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# On the Positive Pell Equation $y^2 = 23x^2 + 13$

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***Abstract: The binary quadratic Diophantine equation represented by the positive pellian $y^2 = 23x^2 + 13$ is analyzed for its non-zero distinct solutions. A few interesting relations among the solutions are given. Further, employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas, parabolas and Pythagorean triangle.***
***Keywords: Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation***.

## I. INTRODUCTION

The binary quadratic equations of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer has been selected by various mathematicians for its non-trivial integer solutions when D takes different integral values[1-4].For an extensive review of various problems, one may refer[5-10]. In this communication, yet another interesting equation given by $y^2 = 23x^2 + 13$ is considered and infinitely many integer solutions are obtained. A few interesting properties among the solutions are presented.

## II. METHOD OF ANALYSIS

The positive Pell equation representing hyperbola under consideration is,

$$y^2 = 23x^2 + 13 \tag{1}$$

The smallest positive integer solutions of (1) are,

$$x_0 = 1, y_0 = 6 \quad D = 23$$

The pellian equation is

$$y^2 = 23x^2 + 1 \tag{2}$$

The initial solution of pellian equation is

$$\tilde{x}_0 = 5, \tilde{y}_0 = 24,$$

The general solution $(x_n, y_n)$ of (2) is given by,

$$\tilde{x}_n = \frac{1}{2\sqrt{23}} g_n, \ \tilde{y}_n = \frac{1}{2} f_n$$

Where,

$$f_n = (24+5\sqrt{23})^{n+1} + (24-5\sqrt{23})^{n+1}$$

$$g_n = (24+5\sqrt{23})^{n+1} - (24-5\sqrt{23})^{n+1}$$

Applying Brahmagupta lemma between $(x_0, y_0)$ and $(\tilde{x}_n, \tilde{y}_n)$ the other integer solution of (1) are given by,

$$x_{n+1} = \frac{1}{2} f_n + \frac{6}{2\sqrt{23}} g_n$$

$$y_{n+1} = \frac{6}{2} f_n + \frac{23}{2\sqrt{23}} g_n$$

The recurrence relation satisfied by the solution $x$ and $y$ are given by,

$$x_{n+3} - 48x_{n+2} + x_{n+1} = 0$$

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$$y_{n+3} - 48y_{n+2} + y_{n+1} = 0 \qquad n=0,1,2,3\ldots..$$

Some numerical examples of $x$ and $y$ satisfying (1) are given in the Table 1 below,

Table 1: Examples

| n | $x_n$ | $y_n$ |
|---|---|---|
| 0 | 1 | 6 |
| 1 | 54 | 259 |
| 2 | 2591 | 12426 |
| 3 | 124314 | 596189 |
| 4 | 5964481 | 28604646 |

From the above table, we observe some interesting relations among the solutions which are presented below.

*A.* $x_n$ values are odd and even alternatively.

*B.* $y_n$ values are even and odd alternatively.

*C.* Each of the following expression is a nasty number:

