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On the special Diophantine Equation $y^2 = 7x^2 + 9^t, t \geq 0$

Dr. G. Sumathi [1], S. Maheswari [2]
[1]Assitant Professor, [2]M.Phil Scholar, PG & Research Department of Mathematics, Shrimati Indira Gandhi college, Trichy-2, Tamilnadu, India.

Abstract: The binary quadratic equation $y^2 = 7x^2 + 9^t, t \geq 0$ ***representing hyperbola is considered for finding its integer solutions. A few interesting properties among the solutions are presented. Also, we present infinitely many positive integer solutions in terms of generalized Fibonacci sequences of numbers, generalized Lucas sequences of numbers.***
Keywords: Binary quadratic integral solution, generalized Fibonacci sequences of numbers, generalized Lucas sequences of numbers.

AMS Mathematics subject classification: 11D09
Notations

$GF_n(K,S)$: Generalized Fibonacci sequences of rank n.

$GL_n(K,S)$: Generalized Lucas sequences of rank n.

$$t_{m,n} = n\left(1 + \frac{(n-1)(m-2)}{2}\right)$$

$$p_n^m = \frac{[n(n+1)((m-2)(n+(5-m))]}{6}$$

$$\Pr_n = n(n+1)$$

$$Ct_{m,n} = \frac{mn(n+1)}{2} + 1$$

I. INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D non-square positive integer has been studied by various mathematician for its non-trivial integral solutions. When D takes different integral values [1,2,4]. In [3] infinitely many Pythagorean triangles in each of which hypotenuse is four time the product of the generators added with unity are obtained by employing the non-integral solutions of binary quadratic equation $y^2 = 3x^2 + 1$. In [5] a, special Pythagorean triangle is obtained by employing the integral solution of $y^2 = 182x^2 + 14$. In [6] different pattern of infinitely many Pythagorean triangles are obtained by employing the non-integral solutions of $y^2 = 14x^2 + 4$. In this context one may also refer [7,8]. These results have motivated us to search for the integral solutions of yet another binary quartic equation is $y^2 = 7x^2 + 9^t, t \geq 0$ representing a hyperbola. A few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration a few patterns of Pythagorean triangles are obtained.

II. METHOD OF ANALYSIS

Consider the binary quadratic equation

$$y^2 = 7x^2 + 9^t, t \geq 0 \qquad (1)$$

with least positive integer solution of (1),

$$x_0 = 3(3^t), y_0 = 8(3^t), D = 7$$

To obtain the other solutions of (1) consider the pell equation

$$y^2 = 7x^2 + 1 \qquad (2)$$

whose general solution $(\tilde{x}_n, \tilde{y}_n)$ is represented by,

$$\tilde{x}_n = \frac{1}{2\sqrt{7}} g_n, \tilde{y}_n = \frac{1}{2} f_n$$

In which,

$$f_n = (8+3\sqrt{7})^{n+1} + (8-3\sqrt{7})^{n+1}$$

$$g_n = (8+3\sqrt{7})^{n+1} - (8-3\sqrt{7})^{n+1}$$

Applying Brahmagupta lemma between (x_0, y_o) and $(\tilde{x}_n, \tilde{y}_n)$ the general solution of equation (1) are given to be

$$x_{n+1} \frac{3(3^t)}{2} f_n + \frac{4(3^t)}{\sqrt{7}} g_n \quad ,$$

$$y_{n+1} = 4(3^t) f_n + \frac{3\sqrt{7}(3^t)}{2} g_n \quad ,$$

The recurrence relation satisfied by the values x_{n+1} and y_{n+1} are respectively,

$$x_{n+3} - 16x_{n+2} + x_{n+1} = 0 \quad ,$$

$$y_{n+3} - 16y_{n+2} + y_{n+1} = 0 \quad ,$$

A few numerical examples are in the table 1 below:

