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# Observations on the Hyperbola, $y^2 = 14x^2 + 16t, t \geq 0$

$$y^2 = 14x^2 + 16t, t \geq 0$$

G. Sumathi<sup>1</sup>, A. Niranjani<sup>2</sup>

<sup>1</sup>Assistant Professor, <sup>2</sup>M.Phil Scholar, PG and Research Department of Mathematics, Shrimati Indira Gandhi College, Trichy-2, Tamil Nadu, India.

**Abstract:** The binary quadratic equation  $y^2 = 14x^2 + 16t$  representing hyperbola is considered for finding its integer solutions. A few interesting properties among the solutions are presented. Also, we present infinitely many positive integer solutions in terms of Generalized Fibonacci sequences of numbers, Generalized Lucas sequences of numbers.

**Keywords:** Binary Quadratic Integral Solutions, Generalized Fibonacci Sequences of Numbers, Generalized Lucas Sequences of Numbers, Integral Solutions.

AMS Mathematics Subject Classification:11D09

Notations

$GF_n(k, s)$ : Generalized Fibonacci Sequences of rank n.

$GL_n(k, s)$ : Generalized Lucas Sequences of rank n.

$$t_{m,n} = n \left( I + \frac{(n-1)(m-2)}{2} \right)$$

$$P_n^m = \frac{[n(n+1)((m-2)(n+(5-m)))]}{6}$$

$$Pr_n = n(n+1)$$

$$Ct_{m,n} = \frac{mn(n+1)}{2} + 1$$

$$S_n = 6n(n-1) + 1$$

## I. INTRODUCTION

The binary quadratic equation of the form  $y^2 = Dx^2 + 1$  where D non-square positive integer has been studied by various mathematics for its non-trivial integer solutions.

when D takes different integral values [1,2,4]. In [3] infinitely many Pythagorean triangles in each of which hypotenuse is four times the product of the generators added with unity are obtained by employing the  $y^2 = 14x^2 + 1$ . In [5] a special Pythagorean triangles is obtained by employing the integral solutions of  $y^2 = 182x^2 + 14t$ .

In [6] different patterns of infinitely many Pythagorean triangles are obtained by employing the non-integral solutions of  $y^2 = 14x^2 + 4$ . In this context one may also refer [7,8]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation  $y^2 = 14x^2 + 16t$  representing a hyperbola. A few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration a few patterns of Pythagorean triangles are obtained.

## II. METHOD OF ANALYSIS

Consider the binary quadratic equation,

$$y^2 = 14x^2 + 16^t, t \geq 0 \tag{1}$$

with least positive integer solutions of (1),

$$x_0 = 4(4^t), y_0 = 15(4^t), D = 14$$

To obtain the other solutions of (1),

Consider the Pellian equation is

$$y^2 = 14x^2 + 1 \tag{2}$$

The initial solution of Pellian equation is

$$\tilde{x}_0 = 4(4^t), \tilde{y}_0 = 15(4^t)$$

The general solution  $(x_n, y_n)$  of (2) is given by,

$$\tilde{x}_n = \frac{1}{2\sqrt{14}} g_n, \tilde{y}_n = \frac{1}{2} f_n$$

where,

$$f_n = (15 + 4\sqrt{14})^{n+1} + (15 - 4\sqrt{14})^{n+1}$$

$$g_n = (15 + 4\sqrt{14})^{n+1} - (15 - 4\sqrt{14})^{n+1}$$

Applying Brahmagupta lemma between  $(x_0, y_0)$  and  $(\tilde{x}_n, \tilde{y}_n)$  the general solutions of equation (1) are found to be,

$$x_{n+1} = 2 \cdot 4^t f_n + \frac{15}{2\sqrt{14}} 4^t g_n \tag{3}$$

$$y_{n+1} = \frac{15}{2} \cdot 4^t f_n + 2\sqrt{14} 4^t g_n \tag{4}$$

Thus (3) and (4) represent the non-zero distinct integer solutions of (1)

The recurrence relation satisfied by the solution  $x$  and  $y$  are given by

$$x_{n+3} - 30x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 30y_{n+2} + y_{n+1} = 0$$

Some numerical examples of  $x$  and  $y$  satisfying (1) are given in the Table 1 below,

