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A Numerical Solution of First order Simultaneous Fuzzy Differential Equations by Sixth Order Runge-Kutta Method

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Abstract: In this paper we introduce a new technique for getting the approximate solution of "First order Simultaneous Fuzzy differential equation by sixth order Runge-Kutta method" based on Seikkala derivative of fuzzy process [10] in order to increase the order of the accuracy of the solutions. This proposed method is discussed in details followed by a complete error analysis.
Keywords: Cauchy Problem - Runge-Kutta sixth order Method – First order simultaneous fuzzy differential equation – Error Analysis

I. INTRODUCTION

The concept of fuzzy derivatives was first introduced by S.L. Chang and L.A. Zadeh [6], it was followed up by D. Dubois and Prade [7] who used the extension principle in their approach. Puri and D.A. Ralesec [15] and R. Goetschel and W. Voxman [9] contributed towards the differential of fuzzy functions.

The fuzzy differential equation and initial value problems were extensively studied by O. Kaleva [10, 11] and by S. Seikkala [16]. Numerical solution of fuzzy differential equations has been introduced by M. Ma, M. Friedman, A. Kandel [13] through Euler method and by S. Abbasbandy and T. Allahviranloo [1] by Taylor method. Runge-Kutta methods have also been studied by authors [2, 14].

In this paper organized as follows: In Section 2, some basic definitions and results on fuzzy numbers and fuzzy derivatives. Section 3 contains the definition of fuzzy Cauchy problem with initial conditions. Section 4, discussed about sixth order Runge-Kutta method and defined to solve the first order simultaneous fuzzy differential equation with initial value problem. The proposed method is illustrated and solved the numerical example in section 5, also the result is compared with Euler's method and Runge-Kutta fourth order method with the approximation solution by Runge-Kutta sixth order method.

II. PRELIMINARIES

Consider the first order simultaneous differential equation

$$\frac{dy}{dt} = f(t, y, z) \quad \& \quad \frac{dz}{dt} = g(t, y, z), \ t_0 \le t \le b, \text{ initial conditions } y(t_0) = y_0,\ z(t_0) = z_0 \qquad \text{. . . (2.1)}$$

The basis of all Runge-Kutta methods is to express the difference between the value of y at t_{n+1} and t_n as

$$y_{n+1} - y_n = \sum_{i=0}^{m} w_i k_i \text{ ; where } w_i \text{ are constant i and}$$

$$k_i = hf(t_n+a_ih,\ y_n + \sum_{j=1}^{i-1} c_{ij} k_j)$$

Most efforts to increase the order of accuracy of the Runge-Kutta methods have been accomplished by increasing the number of Taylor's series terms used and thus the number of functional evaluations required [5]. The method proposed by Goeken. D and Johnson. O [8] introduces new terms involving higher order derivatives of 'f' in the Runge-Kutta k_i terms (i > 1) to obtain a higher order of accuracy without a corresponding increase in evaluations of f, but with the addition of evaluations of f^1.

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Consider

$y(t_{n+1}) = y(t_n) + w_1k_1 + w_2k_2 + w_3k_3 + w_4k_4 + w_5k_5 + w_6k_6 + w_7k_7$

where

$k_1 = h\, f(t_n, y(t_n))$

$k_2 = h\, f(t_n+c_2h, y(t_n)+a_{21}k_1)$

$k_3 = h\, f(t_n+c_3h, y(t_n)+a_{31}k_1+a_{32}k_2)$

$k_4 = h\, f(t_n+c_4h, y(t_n)+a_{41}k_1+a_{42}k_2+a_{43}k_3)$

$k_5 = h\, f(t_n+c_5h, y(t_n)+a_{51}k_1+a_{52}k_2+a_{53}k_3+a_{54}k_4)$

$k_6 = h\, f(t_n+c_6h, y(t_n)+a_{61}k_1+a_{62}k_2+a_{63}k_3+a_{64}k_4+a_{65}k_5+a_{66}k_6)$

$k_7 = h\, f(t_n+c_7h, y(t_n)+a_{71}k_1+a_{72}k_2+a_{73}k_3+a_{74}k_4 + a_{75}k_5+a_{76}k_6 + a_{77}k_7)$ . . . (2.2)