Table 1: Numerical Examples

n	x_{n+1}	y_{n+1}
0	$48(3^t)$	$127(3^t)$
1	$765(3^t)$	$2024(3^t)$
2	$12192(3^t)$	$32257(3^t)$
3	$194307(3^t)$	$514088(3^t)$
4	$3096720(3^t)$	$8193151(3^t)$
5	$49353213(3^t)$	$130576328(3^t)$
6	$786554688(3^t)$	$2081028097(3^t)$

A. A Few Interesting Relations Between The Solutions Are Given Below

1) $x_{n+2} - 8x_{n+1} - 3y_{n+1} = 0$
2) $x_{n+1} - 8x_{n+2} + 3y_{n+2} = 0$
3) $8x_{n+1} - 127x_{n+2} + 3y_{n+3} = 0$
4) $48y_{n+1} - x_{n+3} + 127x_{n+1} = 0$
5) $x_{n+1} + 6y_{n+2} - x_{n+3} = 0$
6) $48y_{n+3}127x_{n+3} + x_{n+1} = 0$
7) $3y_{n+1} - 8x_{n+3} + 127x_{n+2} = 0$
8) $3y_{n+2} + 8x_{n+2} - x_{n+3} = 0$
9) $x_{n+2} - 8x_{n+3} + 3y_{n+3} = 0$
10) $127y_{n+1} - y_{n+3} + 336x_{n+1} = 0$
11) $42x_{n+2} - y_{n+3} + y_{n+1} = 0$
12) $336x_{n+3} - 127y_{n+3} + y_{n+1} = 0$
13) $y_{n+2} - 21x_{n+1} - 8y_{n+1} = 0$
14) $21x_{n+2} - 8y_{n+2} + y_{n+1} = 0$
15) $21x_{n+3} - 127y_{n+2} + 8y_{n+1} = 0$
16) $127y_{n+2} - 8y_{n+3} + 21x_{n+1} = 0$
17) $8y_{n+2} - y_{n+3} + 21x_{n+2} = 0$
18) $y_{n+2} - 8y_{n+3} + 21x_{n+3} = 0$
19) $x_{n+1} - 16x_{n+2} + x_{n+3} = 0$
20) $y_{n+1} - 16y_{n+2} + y_{n+3} = 0$

B. Each Of The Following Expressions Represents A Nasty Number

1) $\frac{1}{3(3^t)}\left[96x_{2n+3} - 1524x_{2n+2} + 36(3^t)\right]$
2) $\frac{1}{3(3^t)}\left[6x_{2n+4} - 1518x_{2n+2} + 36(3^t)\right]$
3) $\frac{1}{3^t}\left[96y_{2n+2} - 252x_{2n+2} + 12(3^t)\right]$
4) $\frac{1}{3^t}\left[12y_{2n+3} - 504x_{2n+2} + 12(3^t)\right]$
5) $\frac{1}{127(3^t)}\left[96y_{2n+4} - 64260x_{2n+2} + 1524(3^t)\right]$
6) $\frac{1}{3(3^t)}\left[1524x_{2n+4} - 24288x_{2n+3} + 36(3^t)\right]$