Table 1: Numerical Examples

N	$x_n$	$y_n$
0	$4(4^t)$	$15(4^t)$
1	$3596(4^t)$	$13455(4^t)$
2	$107760(4^t)$	$403201(4^t)$
3	$3229204(4^t)$	$12082575(4^t)$
4	$96230160(4^t)$	$36006479(4^t)$

A. A Few Interesting Relations Among The Solutions Are Given Below

- 1)  $8x_{n+1} - 240x_{n+2} + 8x_{n+3} = 0$
- 2)  $1118x_{n+2} - 33750x_{n+1} + 8y_{n+1} = 0$
- 3)  $399842x_{n+1} + 13358x_{n+2} + 8y_{n+2} = 0$
- 4)  $30x_{n+1} - 898x_{n+2} + 8y_{n+3} = 0$
- 5)  $240y_{n+1} - 2x_{n+3} - 898x_{n+1} = 0$
- 6)  $30x_{n+1} - 30x_{n+3} + 240y_{n+2} = 0$
- 7)  $2x_{n+1} - 898x_{n+3} + 240y_{n+3} = 0$
- 8)  $30x_{n+2} - 2x_{n+1} - 8y_{n+2} = 0$
- 9)  $30y_{n+3} - 898y_{n+2} - 112x_{n+1} = 0$
- 10)  $1399080x_{n+1} - 374882y_{n+1} + 2y_{n+3} = 0$
- 11)  $30x_{n+2} - 2x_{n+3} + 8y_{n+2} = 0$
- 12)  $2x_{n+2} - 30x_{n+3} + 8y_{n+3} = 0$
- 13)  $30x_{n+3} - 898x_{n+2} - 8y_{n+1} = 0$
- 14)  $30y_{n+3} - 2y_{n+1} - 112x_{n+2} = 0$
- 15)  $30y_{n+3} - 30y_{n+1} - 3360x_{n+2} = 0$
- 16)  $2y_{n+3} - 30y_{n+2} - 112x_{n+2} = 0$
- 17)  $898y_{n+2} - 30y_{n+1} - 112x_{n+3} = 0$
- 18)  $898y_{n+3} - 2y_{n+1} - 3360x_{n+3} = 0$
- 19)  $448y_{n+1} - 13440y_{n+2} + 448y_{n+3} = 0$
- 20)  $112x_{n+3} + 30y_{n+1} - 898y_{n+2} = 0$

B. Each Of The Following Expression Is A Nasty Number

- 1)  $\frac{1}{4.4^t} [180x_{2n+3} - 5388x_{2n+2} + 48.4^t]$
- 2)  $\frac{1}{120.4^t} [180x_{2n+4} - 161460x_{2n+2} + 1440.4^t]$
- 3)  $\frac{1}{4^t} [180y_{2n+2} - 672x_{2n+2} + 12.4^t]$
- 4)  $\frac{1}{15.4^t} [180y_{2n+3} - 20160x_{2n+2} + 180.4^t]$
- 5)  $\frac{1}{449.4^t} [3180y_{2n+4} - 604128x_{2n+2} + 3592.4^t]$
- 6)  $\frac{1}{4.4^t} [3592x_{2n+4} - 161460x_{2n+3} + 48.4^t]$

