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Energy Renormalization on the Sierpinski Pentagon Using Electrical Transform

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Abstract* — In this paper the energy renormalization factor (resistence of the network) on the fractal Sierpinski Pentagon (Pentagasket) is described by the electrical network interpretation. First the Pentagasket-D_5 is obtained by the method of iterated function system. The Delta-wye transforms is iteratively applied on the Pentagasket-D_5 fractal to get the energy renormalization factor.

***Keywords*— Energy renormalization factor, and Pentagasket, Delta-wye transform, Resistance**

I. INTRODUCTION

The term "fractal" was first used by mathematician Benoit Mandelbrot in 1975. The formal mathematical definition of fractal is defined by Benoit Mandelbrot says that a fractal is a set for which the Hausdorff Besicovich dimension strictly exceeds the topological dimension. However, this is a very abstract definition. Generally, we can define a fractal as a rough or fragmented geometric shape that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole. Fractals are generally self-similar and independent of scale. Many people are fascinated by the beautiful images termed fractals. Extending beyond the typical perception of mathematics as a body of complicated, boring formulas, fractal geometry mixes art with mathematics to demonstrate that equations are more than just a collection of numbers. What makes fractals even more interesting is that they are the best existing mathematical descriptions of many natural forms, such as coastlines, mountains or parts of living organisms.The Electrical network on fractals was first constructed by Robert S. Strichartz. The Harmonic extension on the Sierpinski gasket was analyzed by Zhigang Feng and Li Wang. In this we consider the Sierpinski Pentagon for the analysis of energy renormalization using the electrical network Delta-Wye transform. We construct networks with the property that every resistor in a network of a given order is a scalar multiple of the corresponding resistor in the next order network. The Sierpiski pentagon is one of the simplest examples of a nested fractal.

II. ELECTICAL TRANSFORM - Δ-Y TRANSFORM

This transform is a mathematical technique to simplify the analysis of an electrical network. The name is derived from the shape of the circuit diagrams, which look respectively like the letter Y and the greek capital letter Δ. Three resistors are used in the three sides of the both Y and Δ circuits. The resistors show resistance against the current passing through them and it is denoted by the letter 'R'. The basic idea of the Δ-Y transform is, any triangular arrangement of resistors Ra,Rb and Rc within a larger circuit can be replaced by a star, with resistances R1,R2 and R3, such that all the resistances between any two points among the three vertices in the triangle and the star are the same.

The following system equations are used to transform Δ-load to Y-load circuit.

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$
$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \qquad ------------------(1.1)$$

The general idea to compute is,

$$R_\Delta = \frac{R_P}{R_{opposite}} \quad Where\ R_P = R_1R_2 + R_2R_3 + R_3R_1$$

R_P is the sum of the products of all pairs of resistances in the Y circuit and R_{opposite} is the resistance of the vertex in the Y circuit which is opposite the edge with R_Δ.

The formulas to transform Y-load to Δ-load circuit is obtained by,

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \qquad \text{-----------------(1.2)}$$

III. THE PENTAGON SIERPINSKI FRACTAL

The Sierpinski's fractal is named after the Polish mathematician Waclaw Sierpinski who described some of its interesting properties in 1916. It is one of the simplest fractal shapes in existence. The construction of the pentagasket (P_0) fractal is similar to the construction of the Sierpinski gasket by scaling a triangle and translating three copies. We scale a pentagon and translate five similar copies of itself. We place the five smaller pentagons inside the larger pentagon. The Sierpinski pentagon can be obtained as an iterated function system construction. The IFS consisting of a scaling factor by r = $\frac{3-\sqrt{5}}{2}$ =0.382 and an appropriate translation for each of the five functions. The Sierpinski pentagon is the limiting set for this construction.
paragraphs must be indented. All paragraphs must be justified, i.e. both left-justified and right-justified.

$$F_1\begin{pmatrix} x \\ y \end{pmatrix} = 0.382\begin{pmatrix} x \\ y \end{pmatrix},\quad F_2\begin{pmatrix} x \\ y \end{pmatrix} = 0.382\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0.618 \\ 0 \end{pmatrix},\quad F_3\begin{pmatrix} x \\ y \end{pmatrix} = 0.382\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0.809 \\ 0.588 \end{pmatrix},$$