7) $\frac{1}{8(3^t)}\left[1524y_{2n+2} - 252x_{2n+3} + 96(3^t)\right]$

8) $\frac{1}{3^t}\left[1524y_{2n+3} - 4032x_{2n+3} + 12(3^t)\right]$

9) $\frac{1}{8(3^t)}\left[1524y_{2n+4} - 64260x_{2n+3} + 96(3^t)\right]$

10) $\frac{1}{127(3^t)}\left[24288y_{2n+2} - 252x_{2n+4} + 1524(3^t)\right]$

11) $\frac{1}{3^t}\left[3036y_{2n+3} - 504x_{2n+4} + 12(3^t)\right]$

12) $\frac{1}{3^t}\left[24288y_{2n+4} - 64260x_{2n+4} + 12(3^{t)}\right]$

13) $\frac{1}{3^t}\left[192y_{2n+2} - 12y_{2n+3} + 12(3^t)\right]$

14) $\frac{1}{16(3^t)}\left[3060y_{2n+2} - 12y_{2n+4} + 192(3^t)\right]$

15) $\frac{1}{3(3^t)}\left[9180y_{2n+3} - 576y_{2n+4} + 36(3^t)\right]$

C. Each Of The Following Expressions Is A Cubical Integer

1) $\frac{1}{3(3^t)}[16x_{3n+4} - 254x_{3n+3} + 48x_{n+2} - 762x_{n+1}]$

2) $\frac{1}{3^t}[16y_{3n+3} - 42x_{3n+3} + 48y_{n+1} - 126x_{n+1}]$

3) $\frac{1}{3^t}[2y_{3n+4} - 84x_{3n+3} + 6y_{n+2} - 252x_{n+1}]$

4) $\frac{1}{127(3^t)}[16y_{3n+5} - 10710x_{3n+3} + 48y_{n+3} - 32130x_{n+1}]$

5) $\frac{1}{3(3^t)}[254x_{3n+5} - 4048x_{3n+4} + 762x_{n+3} - 12144x_{n+2}]$

6) $\frac{1}{8(3^t)}[254y_{3n+3} - 42x_{3n+4} + 762y_{n+1} - 126x_{n+2}]$

7) $\frac{1}{3^t}[254y_{3n+4} - 672x_{3n+4} + 762y_{n+2} - 2016x_{n+2}]$

8) $\frac{1}{8(3^t)}[254y_{3n+5} - 10710x_{3n+4} + 762y_{n+3} - 10710x_{n+2}]$

9) $\frac{1}{127(3^t)}[4048y_{3n+3} - 42x_{3n+5} + 12144y_{n+1} - 126x_{n+3}]$

10) $\frac{1}{3^t}[506y_{3n+4} - 84x_{3n+5} + 1518y_{n+2} - 252x_{n+3}]$

11) $\frac{1}{3^t}[4048y_{3n+5} - 10710x_{3n+5} + 12144y_{n+3} - 32130x_{n+3}]$

12) $\frac{1}{3^t}[32y_{3n+3} - 2y_{3n+4} + 96y_{n+1} - 6y_{n+2}]$

13) $\frac{1}{16(3^t)}[510y_{3n+3} - 2y_{3n+5} + 1530y_{n+1} - 6y_{n+3}]$

14) $\frac{1}{3(3^t)}[1530y_{3n+4} - 96y_{3n+5} + 4590y_{n+2} - 288y_{n+3}]$

D. Each Of The Following Expressions Represent A Bi-Quadratic Integer

1) $\frac{1}{3(3^t)}[16x_{4n+5} - 254x_{4n+4} + 64x_{2n+3} - 1016x_{2n+2} + 18(3^t)]$

2) $\frac{1}{3(3^t)}[x_{4n+6} - 253x_{4n+4} + 4x_{2n+4} - 1012x_{2n+2} + 18(3^t)]$

3) $\frac{1}{3^t}[16y_{4n+4} - 42x_{4n+4} + 64y_{2n+2} - 168x_{2n+2} + 3^t]$

4) $\frac{1}{3^t}[2y_{4n+5} - 84x_{4n+4} + 8y_{2n+3} - 336x_{2n+2} + 3^t]$

5) $\frac{1}{127(3^t)}[16y_{4n+6} - 10710x_{4n+4} + 64y_{2n+4} - 42480x_{2n+2} + 762(3^t)]$

6) $\frac{1}{3(3^t)}[254x_{4n+6} - 4048x_{4n+5} + 1016x_{2n+4} - 16192x_{2n+3} + 18(3^t)]$

7) $\frac{1}{8(3^t)}[254y_{4n+4} - 42x_{4n+5} + 1016y_{n+1} - 168x_{n+2} + 48(3^t)]$