$$7) \frac{1}{15.4^t} [3592y_{2n+2} - 672x_{2n+3} + 180.4^t]$$

$$8) \frac{1}{4^t} [3592y_{2n+3} - 20160x_{2n+3} + 12.4^t]$$

$$9) \frac{1}{15.4^t} [3592y_{2n+4} - 604128x_{2n+3} + 180.4^t]$$

$$10) \frac{1}{449.4^t} [161460y_{2n+2} - 672x_{2n+4} + 3592.4^t]$$

$$11) \frac{1}{15.4^t} [161460y_{2n+3} - 20160x_{2n+4} + 180.4^t]$$

$$12) \frac{1}{4^t} [161460y_{2n+4} - 604128x_{2n+4} + 12.4^t]$$

$$13) \frac{1}{4.4^t} [1440y_{2n+2} - 48x_{2n+3} + 48.4^t]$$

$$14) \frac{1}{120.4^t} [43152y_{2n+2} - 48y_{2n+4} + 1440.4^t]$$

$$15) \frac{1}{4.4^t} [43152y_{2n+3} - 1440y_{2n+4} + 48.4^t]$$

C. Each Of The Following Expressions Is A Cubical Integer

$$1) \frac{1}{4.4^t} [898x_{3n+3} - 30x_{3n+4} + 2694x_{n+1} - 90x_{n+2}]$$

$$2) \frac{1}{120.4^t} [30x_{3n+5} - 26910x_{3n+3} + 90x_{n+3} - 80730x_{n+1}]$$

$$3) \frac{1}{4^t} [30y_{3n+3} - 112x_{3n+3} + 90y_{n+1} - 336x_{n+1}]$$

$$4) \frac{1}{15.4^t} [30y_{3n+4} - 3360x_{3n+3} + 90y_{n+2} - 10080x_{n+1}]$$

$$5) \frac{1}{449.4^t} [30y_{3n+5} - 100688x_{3n+3} + 90y_{n+3} - 302064x_{n+1}]$$

$$6) \frac{1}{4.4^t} [898x_{3n+5} - 26910x_{3n+4} + 2694x_{n+3} - 80730x_{n+2}]$$

$$7) \frac{1}{15.4^t} [898y_{3n+3} - 112x_{3n+4} + 2694y_{n+1} - 36x_{n+2}]$$

$$8) \frac{1}{4^t} [898y_{3n+4} - 3360x_{3n+4} + 2694y_{n+2} - 10080x_{n+2}]$$

$$9) \frac{1}{15.4^t} [898y_{3n+5} - 100688x_{3n+4} + 2694y_{n+3} - 302064x_{n+2}]$$

$$10) \frac{1}{449.4^t} [26910y_{3n+} - 112x_{3n+5} + 80730y_{n+1} - 336x_{n+3}]$$

$$11) \frac{1}{15.4^t} [26910y_{3n+4} - 3360x_{3n+5} + 80730y_{n+2} - 10080x_{n+3}]$$

$$12) \frac{1}{4^t} [26910y_{3n+5} - 100688x_{3n+5} + 80730y_{n+3} - 302064x_{n+3}]$$

$$13) \frac{1}{4.4^t} [240y_{3n+3} - 8y_{3n+4} + 720y_{n+1} - 24y_{n+2}]$$

$$14) \frac{1}{120.4^t} [7192y_{3n+3} - 8y_{2n+5} + 21576y_{n+1} - 24y_{n+3}]$$

$$15) \frac{1}{4.4^t} [7192y_{3n+4} - 240y_{3n+5} + 21576y_{n+2} - 720y_{n+3}]$$

D. Each Of The Following Expressions Is A Biquadratic Integer

$$1) \frac{1}{4.4^t} [30x_{4n+5} - 898x_{4n+4} + 120x_{2n+3} - 3592x_{2n+2} + 24.4^t]$$

$$2) \frac{1}{120.4^t} [30x_{4n+6} - 26910x_{4n+4} + 120x_{2n+4} - 107640x_{2n+2} + 720.4^t]$$

$$3) \frac{1}{15.4^t} [30y_{4n+5} - 3360x_{4n+4} + 120y_{2n+3} - 13440x_{2n+2} + 90.4^t]$$

$$4) \frac{1}{4^t} [30y_{4n+4} - 112x_{4n+4} + 120y_{2n+2} - 448x_{2n+2} + 6.4^t]$$

$$5) \frac{1}{449.4^t} [30y_{4n+6} - 100688x_{4n+4} + 120y_{2n+4} - 402752x_{2n+2} + 2694.4^t]$$

$$6) \frac{1}{4.4^t} [898x_{4n+6} - 26910x_{4n+5} + 3592x_{2n+4} - 107640x_{2n+3} + 24.4^t]$$

$$7) \frac{1}{15.4^t} [898y_{4n+4} - 112x_{4n+5} + 3592y_{2n+2} + 448x_{2n+3} + 90.4^t]$$

$$8) \frac{1}{4^t} [898y_{4n+5} - 3360x_{4n+5} + 3592y_{2n+3} - 13440x_{2n+3} + 6.4^t]$$

$$9) \frac{1}{15.4^t} [898y_{4n+6} - 100688x_{4n+5} + 3592y_{2n+4} - 402752x_{2n+3} + 90.4^t]$$

$$10) \frac{1}{449.4^t} [26910y_{4n+4} - 112x_{4n+6} + 107640y_{2n+2} - 448x_{2n+4} + 2694.4^t]$$

