_{5n+6}\right]$

8) $\frac{1}{2^t}\left[21140y_{n+2}-121440x_{n+2}+10570y_{3n+4}-60720x_{3n+4}+2114y_{5n+6}-12144x_{5n+6}\right]$

9) $\frac{1}{23(2)^t}\left[21140y_{n+3}-5583600x_{n+2}+10570y_{3n+5}-2791800x_{3n+4}+2114y_{5n+7}-558360x_{5n+6}\right]$

10) $\frac{1}{1057(2)^t}\left[971980y_{n+1}-2640x_{n+3}+485990y_{3n+3}-1320x_{3n+5}+97198y_{5n+5}-264x_{5n+7}\right]$

11) $\frac{1}{2^t}\left[84520y_{n+2}-5280x_{n+3}+21130y_{3n+4}-2640x_{3n+5}+4226y_{5n+6}-528x_{5n+7}\right]$

12) $\frac{1}{2^t}\left[971980y_{n+3}-5583600x_{n+3}+485990y_{3n+5}-2791800x_{3n+5}+97198y_{5n+7}-558360x_{5n+7}\right]$

13) $\frac{1}{2^t}\left[920y_{n+1}-20y_{n+2}+460y_{3n+3}-10y_{3n+4}+92y_{5n+5}-2y_{5n+6}\right]$

14) $\frac{1}{8(2)^t}\left[21150\ y_{n+1}-10\ y_{n+3}+10575\ y_{3n+3}-5y_{3n+5}+2115\ y_{5n+5}-y_{5n+7}\right]$

15) $\frac{1}{2^t}\left[42300\ y_{n+2}-920\ y_{n+3}+21150\ y_{3n+4}-460\ y_{3n+5}+4230\ y_{5n+6}-92\ y_{5n+7}\right]$

III. REMARKABLE OBSERVATIONS

1) *The solutions of (1) in terms of special integers namely, generalized Fibonacci* GF_n *and*

Lucas GL_n are exhibited below.

$$x_{n+1}=2(2)^t GL_{n+1}(46,-1)+92(2)^t GF_{n+1}(46,-1)$$

$$y_{n+1}=\frac{23(2)^t}{2}GL_{n+1}(46,-1)+528(2)^t GF_{n+1}(46,-1)$$

2) Employing The Linear Combinations Among The Solutions Of (1), One May Generate Integer Solutions For Other Choices Of Hyperbola Which Are Presented In The Table 2 Below

Table: 2 Hyperbolas

S. No	Hyperbola	(X,Y)
1	$X^2-33Y^2=64(2^t)^2$	$(46x_{n+2}-2114x_{n+1}, 368x_{n+1}-8x_{n+2})$
2	$X^2-33Y^2=135424(2^t)^2$	$(46x_{n+3}-97198x_{n+1}, 16920x_{n+1}-8x_{n+3})$
3	$X^2-33Y^2=4(2^t)^2$	$(46y_{n+1}-264x_{n+1}, 46x_{n+1}-8y_{n+1})$
4	$X^2-33Y^2=2116(2^t)^2$	$(46y_{n+2}-12144x_{n+1}, 2114x_{n+1}-8y_{n+2})$
5	$X^2-33Y^2=4468996(2^t)^2$	$(46y_{n+3}-558360x_{n+1}, 97198x_{n+1}-8y_{n+3})$
6	$X^2-33Y^2=64(2^t)^2$	$(2114x_{n+3}-97198x_{n+2}, 16920x_{n+2}-368x_{n+3})$
7	$X^2-33Y^2=2116(2^t)^2$	$(2114y_{n+1}-264x_{n+2}, 46x_{n+2}-368y_{n+1})$
8	$X^2-33Y^2=4(2^t)^2$	$(2114y_{n+2}-12144x_{n+2}, 2114x_{n+2}-368y_{n+2})$
9	$X^2-33Y^2=2116(2^t)^2$	$(2114y_{n+3}-558360x_{n+2}, 97198x_{n+2}-368y_{n+3})$
10	$X^2-33Y^2=4468996(2^t)^2$	$(97198y_{n+1}-264x_{n+3}, 46x_{n+3}-16920y_{n+1})$
11	$X^2-33Y^2=2116(2^t)^2$	$(97198y_{n+2}-12144x_{n+3}, 2114x_{n+3}-16920y_{n+2})$
12	$X^2-33Y^2=4(2^t)^2$	$(97198y_{n+3}-558360x_{n+3}, 97198x_{n+3}-16920y_{n+3})$
13	$33X^2-Y^2=2112(2^t)^2$	$(368y_{n+1}-8y_{n+2}, 46y_{n+2}-2114y_{n+1})$
14	$33X^2-Y^2=4468992(2^t)^2$	$(16920y_{n+1}-8y_{n+3}, 46y_{n+3}-97198y_{n+1})$
15	$33X^2-Y^2=2112(2^t)^2$	$(16920y_{n+2}-368y_{n+3}, 2114y_{n+3}-97198y_{n+2})$