1) $\frac{6}{13}\left[26 + 12y_{2n+2} - 46x_{2n+2}\right]$

2) $\frac{6}{65}\left[130 + 12x_{2n+3} - 518x_{2n+2}\right]$

3) $\frac{6}{1560}\left[3120 + 6x_{2n+4} - 12426x_{2n+2}\right]$

4) $\frac{6}{312}\left[624 + 12y_{2n+3} - 2484x_{2n+2}\right]$

5) $\frac{6}{14963}\left[29926 + 12y_{2n+4} - 119186x_{2n+2}\right]$

6) $\frac{6}{312}\left[624 + 518y_{2n+2} - 46x_{2n+3}\right]$

7) $\frac{6}{14963}\left[29926 + 24852y_{2n+2} - 46x_{2n+4}\right]$

8) $\frac{6}{1495}\left[2990 + 2484y_{2n+2} - 46y_{2n+3}\right]$

9) $\frac{6}{71760}\left[143520 + 119186y_{2n+2} - 46y_{2n+4}\right]$

10) $\frac{6}{13}\left[26 + 518y_{2n+3} - 2484x_{2n+3}\right]$

11) $\frac{6}{65}\left[130 + 518x_{2n+4} - 24852x_{2n+3}\right]$

12) $\frac{6}{312}\left[624 + 518y_{2n+4} - 119186x_{2n+3}\right]$

13) $\frac{6}{156}\left[312 + 12426y_{2n+3} - 1242x_{2n+4}\right]$

14) $\frac{6}{13}\left[26+24852y_{2n+4}-119186x_{2n+4}\right]$

15) $\frac{6}{1495}\left[2990+119186y_{2n+3}-2484y_{2n+4}\right]$

*D. Each Of The Following Expressions Is A Cubical Integer*

1) $\frac{1}{13}\left[12y_{3n+3}-46x_{3n+3}+36y_{n+1}-138x_{n+1}\right]$

2) $\frac{1}{65}\left[12x_{3n+4}-518x_{3n+3}+36x_{n+2}-1554x_{n+1}\right]$

3) $\frac{1}{1560}\left[6x_{3n+5}-12426x_{3n+3}+18x_{n+3}-37278x_{n+1}\right]$

4) $\frac{1}{132}\left[12y_{3n+4}-2484x_{3n+3}+36y_{n+2}-7452x_{n+1}\right]$

5) $\frac{1}{14963}\left[12y_{3n+5}-119186x_{3n+3}+36y_{n+3}-357558x_{n+1}\right]$

6) $\frac{1}{312}\left[518y_{3n+3}-46x_{3n+4}+1554y_{n+1}-138x_{n+2}\right]$

7) $\frac{1}{14963}\left[24852y_{3n+3}-46x_{3n+5}+149112y_{n+1}-138x_{n+3}\right]$

8) $\frac{1}{1495}\left[2484y_{3n+3}-46y_{3n+4}+7452y_{n+1}-138y_{n+2}\right]$

9) $\frac{1}{71760}\left[119186y_{3n+3}-46y_{3n+5}+357558y_{n+1}-138y_{n+3}\right]$

10) $\frac{1}{65}\left[518x_{3n+5}-24852x_{3n+4}+1554x_{n+3}-74556x_{n+2}\right]$

11) $\frac{1}{13}\left[518y_{3n+4}-2484x_{3n+4}+1554y_{n+2}-7452x_{n+2}\right]$

12) $\frac{1}{312}\left[518y_{3n+5}-119186x_{3n+4}+1554y_{n+3}-357558x_{n+2}\right]$

13) $\frac{1}{156}\left[12426y_{3n+4\backslash}-1242x_{3n+5}+37278y_{n+2}-3726x_{n+3}\right]$

14) $\frac{1}{13}\left[24852y_{3n+5\backslash}-119186x_{3n+5}+74556y_{n+3}-357558x_{n+3}\right]$

15) $\frac{1}{1495}\left[119186y_{3n+4\backslash}-2484y_{3n+5}+357558y_{n+2}-7452y_{n+3}\right]$

*E. Each Of The Following Expressions Is A Biquadratic Integer*

1) $\frac{1}{13}\left[12y_{4n+4}-46x_{4n+4}+48y_{2n+2}-184x_{2n+2}+78\right]$

2) $\frac{1}{65}\left[12x_{4n+5} - 518x_{4n+4} + 48x_{2n+3} - 2072x_{2n+2} + 390\right]$

3) $\frac{1}{1560}\left[6x_{4n+6} - 12426x_{4n+4} + 24x_{2n+4} - 49704x_{2n+2} + 9360\right]$

4) $\frac{1}{1312}\left[12y_{4n+5} - 2484x_{4n+4} + 48y_{2n+3} - 9936x_{2n+2} + 1872\right]$

5) $\frac{1}{14963}\left[12y_{4n+6} - 119186x_{4n+4} + 48y_{2n+4} - 476744x_{2n+2} + 89778\right]$

6) $\frac{1}{312}\left[518y_{4n+4} - 46x_{4n+5} + 2708y_{2n+2} - 184x_{2n+3} + 1872\right]$

7) $\frac{1}{14963}\left[24852y_{4n+4} - 46x_{4n+6} + 99408y_{2n+2} - 184x_{2n+4} + 89778\right]$

8) $\frac{1}{1495}\left[2484y_{4n+4} - 46y_{4n+5} + 9936y_{2n+2} - 184y_{2n+3} + 8970\right]$