8) $\frac{1}{3^t}[254y_{4n+5} - 672x_{4n+5} + 1016y_{2n+3} - 672x_{2n+3} + 3^t]$

9) $\frac{1}{8(3^t)}[254y_{4n+6} - 10710x_{4n+5} + 1016y_{2n+4} - 4284x_{2n+3} + 48(3^t)]$

10) $\frac{1}{127(3^t)}[4048y_{4n+4} - 42x_{4n+6} + 16192y_{2n+2} - 168x_{2n+4} + 762(3^t)]$

11) $\frac{1}{3^t}[506y_{4n+5} - 84x_{4n+6} + 2024y_{2n+3} - 336x_{2n+4} + 3^t]$

12) $\frac{1}{3^t}[4048y_{4n+6} - 10710x_{4n+6} + 16192y_{2n+4} - 42840x_{2n+4} + 3^t]$

13) $\frac{1}{3^t}[32y_{4n+4} - 2y_{4n+5} + 128y_{2n+2} - 8y_{2n+3} + 3^t]$

14) $\frac{1}{16(3^t)}[510y_{4n+4} - 2y_{4n+6} + 2040y_{2n+2} - 8y_{2n+4} + 96(3^t)]$

15) $\frac{1}{3(3^t)}[1530y_{4n+5} - 96y_{4n+6} + 6120y_{n+2} - 384y_{n+3} + 18(3^t)]$

E. Each Of The Following Expression Represents A Quantic Integer

1) $\frac{1}{3(3^t)}[16x_{5n+6} - 254x_{5n+5} + 80x_{3n+4} - 1270x_{3n+3} + 160x_{n+2} - 2540x_{n+1}]$

2) $\frac{1}{3(3^t)}[x_{5n+7} - 253x_{5n5} + 5x_{3n+5} - 1265x_{3n+3} + 10x_{n+3} - 2530x_{n+1}]$

3) $\frac{1}{3^t}[16y_{5n+5} - 21x_{5n+5} + 80y_{3n+3} - 210x_{3n+3} + 160y_{n+1} - 420x_{n+1}]$

4) $\frac{1}{3^t}[2y_{5n+6} - 84x_{5n+5} + 10y_{3n+4} - 420x_{3n+3} + 20y_{n+2} - 840x_{n+1}]$

5) $\frac{1}{127(3^t)}[16y_{5n+7} - 10710x_{5n+5} + 80y_{3n+5} - 53550x_{3n+3} + 160y_{n+3} - 107100x_{n+1}]$

6) $\frac{1}{3(3^t)}[254x_{5n+7} - 4048x_{5n+6} + 1270x_{3n+5} - 20240x_{3n+4} + 2540x_{n+3} - 40480x_{n+2}]$

7) $\frac{1}{8(3^t)}[254y_{5n+5} - 42x_{5n+6} + 1270y_{3n+3} - 210x_{3n+4} + 2540y_{n+1} - 420x_{n+2}]$

8) $\frac{1}{3^t}[254y_{5n+6} - 672x_{5n+7} + 1270y_{3n+4} - 3360x_{3n+4} + 2540y_{n+2} - 10020x_{n+2}]$

9) $\frac{1}{8(3^t)}[254y_{5n+7} - 10710x_{5n+6} + 1270y_{3n+5} - 53550x_{3n+4} - 460y_{n+3} - 107100x_{n+2}]$

10) $\frac{1}{127(3^t)}[4048y_{5n+5} - 42x_{5n+7} + 20240y_{3n+3} - 210x_{3n+5} + 40480y_{n+1} - 420x_{n+3}]$

11) $\frac{1}{3^t}[506y_{5+6} - 84x_{5n+7} + 2530y_{3n+4} - 420x_{3n+5} + 5060y_{n+2} - 840x_{n+3}]$

12) $\frac{1}{3^t}[4048y_{5n+7} - 10710x_{5n+7} + 20240y_{3n+5} - 53550x_{3n+5} + 40480y_{n+3} - 107100x_{n+3}]$