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3) *Employing Linear Combinations Among The Solutions Of (1), One May Generate Integer Solutions For Other Choices Of Parabola Which Are Presented In The Table 3 Below*

Table: 3 Parabolas

S. No	Parabola	(X,Y)
1	$2^t X - 132Y^2 = 8(4)^t$	$(46x_{2n+3} - 2114x_{2n+2}, 368x_{n+1} - 8x_{n+2})$
2	$184(2)^t X - 33Y^2 = 67712(4)^t$	$(46x_{2n+4} - 97198x_{2n+2}, 16920x_{n+1} - 8x_{n+3})$
3	$2^t X - 33Y^2 = 2(4)^t$	$(46y_{2n+2} - 264x_{2n+2}, 46x_{n+1} - 8y_{n+1})$
4	$23(2)^t X - 33Y^2 = 1058(4)^t$	$(46y_{2n+3} - 12144x_{2n+2}, 2114x_{n+1} - 8y_{n+2})$
5	$1057(2)^t X - 33Y^2 = 2234498(4)^t$	$(46y_{2n+4} - 558360x_{2n+2}, 97198x_{n+1} - 8y_{n+3})$
6	$4(2)^t X - 33Y^2 = 32(4)^t$	$(2114x_{2n+4} - 97198x_{2n+3}, 16920x_{n+2} - 368x_{n+3})$
7	$23(2)^t X - 33Y^2 = 1058(2)^t$	$(2114y_{2n+2} - 264x_{2n+3}, 46x_{n+2} - 368y_{n+1})$
8	$2^t X - 33Y^2 = 2(4)^t$	$(2114y_{2n+3} - 12144x_{2n+3}, 2114x_{n+2} - 368y_{n+2})$
9	$23(2)^t X - 33Y^2 = 1058(4)^t$	$(2114y_{2n+4} - 558360x_{2n+3}, 97198x_{n+2} - 368y_{n+3})$
10	$1057(2)^t X - 33Y^2 = 2234498(4)^t$	$(97198y_{2n+2} - 264x_{2n+4}, 46x_{n+3} - 16920y_{n+1})$
11	$23(2)^t X - 33Y^2 = 1058(4)^t$	$(97198y_{2n+3} - 12144x_{2n+4}, 2114x_{n+3} - 16920y_{n+2})$
12	$2^t X - 33Y^2 = 2(4)^t$	$(97198y_{2n+4} - 558360x_{2n+4}, 97198x_{n+3} - 16920y_{n+3}$
13	$132(2)^t X - Y^2 = 1056(4)^t$	$(368y_{2n+2} - 8y_{2n+3}, 46y_{n+2} - 2114y_{n+1})$
14	$6072(2)^t X - Y^2 = 2234496(4)^t$	$(16920y_{2n+2} - 8y_{2n+4}, 46y_{n+3} - 97198y_{n+1})$
15	$132(2)^t X - Y^2 = 1056(4)^t$	$(16920y_{2n+3} - 368y_{2n+4}, 2114y_{n+3} - 97198y_{n+2})$

4) *Employing the following solution* (x, y), *each of the following expressions among the special polygonal, pyramidal, star, pronic and centered polygonal number is congruent to under modulo 4.*

$$\left(\frac{3P_{y-2}^{3}}{t_{3,y-2}}\right)^{2}-\left(\frac{198P_{x}^{3}}{\Pr_{x+1}}\right)^{2}$$

$$\left(\frac{12P_{y}^{5}}{S_{y+1}-1}\right)^{2}-\left(\frac{1188P_{x-2}^{3}}{S_{x-2}-1}\right)^{2}$$

$$\left(\frac{P_{y}^{5}}{t_{3,y}}\right)^{2}-\left(\frac{396P_{x}^{5}}{S_{x+1}-1}\right)^{2}$$

$$\left(\frac{2P_{y}^{5}}{Ct_{4,y}-1}\right)^{2}-\left(\frac{198P_{x}^{5}}{Ct_{6,x}}\right)^{2}$$

$$\left(\frac{4Pt_{n-3}}{P_{n-3}^{3}}\right)^{2}-\left(\frac{132P_{x}^{5}}{Ct_{4,x}-1}\right)^{2}$$

IV. CONCLUSION

To conclude, one may search for other choices of positive Pell equations for finding their integer solutions with suitable properties.