$$11) \frac{1}{15.4^t} [26910y_{4n+5} - 3360x_{4n+6} + 107640y_{2n+4} - 402752x_{2n+4} + 90.4^t]$$

$$12) \frac{1}{4.4^t} [240y_{4n+4} - 8y_{4n+5} + 960y_{2n+2} - 32y_{2n+3} + 24.4^t]$$

$$13) \frac{1}{120.4^t} [7192y_{4n+4} - 8y_{4n+6} + 28768y_{2n+2} - 32y_{2n+4} + 720.4^t]$$

$$14) \frac{1}{4.4^t} [7192y_{4n+5} - 240y_{4n+6} + 28768y_{2n+3} - 960y_{2n+4} + 24.4^t]$$

E. Each Of The Following Expression Is A Quintic Integer

- 1)  $\frac{1}{4.4^t} [30x_{5n+6} - 898x_{5n+5} - 4490x_{3n+3} + 150x_{n+4} - 8980x_{n+1} + 300x_{n+2}]$
- 2)  $\frac{1}{120.4^t} [30x_{5n+7} - 26910x_{5n+5} + 150x_{3n+5} - 134550x_{3n+3} + 300x_{n+3} - 269100x_{n+1}]$
- 3)  $\frac{1}{4^t} [30y_{5n+5} - 112x_{5n+5} - 560x_{3n+3} + 150y_{3n+2} - 1120x_{n+1} + 300y_{n+1}]$
- 4)  $\frac{1}{15.4^t} [480y_{n+1} - 3360x_{5n+5} + 150y_{3n+4} - 16800x_{3n+3} - 33600x_{n+1} - 150y_{n+2}]$
- 5)  $\frac{1}{449.4^t} [30y_{5n+7} - 100688x_{5n+5} + 150y_{3n+4} - 503440x_{3n+3} + 300y_{n+3} - 2013760x_{n+1}]$
- 6)  $\frac{1}{15.4^t} [898y_{5n+5} - 112x_{5n+6} + 4490y_{3n+3} - 560x_{3n+4} + 8980y_{n+1} - 1120x_{n+2}]$
- 7)  $\frac{1}{4.4^t} [898x_{5n+7} - 26910x_{5n+6} + 4490x_{3n+5} - 134550x_{3n+4} + 8980x_{n+3} - 269100x_{n+2}]$
- 8)  $\frac{1}{4^t} [898y_{5n+6} - 3360x_{5n+6} + 4490y_{3n+4} - 16800x_{3n+4} + 8980y_{n+2} - 33600x_{n+2}]$
- 9)  $\frac{1}{15.4^t} [898y_{5n+7} - 100688x_{5n+6} + 4490y_{3n+5} - 503440x_{3n+4} + 8980y_{n+3} - 2013760x_{n+2}]$
- 10)  $\frac{1}{449.4^t} [26910y_{5n+5} - 112x_{5n+7} + 134550y_{3n+3} - 560x_{3n+5} + 269100y_{n+1} - 2240x_{n+3}]$
- 11)  $\frac{1}{15.4^t} [26910y_{5n+6} - 3360x_{5n+7} + 134550y_{n+4} - 16800x_{3n+5} + 269100y_{n+2} - 33600x_{n+3}]$
- 12)  $\frac{1}{4^t} [26910y_{5n+7} - 100688x_{5n+7} + 134550y_{n+5} - 503440x_{3n+5} + 269100y_{n+3} - 2013760x_{n+3}]$
- 13)  $\frac{1}{4.4^t} [240y_{5n+5} - 8y_{5n+6} + 1200y_{3n+3} - 40y_{3n+4} - 1320y_{n+2} + 3640y_{n+1}]$
- 14)  $\frac{1}{120.4^t} [7192y_{5n+5} - 8y_{5n+7} + 3596y_{3n+3} - 40y_{3n+4} + 104284y_{n+1} - 80y_{n+3}]$
- 15)  $\frac{1}{4.4^t} [7192y_{5n+6} - 240y_{5n+7} + 35960y_{3n+4} - 1200y_{3n+5} + 71920y_{n+2} - 2400y_{n+3}]$

16) The solution of (1) in terms of special integers namely, generalized Fibonacci