9) $\frac{1}{71760}\left[11986y_{4n+4} - 46y_{4n+6} + 47664y_{2n+2} - 184y_{2n+4} + 430560\right]$

10) $\frac{1}{65}\left[518x_{4n+6} - 24852x_{4n+5} + 2072x_{2n+4} - 99408x_{2n+3} + 390\right]$

11) $\frac{1}{13}\left[518y_{4n+5} - 2484x_{4n+5} + 2072y_{2n+3} - 9836x_{2n+3} + 78\right]$

12) $\frac{1}{312}\left[518y_{4n+6} - 119186x_{4n+5} + 2072y_{2n+4} - 476744x_{2n+3} + 1872\right]$

13) $\frac{1}{156}\left[12426y_{4n+5} - 1242x_{4n+6} + 49704y_{2n+3} - 4968x_{2n+4} + 936\right]$

14) $\frac{1}{13}\left[24852y_{4n+6} - 119186x_{4n+6} + 99408y_{2n+4} - 476744x_{2n+4} + 78\right]$

15) $\frac{1}{1495}\left[119186y_{4n+5} - 2484y_{4n+6} + 476744y_{2n+3} - 9936y_{2n+4} + 8970\right]$

### F. *Each Of The Following Expression Is A Quintic Integer*

1) $\frac{1}{13}\left[12y_{5n+5} - 46x_{5n+5} + 60y_{3n+3} - 230x_{3n+3} + 120y_{n+1} - 460x_{n+1}\right]$

2) $\frac{1}{65}\left[12x_{5n+6} - 518x_{5n+5} + 60x_{3n+4} - 2590x_{3n+3} + 120x_{n+2} + 1036x_{n+1}\right]$

3) $\frac{1}{1560}\left[6x_{5n+7} - 12426x_{5n+5} - 30x_{3n+5} - 62130x_{3n+3} + 60x_{n+3} - 124260x_{n+1}\right]$

4) $\frac{1}{312}\left[12y_{5n+6} - 2484x_{5n+5} + 60y_{3n+4} - 12420x_{3n+3} + 120y_{n+2} - 24840x_{n+1}\right]$

5) $\frac{1}{14963}\left[12y_{5n+7} - 119186x_{5n+5} + 60y_{3n+5} - 595930x_{3n+3} + 120y_{n+3} - 1191860x_{n+1}\right]$

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6) $\frac{1}{312}\left[518y_{5n+5}-46x_{5n+6}+2590y_{3n+3}-230x_{3n+4}+5180y_{n+1}-460x_{n+2}\right]$

7) $\frac{1}{14963}\left[24852y_{5n+5}-46x_{5n+7}+124260y_{3n+3}-230x_{3n+5}+248520y_{n+1}-460x_{n+3}\right]$

8) $\frac{1}{1495}\left[2484y_{5n+5}-46y_{5n+7}+12420y_{3n+3}-230y_{3n+5}+24840y_{n+1}-460y_{n+2}\right]$

9) $\frac{1}{71760}\left[119186y_{5n+5}-46y_{5n+7}+595930y_{3n+3}-230y_{3n+5}+1191860y_{n+1}-460y_{n+3}\right]$

10) $\frac{1}{65}\left[518x_{5n+7}-24852x_{5n+6}+2590x_{3n+5}-124260x_{3n+4}+5180x_{n+3}-248520x_{n+2}\right]$

11) $\frac{1}{13}\left[518y_{5n+6}-2484x_{5n+6}+2590y_{3n+4}-12420x_{3n+4}+5180y_{n+2}-24840x_{n+2}\right]$

12) $\frac{1}{312}\left[518y_{5n+7}-119186x_{5n+6}+2990y_{3n+5}-595930x_{3n+4}+5180y_{n+3}-1191860x_{n+2}\right]$

13) $\frac{1}{156}\left[12426y_{5n+6}-1242x_{5n+7}+62130y_{3n+4}-6210x_{3n+5}+124260y_{n+2}-12420x_{n+3}\right]$

14) $\frac{1}{13}\left[24852y_{5n+7}-119186x_{5n+7}+124260y_{3n+5}-595930x_{3n+5}+248520y_{n+3}-1191860x_{n+3}\right]$