13) $\frac{1}{3^t}[32y_{5n+5} - 2y_{5n+6} + 160y_{3n+3} - 10y_{3n+4} + 320y_{n+1} - 25y_{n+2}]$

14) $\frac{1}{16(3^t)}[510y_{5n+5} - 2y_{5n+7} + 2550y_{n+3} - 10y_{3n+5} + 5100y_{n+1} - 25y_{n+3}]$

15) $\frac{1}{3(3^t)}[1530y_{5n+6} - 96y_{5n+7} + 7650y_{3n+4} - 480y_{3n+5} + 15300y_{n+2} - 960y_{n+3}]$

F. The solution of (1) in terms of special integers namely , generalized fibonacci sequence and generalized Lucas sequence are exhibited below

$$x_{n+1} = \frac{3(3^t)}{2} GL_{n+1}(16,-1) + 24(3^t) GF_{n+1}(16,-1)$$

$$y_{n+1} = 4(3^t) GL_{n+1}(16,-1) + 63(3^t) GF_{n+1}(16,-1)$$

III. REMARKABLE OBSERVATION

Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbolas which are presented in table 2 below

Table 2: Hyperbola

S.NO	Hyperbola	(X,Y)
1	$4X^2 - 63Y^2 = 36(3^{2t})$	$(8x_{n+2} - 127x_{n+1}, 32x_{n+1} - 2x_{n+2})$
2	$64X^2 - 63Y^2 = 2304(3^{2t})$	$(x_{n+3} - 253x_{n+1}, 255x_{n+1} - x_{n+3})$
3	$4X^2 - 14Y^2 = 4(3^{2t})$	$(8y_{n+1} - 21x_{n+1}, 8x_{n+1} - 3y_{n+1})$
4	$16X^2 - 7Y^2 = 64(3^{2t})$	$(2y_{n+2} - 84x_{n+1}, 127x_{n+1} - 3y_{n+2})$
5	$X^2 - 7Y^2 = 64516(3^{2t})$	$(16y_{n+3} - 10710x_{n+1}, 4048x_{n+1} - 6y_{n+3})$
6	$X^2 - 63Y^2 = 36(3^{2t})$	$(254x_{n+3} - 4048x_{n+2}, 510x_{n+2} - 32x_{n+3})$
7	$X^2 - 448Y^2 = 256(3^{2t})$	$(254y_{n+1} - 42x_{n+2}, 2x_{n+2} - 12y_{n+1})$
8	$4X^2 - 28Y^2 = 4(3^{2t})$	$(127y_{n+2} - 336x_{n+2}, 127x_{n+2} - 48y_{n+2})$
9	$X^2 - 7Y^2 = 256(3^{2t})$	$(254y_{n+3} - 10710x_{n+2}, 4048x_{n+2} - 96y_{n+3})$
10	$X^2 - 7Y^2 = 64516(3^{2t})$	$(4048y_{n+1} - 42x_{n+3}, 16x_{n+3} - 1530y_{n+1})$
11	$64X^2 - 7Y^2 = 256(3^{2t})$	$(506y_{n+2} - 84x_{n+3}, 254x_{n+3} - 1530y_{n+2})$
12	$X^2 - 28Y^2 = 4(3^{2t})$	$(4048y_{n+3} - 1530y_{n+2}, 2024x_{n+3} - 765y_{n+3})$
13	$63X^2 - Y^2 = 252(3^{2t})$	$(32y_{n+1} - 2y_{n+2}, 16y_{n+2} - 254y_{n+1})$
14	$252X^2 - 256Y^2 = 258048(3^{2t})$	$(510y_{n+1} - 2y_{n+3}, 2y_{n+3} - 506y_{n+1})$
15	$63X^2 - 9Y^2 = 2268(3^{2t})$	$(1530y_{n+2} - 96y_{n+3}, 254y_{n+3} - 4048y_{n+2})$