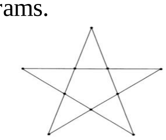
$$F_4\begin{pmatrix} x \\ y \end{pmatrix} = 0.382\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0.309 \\ 0.951 \end{pmatrix},\quad F_5\begin{pmatrix} x \\ y \end{pmatrix} = 0.38\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -0.191 \\ 0.588 \end{pmatrix} \qquad \text{-------------------------(2.1)}$$

The Sierpinski pentagon is self similar with five non overlapping copies of itself, each scaled by the factor r<1, hence the similarity dimension is

$$d = \frac{\log(1/n)}{\log(1/r)} = \frac{\log(1/5)}{\log(2/(3-\sqrt{5}))} = 1.6723.$$

We denote Sierpinski pentagon by P, the $P = \bigcup_{i=1}^{5} F_i(P)$, where P is the self similar set with respect to {F_1, F_2, F_3, F_4, F_5 }. The Pentagasket and V_1 of the pentagasket are shown in the following diagrams.

Sierpinski pentagasket



Pentagon-V_1

A. Graph Energy And Energy Renormalization

The initial energy of the pentagasket is given of the form

$$E_0(u) = a((x_0-x_1)^2+(x_0-x_2)^2)+b(x_1-x_2)^2 \text{--(2.2)}$$

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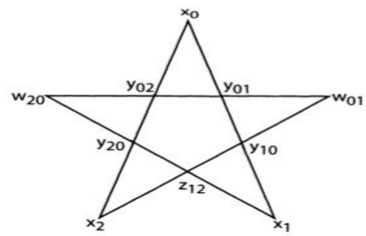

Sierpinski pentagasket

Without loss of generality we consider a=1 where b is not predictable previously. Also we consider the factor r_i is equal to one initially. Then we have

$$\begin{aligned} E_1(\tilde{u}) = {} & (x_0 - y_{01})^2 + (x_0 + y_{02})^2 + b(y_{01} - y_{02})^2 + (w_{01} - y_{01})^2 + (w_{01} - y_{10})^2 + b(y_{01} - y_{10})^2 \\ & + (w_{20} - y_{02})^2 + (w_{20} - y_{20})^2 + b(y_{02} - y_{20})^2 + (x_1 - y_{10})^2 + (x_1 - z_{12})^2 + b(y_{10} - z_{12})^2 \\ & + (x_2 - y_{20})^2 + (x_2 - z_{12})^2 + b(z_{20} - z_{12})^2 \qquad --------------- (2.3) \end{aligned}$$

This equation leads a system in the seven variables $y_{01}, y_{10}, y_{02}, y_{20}, z_{12}, w_{01}, w_{20}$. And their solution will consists of the parameter b. By substituting these solution in the above equation (2.3), we get an expression as the equation (2.2) with the parameters a_1 and b_1, and we obtain the renormalization equation b = $\frac{b_1}{a_1}$, which is a quadratic equation in terms of b with only one positive root. The solution of the mean value equations yields the harmonic extension algorithm where $r_i = \frac{1}{a_1}$. The relation between the energy functions from the equations (2.2) and (2.3) is given by

$E_1(\tilde{u}) = c_0 E_0(u)$---(2.4)

where c_0 is a renormalization factor and for the Sierpinski pentagon fractal it's value is given as c_0=0.4611(approximately).
We use the Δ-Y transform in the following Sierpinski pentagon diagram 2(a), to find the equivalent network with three vertices. We consider the resistence r_1 in the outer sides of the star pentagon and r_2 is the resistence in the inner sides of the pentagon. The five Y transforms are formed from the five Delta shapes in the corners of the pentagon star. In figure 2(a)(i), applying the Delta-Wye transform the figure 2(a)(ii) is obtained.