15) $\frac{1}{1495}\left[119186y_{5n+6}-2484y_{5n+7}+595930y_{3n+4}-12420y_{3n+5}+1191860y_{n+2}-24840y_{n+3}\right]$

### *G. Relations Among The Solutions Are Given Below*

1) $x_{n+2}=5y_{n+1}+24x_{n+1}$
2) $x_{n+3}=240y_{n+1}+1151x_{n+1}$
3) $y_{n+2}=24y_{n+1}+115x_{n+1}$
4) $y_{n+3}=1151y_{n+1}+5520x_{n+1}$
5) $x_{n+3}=48x_{n+2}-x_{n+1}$
6) $5y_{n+2}=24x_{n+2}-x_{n+1}$
7) $5y_{n+3}=1151x_{n+2}-24x_{n+1}$
8) $10y_{n+2}=x_{n+3}-x_{n+1}$
9) $240y_{n+3}=1151x_{n+3}-x_{n+1}$
10) $24y_{n+3}=1151y_{n+2}+115x_{n+1}$
11) $24x_{n+3}=5y_{n+1}+1151x_{n+2}$
12) $24y_{n+2}=y_{n+1}+115x_{n+2}$
13) $24y_{n+3}=24y_{n+1}+5520x_{n+2}$
14) $1151y_{n+2}=24y_{n+1}+115x_{n+3}$
15) $1151y_{n+3}=y_{n+1}+5520x_{n+3}$

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16) $115y_{n+3} = 5520y_{n+2} - 115y_{n+1}$
17) $5y_{n+2} = x_{n+3} - 24x_{n+2}$
18) $5y_{n+3} = 24x_{n+3} - x_{n+2}$
19) $y_{n+3} = 24y_{n+2} + 115x_{n+2}$
20) $24y_{n+3} = y_{n+2} + 115x_{n+3}$

## III.REMARKABLE OBSERVATION

*A.* Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbolas which are presented in table 2 below

Table 2: Hyperbola

| S.NO | Hyperbola | (X,Y) |
|---|---|---|
| 1 | $Y^2 - 23X^2 = 676$ | $(12x_{n+1} - 2y_{n+1}, 12y_{n+1} - 46x_{n+1})$ |
| 2 | $Y^2 - 23X^2 = 169000$ | $(108x_{n+1} - 2x_{n+2}, 12x_{n+2} - 518x_{n+1})$ |
| 3 | $Y^2 - 23X^2 = 9734400$ | $(2591x_{n+1} - x_{n+3}, 6x_{n+3} - 12426x_{n+1})$ |
| 4 | $Y^2 - 23X^2 = 3896376$ | $(518x_{n+1} - 2y_{n+2}, 12y_{n+2} - 2484x_{n+1})$ |
| 5 | $Y^2 - 23X^2 = 895565476$ | $(24852x_{n+1} - 2y_{n+3}, 12y_{n+3} - 119186x_{n+1})$ |
| 6 | $Y^2 - 23X^2 = 389376$ | $(12x_{n+2} - 108y_{n+1}, 518y_{n+1} - 46x_{n+2})$ |
| 7 | $Y^2 - 23X^2 = 895565476$ | $(12x_{n+3} - 5182y_{n+1}, 24852y_{n+1} - 46x_{n+3})$ |
| 8 | $Y^2 - 23X^2 = 8940100$ | $(12y_{n+2} - 518y_{n+1}, 2484y_{n+1} - 46y_{n+2})$ |
| 9 | $Y^2 - 23X^2 = 5149497600$ | $(6y_{n+3} - 12426y_{n+1}, 59593y_{n+1} - 23y_{n+3})$ |
| 10 | $Y^2 - 23X^2 = 16900$ | $(5182x_{n+2} - 24852x_{n+3}, 518x_{n+3} - 24852x_{n+2})$ |
| 11 | $Y^2 - 23X^2 = 676$ | $(518x_{n+2} - 108y_{n+2}, 518y_{n+2} - 2484x_{n+2})$ |
| 12 | $Y^2 - 23X^2 = 389376$ | $(24852x_{n+2} - 108y_{n+3}, 518y_{n+3} - 119186x_{n+2})$ |
| 13 | $Y^2 - 23X^2 = 97344$ | $(259x_{n+3} - 2591y_{n+2}, 12426y_{n+2} - 1242x_{n+3})$ |
| 14 | $Y^2 - 23X^2 = 676$ | $(24852x_{n+3} - 5182y_{n+3}, 24852y_{n+3} - 119186x_{n+3})$ |
| 15 | $Y^2 - 23X^2 = 8940100$ | $(518y_{n+3} - 24852y_{n+2}, 119186y_{n+2} - 2484y_{n+3})$ |