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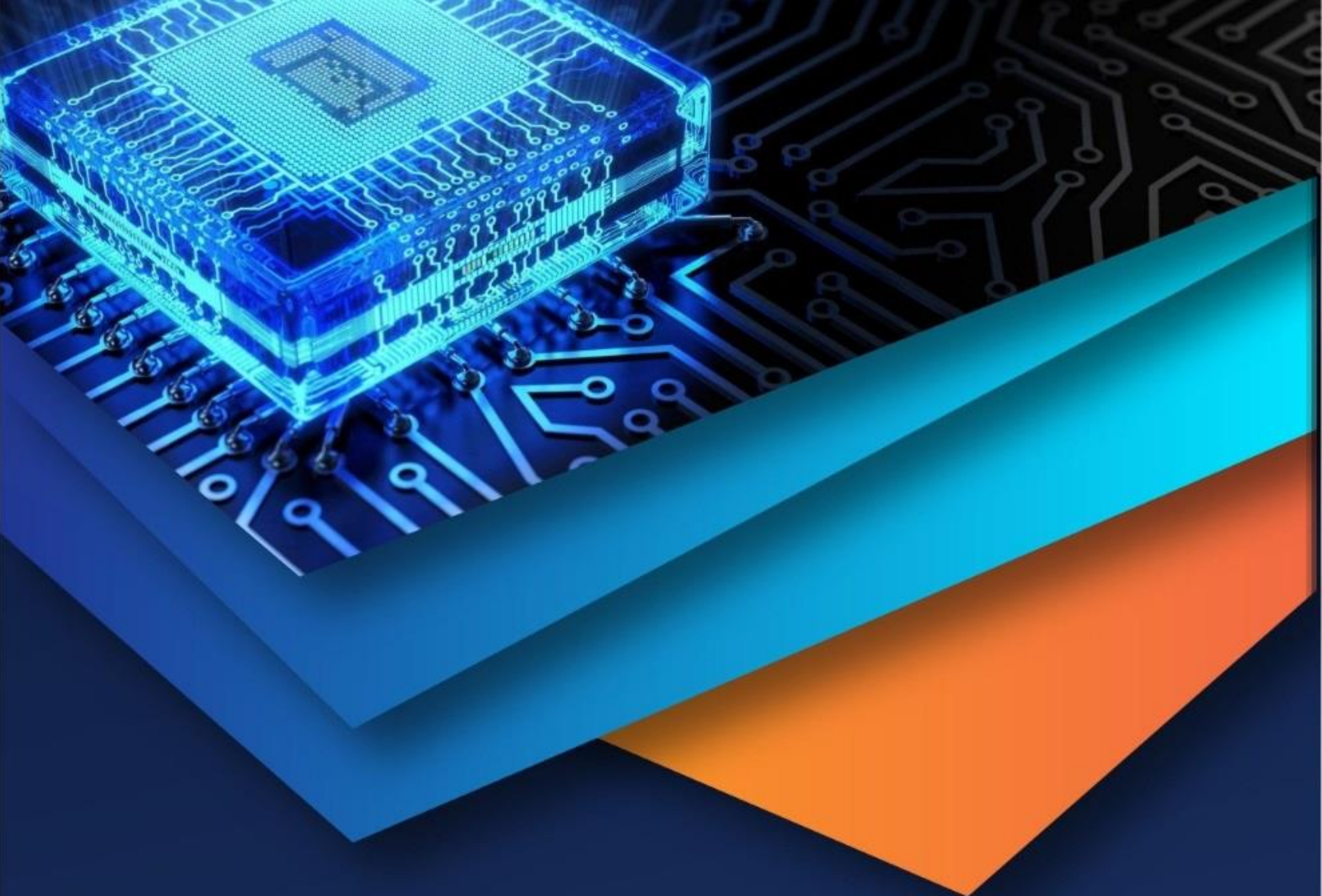

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