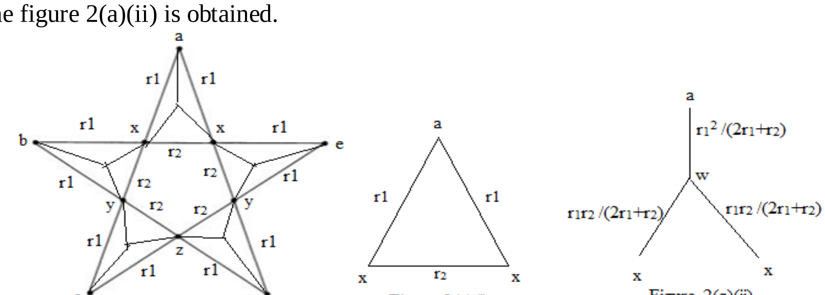

Figure-2(a) Figure-2(a)(i) Figure-2(a)(ii)

Applying the Delta-Wye transform on the resistances of all the five corner triangles and adding the resistors in series the figure-2(b) is obtained and the reistences of all sides are calculated.

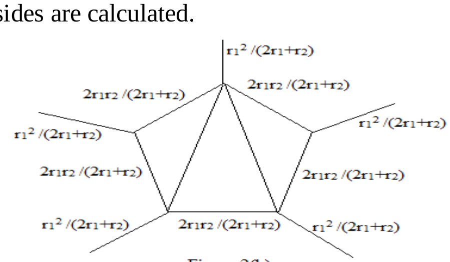

Figure-2(b)

Now from the above diagram-2(b) by cutting and adding resistances in series we get the diagram-2(c) and then again using Δ-Y transform in 2(c) we get the diagram-2(d). And by adding resistances in series in the diagram-2(d) gives the diagram 2(e).

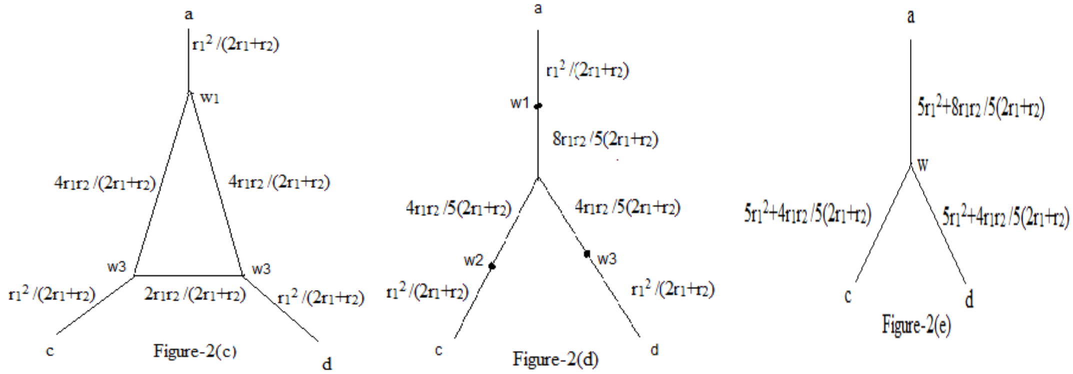

Figure-2(c) Figure-2(d) Figure-2(e)

The renormalization equation describes exactly that the Figure 2(e) must be a multiple of the networt in the figure 2(a)(ii), which yields the renormalization factor. Which yeids the equation

$$\frac{(5r_1^2+4r_1r_2)\big/(5(2r_1+r_2))}{(5r_1^2+8r_1r_2)\big/(5(2r_1+r_2))}=\frac{r_1r_2\big/(2r_1+r_2)}{r_1^2\big/(2r_1+r_2)}$$

$$i.e.,\qquad \frac{(5r_1^2+4r_1r_2)}{(5r_1^2+8r_1r_2)}=\frac{r_1r_2}{r_1^2}$$

$$i.e.,\qquad (5r_1^2+4r_1r_2)r_1=(5r_1^2+8r_1r_2)r_2$$

Simplifying we get, the equation $8r_2^2+r_2r_1-5r_1^2=0$, By considering the r_1 value as 1 unit we get the equation interms of r_2 only and solving for r_2 gives the solution, $r_2=0.7305$.

Thus the value of b is defined by the formula, $b=\frac{1}{r_2}=\frac{1}{0.7305}=1.3689$. This coincide with the value of b in the equation (2.2).

The renormalization factor is obtained by the formula using the value of r_2 is given by, $c_i=\frac{5}{8r_2+5}=\frac{5}{8(0.7305)+5}\approx 0.4611$.