$GF_n$  and Lucas  $GL_n$  are exhibited below,

$$x_{n+1} = 2(4^t)GL_{n+1}(30, -1) + 60(4^t)GF_{n+1}(30, -1)$$

$$y_{n+1} = \frac{15}{2}(4^t)GL_{n+1}(30, -1) + 224GF_{n+1}(30, -1)$$

**III. REMARKABLE OBSERVATION**

- 1) Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbola which are presented in table 2 below

Table 2: Hyperbolas

S.NO	Hyperbola	(X,Y)
1	$X^2 - 224Y^2 = 64(4^t)^2$	$(898\sqrt{14}x_{n+1} - 30\sqrt{14}x_{n+2}, 60x_{n+1} - 2x_{n+2})$
2	$X^2 - 14Y^2 = 57600(4^t)^2$	$(30x_{n+3} - 26910x_{n+1}, 7192x_{n+1} - 8x_{n+3})$
3	$X^2 - Y^2 = 4(4^t)^2$	$(112x_{n+1} - 30y_{n+1}, 30x_{n+1} - 8y_{n+1})$
4	$X^2 - 14X^2 = 900(4^t)^2$	$(30y_{n+2} - 3360x_{n+1}, 898x_{n+1} - 8y_{n+2})$
5	$X^2 - 14Y^2 = 806404(4^t)^2$	$(30y_{n+3} - 100688x_{n+1}, 26910x_{n+1} - 8y_{n+3})$
6	$X^2 - 14Y^2 = 64(4^t)^2$	$(898x_{n+3} - 26910x_{n+2}, 7192x_{n+2} - 240x_{n+3})$
7	$X^2 - 14Y^2 = 900(4^t)^2$	$(898y_{n+1} - 112x_{n+2}, 30x_{n+2} - 240y_{n+1})$
8	$X^2 - 14Y^2 = 4(4^t)^2$	$(898y_{n+2} - 3360x_{n+2}, 898x_{n+2} - 240y_{n+2})$
9	$X^2 - 14Y^2 = 900(4^t)^2$	$(898y_{n+3} - 100688x_{n+2}, 26910x_{n+2} - 240y_{n+3})$
10	$X^2 - 14Y^2 = 806404(4^t)^2$	$(26910y_{n+1} - 112x_{n+3}, 30x_{n+3} - 7192y_{n+1})$
11	$X^2 - 14Y^2 = 900(4^t)^2$	$(26910y_{n+2} - 3360x_{n+3}, 898x_{n+3} - 7192y_{n+2})$
12	$X^2 - 14Y^2 = 4(4^t)^2$	$(26910y_{n+3} - 100688x_{n+3}, 26910x_{n+3} - 7192y_{n+3})$
13	$196X^2 - 14Y^2 = 12544(4^t)^2$	$(240y_{n+1} - 8y_{n+2}, 30y_{n+2} - 898y_{n+1})$
14	$196X^2 - 14Y^2 = 5644800(4^t)^2$	$(7192y_{n+1} - 8y_{n+3}, 30y_{n+3} - 26910y_{n+1})$
15	$3136X^2 - 224Y^2 = 200704(4^t)^2$	$(7192y_{n+2} - 240y_{n+3}, 898y_{n+3} - 26910y_{n+2})$

- 2) Employing linear combination among the solutions of (1), one may generate integer solutions for other choices of parabolas which are presented in table 3 below

Table 3: Parabolas

S.NO	Parabola	(X,Y)
1	$4^t X + 56Y^2 = 8(4^t)^2$	$(898x_{2n+2} - 30x_{2n+3}, 60x_{n+1} - 2x_{n+2})$
2	$120(4^t)X - 14Y^2 = 28800(4^t)^2$	$(30x_{2n+4} - 26910x_{2n+2}, 7192x_{n+1} - 8x_{n+3})$
3	$(4^t)X + Y^2 = 2(4^t)^2$	$(112x_{2n+2} - 30y_{2n+2}, 30x_{n+1} - 8y_{n+1})$
4	$449(4^t)X - 14Y^2 = 403202(4^t)^2$	$(30y_{2n+4} - 100688x_{2n+2}, 26910x_{n+1} - 8y_{n+3})$
5	$4(4^t)X - 14Y^2 = 32(4^t)^2$	$(898x_{2n+4} - 26910x_{2n+3}, 7192x_{n+2} - 240x_{n+3})$
6	$15(4^t)X - 14Y^2 = 450(4^t)^2$	$(30y_{2n+3} - 3360x_{2n+2}, 898x_{n+1} - 8y_{n+2})$
7	$15(4^t)X - 14Y^2 = 450(4^t)^2$	$(898y_{2n+2} - 112x_{2n+3}, 30x_{n+2} - 240y_{n+1})$
8	$(4^t)X - 14Y^2 = 2(4^t)^2$	$(898y_{2n+3} - 3360x_{2n+3}, 898x_{n+2} - 240y_{n+2})$
9	$15(4^t)X - 14Y^2 = 450(4^t)^2$	$(898y_{2n+4} - 100688x_{2n+3}, 26910x_{n+2} - 240y_{n+3})$
10	$449(4^t)X - 14Y^2 = 403202(4^t)^2$	$(26910y_{2n+2} - 112x_{2n+4}, 30x_{n+3} - 7192y_{n+1})$