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1) Employing linear combination among the solutions of (1), one may generate integer solutions for other choices of parabolas which are presented in table 3 below

Table 3: Parabola

S.NO	Parabola	(X,Y)
1	$2(3^t)X-7Y^2=6(3^{2t})$	$(8x_{n+2}-127x_{n+1},32x_{n+1}-2x_{n+2})$
2	$64(3^t)X-7Y^2=384(3^{2t})$	$(x_{n+3}-253x_{n+1},255x_{n+1}-x_{n+3})$
3	$2(3^t)X-28Y^2=2(3^{2t})$	$(8y_{n+1}-21x_{n+1},8x_{n+1}-3y_{n+1})$
4	$16(3^t)X-7Y^2=32(3^{2t})$	$(2y_{n+2}-84x_{n+1},127x_{n+1}-3y_{n+2})$
5	$127(3^t)X-7Y^2=32258(3^{2t})$	$(16y_{n+3}-10710x_{n+1},4048x_{n+1}-6y_{n+3})$
6	$3(3^t)X-7Y^2=18(3^{2t})$	$(254x_{n+3}-4048x_{n+2},510x_{n+2}-32x_{n+3})$
7	$(3^t)X-7Y^2=16(3^{2t})$	$(254y_{n+1}-42x_{n+2},2x_{n+2}-12y_{n+1})$
8	$2(3^t)X-28Y^2=2(3^{2t})$	$(127y_{n+2}-336x_{n+2},127x_{n+2}-48y_{n+2})$
9	$8(3^t)X-7Y^2=128(3^{2t})$	$(254y_{n+3}-10710x_{n+2},4048x_{n+2}-96y_{n+3})$
10	$127(3^t)X-7Y^2=32258(3^{2t})$	$(4048y_{n+1}-42x_{n+3},16x_{n+3}-1530y_{n+1})$
11	$64(3^t)X-7Y^2=128(3^{2t})$	$(506y_{n+2}-84x_{n+3},254x_{n+3}-1530y_{n+2})$
12	$(3^t)X-28Y^2=2(3^{2t})$	$(4048y_{n+3}-10710x_{n+3},2024x_{n+3}-765y_{n+3})$
13	$63(3^t)X-Y^2=126(3^{2t})$	$(32y_{n+1}-2y_{n+2},16y_{n+2}-254y_{n+1})$
14	$252(3^t)X-16Y^2=8064(3^{2t})$	$(510y_{n+1}-2y_{n+3},2y_{n+3}-506y_{n+1})$
15	$63(3^t)X-3Y^2=378(3^{2t})$	$(1530y_{n+2}-96y_{n+3},254y_{n+3}-4048y_{n+2})$

2) Employing the following solutions(x,y), each of the following expressions among the special polygonal, star, pyramidal, pronic and centered polygonal number id congruent under modulo 9

$$\left(\frac{3p_{y-2}^3}{t_{3,y-2}}\right)^2-\left(\frac{42p_x^3}{pr_{x+1}}\right)^2$$

$$\left(\frac{12p_y^5}{s_{y+1}-1}\right)-\left(\frac{252p_{x-2}^3}{s_{x-2}-1}\right)^2$$

$$\left(\frac{p_y^5}{t_{3,y}}\right)^2-\left(\frac{84p_x^5}{s_{x+1}-1}\right)^2$$

$$\left(\frac{2p_y^5}{ct_{4,y}-1}\right)^2-\left(\frac{42p_x^5}{ct_{6,x}}\right)^2$$

$$\left(\frac{4pt_{n-3}}{p_{n-3}^3}\right)^2-\left(\frac{28p_x^5}{ct_{4,x}-1}\right)^2$$

IV. CONCLUSION

In this paper, we have presented infinitely many integer solutions for the negative Pell Equations $y^2 = 7x^2 + 9^t, t \geq 0$. As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of Pell Equations and determine their integer solutions along with suitable properties.

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