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*B.* Employing linear combination among the solutions of (1), one may generate integer solutions for other choices of parabolas which are presented in table 3 below

Table 3: Parabola

| S.NO | Parabola | (X,Y) |
|---|---|---|
| 1 | $13Y-23X^2=676$ | $(12x_{n+1}-2y_{n+1},12y_{2n+2}-46x_{2n+2}+26)$ |
| 2 | $65Y-23X^2=16900$ | $(108x_{n+1}-2x_{n+2},12x_{2n+3}-518x_{2n+2}+130)$ |
| 3 | $1560Y-23X^2=9734400$ | $(2591x_{n+1}-x_{n+3},6x_{2n+4}-12426x_{2n+2}+3120)$ |
| 4 | $312Y-23X^2=389376$ | $(518x_{n+1}-2y_{n+2},12y_{2n+3}-2484x_{2n+2}+624)$ |
| 5 | $14963Y-23X^2=895565476$ | $(24852x_{n+1}-2y_{n+3},12y_{2n+4}-119186x_{2n+2}+29926)$ |
| 6 | $312Y-23X^2=389376$ | $(12x_{n+2}-108y_{n+1},518y_{2n+2}-46x_{2n+3}+624)$ |
| 7 | $14963Y-23X^2=895565476$ | $(12x_{n+3}-5182y_{n+1},24852y_{2n+2}-46x_{2n+4}+29926)$ |
| 8 | $1495Y-23X^2=8940100$ | $(12y_{n+2}-518y_{n+1},2484y_{2n+2}-46y_{2n+3}+2990)$ |
| 9 | $35880Y-23X^2=5149497600$ | $(6y_{n+3}-12426y_{n+1},59593y_{2n+2}-23y_{2n+4}+71760)$ |
| 10 | $65Y-23X^2=16900$ | $(5182x_{n+2}-108x_{n+3},518x_{2n+4}-24852x_{2n+3}+130)$ |
| 11 | $13Y-23X^2=676$ | $(518x_{n+2}-108y_{n+2},518y_{2n+3}-2484x_{2n+3}+26)$ |
| 12 | $312Y-23X^2=389376$ | $(24852x_{n+2}-108y_{n+3},518y_{2n+4}-119186x_{2n+3}+624)$ |
| 13 | $156Y-23X^2=97344$ | $(259x_{n+3}-2591y_{n+2},12426y_{2n+3}-1242x_{2n+4}+312)$ |
| 14 | $13Y-23X^2=676$ | $(24852x_{n+3}-5182y_{n+3},24852y_{2n+4}-119186x_{2n+4}+26)$ |
| 15 | $1495Y-23X^2=8940100$ | $(518y_{n+3}-24852y_{n+2},119186y_{2n+3}-2484y_{2n+4}+2990)$ |

*C. Some Special Cases Among The Solutions Are Given Below*

1) $P_y^{10}(t_{3,x+1})^2=207P_x^6(t_{3,y})^2+13(t_{3,y})^2(t_{3,x+1})^2$

2) $9P_y^6(t_{3,x})^2=23P_x^{10}(t_{3,y+1})^2+13(t_{3,x})^2(t_{3,y+1})^2$

3) $P_y^{10}(t_{3,2x-2})^2=23(6P_{x-1}^4)^2(t_{3,y})^2+13(t_{3,y})^2(t_{3,2x-2})^2$

4) $36P_{y-1}^8(t_{3,x})^2=23P_x^{10}(t_{3,2y-2})^2+13(t_{3,x})^2(t_{3,2y-2})^2$

5) $9P_y^6(t_{3,2x-2})^2=23(36P_{x-1}^8)(t_{3,y+1})^2+13(t_{3,2x-2})^2(t_{3,y+1})^2$

6) $(6P_{y-1}^4)^2(t_{3,x+1})^2=23(3P_x^3)^2(t_{3,2y-2})^2+13(t_{3,x+1})^2(t_{3,2y-2})^2$