IV. CONCLUSIONS

The Electrical transform is a mathematical technique to simplify the analysis of an electrical network. This electrical network interpretation using the Delta-Wye transform can be applied for the renormalization on the other Sierpinski fractals.

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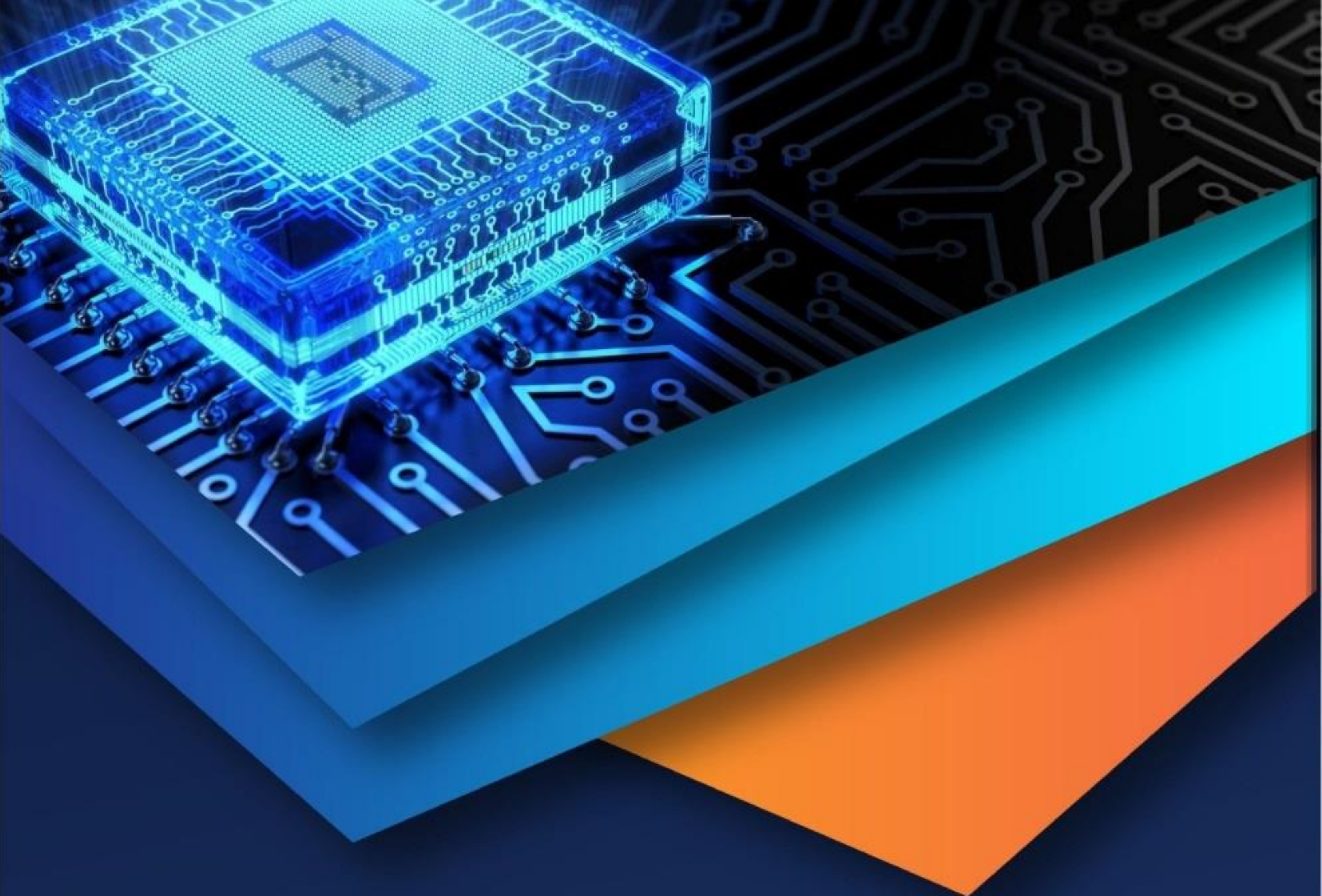

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