11	$15(4^t)X - 14Y^2 = 450(4^t)^2$	$(26910y_{2n+3} - 3360_{2n+4}, 898x_{n+3} - 7192y_{n+2})$
12	$(4^t)X - 14Y^2 = 2(4^t)^2$	$(26910_{2n+4} - 100688x_{2n+4}, 26910x_{n+3} - 7192y_{n+3})$
13	$25320(4^t)X - 14Y^2 = 5644800(4^t)^2$	$(7192y_{2n+2} - 8y_{2n+4}, 30y_{n+3} - 26910y_{n+1})$
14	$784(4^t)X - 14Y^2 = 6272(4^t)^2$	$(240y_{2n+2} - 8y_{2n+3}, 30y_{n+2} - 898y_{n+1})$
15	$56(4^t)X - Y^2 = 448(4^t)^2$	$(7192y_{2n+3} - 240y_{2n+4}, 898y_{n+3} - 26910y_{n+2})$

3) Employing the following solutions (x,y), each of the following expressions among the special polygonal, pyramidal, star, pronic and centered polygonal number is congruent to under modulo 16

$$\left(\frac{3P^3_{y-2}}{t_{3,y-2}}\right)^2 - \left(\frac{84P^3_x}{Pr_{x+1}}\right)^2$$

$$\left(\frac{12P^5_y}{S_{y+1}-1}\right)^2 - \left(\frac{504P^3_{x-2}}{S_{x-2}-1}\right)^2$$

$$\left(\frac{P^5_y}{t_{3,y}}\right)^2 - \left(\frac{168P^5_x}{S_{x+1}-1}\right)^2$$

$$\left(\frac{2P^5_y}{Ct_{4,y}-1}\right)^2 - \left(\frac{84P^5_x}{Ct_{6,x}}\right)^2$$

$$\left(\frac{4Pt_{n-3}}{P_{n-3}^3}\right)^2 - \left(\frac{1848P^5_x}{Ct_{4,x}-1}\right)^2$$

#### IV. CONCLUSION

To conclude, one may search for other choices of positive Pell equations for finding their integer solutions with suitable properties.

#### REFERENCES

- [1] L.E.Dickson, History of Theory of numbers, Vol.2, Chelsea Publishing Company, New York,(1952)
- [2] L.J.Mordell, Diophantine Equations, Academic Press, New York, 1969.
- [3] M.A.Gopalan and G.Janaki, Observation On  $y^2 = 3x^2 + 1$ , Acta Ciancia Indica, XXXIVM(2) (2008) 639-696.
- [4] L.E.Dickson, History of Theory of numbers and Diophantine analysis, Vol 2, Dover publications, Newyork,(2005)
- [5] Dr.G.Sumathi "Observations On the Hyperbola"  $y^2 = 182x^2 + 14$ , Journal of mathematics and Informatics, Vol 11, 73-81, 2017.
- [6] Dr.G.Sumathi "Observations on the Pell equation  $y^2 = 14x^2 + 4$ " International Journal of Creative Research Thoughts, Vol 6, Issue 1, Pp1074-1084, March 2018.
- [7] Dr.G.Sumathi "Observations on the Equation  $y^2 = 312x^2 + 1$ " International Journal of Mathematics Trends and Technology, Vol 50, Issue4, 31-34, Oct 2017.
- [8] Dr.G.Sumathi " Observations on the Hyperbola  $y^2 = 150x^2 + 16$ " International Journal of recent Trends in Engineering and Research, Vol 3, Issue 9, Pp198-206, sep 2017.



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