## IV. CONCLUSION

In this paper, we have presented infinitely many integer solutions for the positive Pell Equations $y^2=23x^2+13$ As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of Pell Equations and determine their integer solutions along with suitable properties

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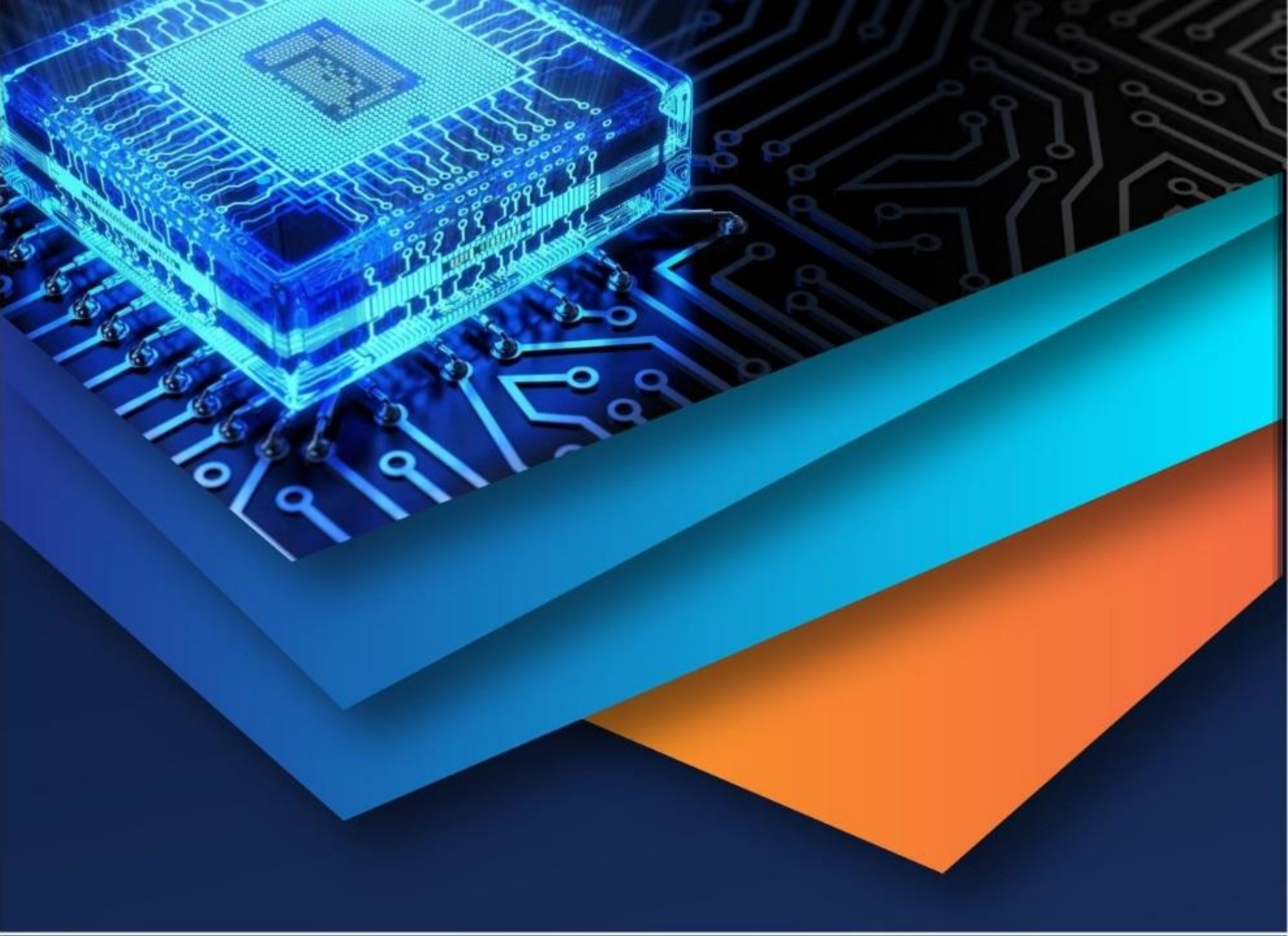

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