Utilizing the Taylor's series expansion techniques, Runge-Kutta method of order sixth is given by,

$$y_{n+1} = y_n + \frac{9k_1 + 64k_3 + 49k_5 + 49k_6 + 9k_7}{180}$$

where

$k_1 = h\, f(t_n, y(t_n))$

$k_2 = h\, f(t_n+ vh, y(t_n)+ vk_1)$

$k_3 = h\, f(t_n+ \frac{h}{2}, y(t_n)+ ((4v – 1)k_1 + k_2)/(8v))$

$k_4 = h\, f(t_n+ \frac{2h}{3}, y(t_n)+((10v – 2)k_1 + 2k_2 + 8vk_3)/(27v))$

$k_5 = h\, f(t_n+(7+4.582576)\frac{h}{14}, y(t_n)+(-((77v–56)+(17v–8)4.582576)k_1–8(7+4.582576)k_2$
$+ 48(7+4.582576)vk_3 – 3(21+4.582576)vk_4)/(392v)$

$k_6 = h\, f(t_n+(7 - 4.582576)\frac{h}{14}, y(t_n)+(-5((287v–56) - (59v–8)4.582576)k_1–40(7-4.582576)k_2$
$+ 320(4.582576)vk_3 + 3(21-121(4.582576))vk_4 + 392(6-4.582576)vk_5) /(1960v)$

$k_7 = h\, f(t_n + h, y(t_n) + (15((30v – 8) - 7v(4.582576))k_1+120k_2 – 40(5 + 7(4.582576))vk_3$
$+ 63(2 + 3(4.582576))vk_4 – 14(49 - 9(4.582576))vk_5 + 70(7 + 4.582576)vk_6)/(180v)$ . . . (2.3)

and

$$z_{n+1} = z_n + \frac{9l_1 + 64l_3 + 49l_5 + 49l_6 + 9l_7}{180}$$

where

$l_1 = h\, g(t_n, z(t_n))$

$l_2 = h\, g(t_n+ vh, z(t_n)+ vl_1)$

$l_3 = h\, g(t_n+ \frac{h}{2}, z(t_n)+ ((4v – 1)l_1 + l_2)/(8v))$

$l_4 = h\, g(t_n+ \frac{2h}{3}, z(t_n)+((10v – 2)l_1 + 2l_2 + 8vl_3)/(27v))$

$l_5 = h\, g(t_n+(7+4.582576)\frac{h}{14}, z(t_n)+(-((77v–56)+(17v–8)4.582576)l_1–8(7+4.582576)l_2$
$+ 48(7+4.582576)vl_3 – 3(21+4.582576)vl_4)/(392v)$

$l_6 = h\, g(t_n+(7 - 4.582576)\frac{h}{14}, z(t_n)+(-5((287v–56) - (59v–8)4.582576)l_1–40(7-4.582576)l_2$

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$+ 320(4.582576)vl_3 + 3(21-121(4.582576))vl_4 + 392(6-4.582576)vl_5) /(1960v)$

$l_7 =$ $h\, g(t_n + h, z(t_n) + (15((30v - 8) - 7v(4.582576))l_1 + 120l_2 - 40(5 + 7(4.582576))vl_3$
$+ 63(2 + 3(4.582576))vl_4 - 14(49 - 9(4.582576))vl_5 + 70(7 + 4.582576)vl_6)/(180v)$

. . . (2.4)

A. Definition – 2.1

A fuzzy number u as a fuzzy subset of R ie) u : R → [0, 1] satisfying the following conditions.

1) u is normal, ie $\exists\, x_0 \in R \ni u(x_0) = 1$
2) u is a convex fuzzy set
   ie) $u(tx + (1-t)y) \geq \min\{u(x), u(y)\}$, $\forall\, t \in [0, 1]$ and $x, y \in R$
3) u is upper semi continuous on R
4) $\overline{\{x \in R, u(x) > 0\}}$ is compact

The set E is the family of fuzzy numbers and arbitrary fuzzy number is represented by an ordered pair of functions $\left(\underline{u}(r), \overline{u}(r)\right)$, $0 \leq r \leq 1$ has satisfies the following requirements

a) $\underline{u}(r)$ is a bounded left continuous non-decreasing function over [0, 1] with respect to any 'r'.
b) $\overline{u}(r)$ is a bounded right continuous non-increasing function over [0, 1] with respect to any 'r'.
c) $\underline{u}(r) \leq \overline{u}(r)$, $0 \leq r \leq 1$, r-level cut is $[u]_r = \{x/u(x) \geq r\}$, $0 \leq r \leq 1$ as a closed & bounded interval denoted by $[u]_r = \left[\underline{u}(r), \overline{u}(r)\right]$ and clearly $[u]_0 = \{x/u(x) > 0\}$ is compact.

B. Definition – 2.2

A triangular fuzzy number u is a fuzzy set in E that is characterised by an ordered triple $(u_l, u_c, u_r) \in R^3$ with $u_l < u_c < u_r$ such that $[u]_0 = [u_l : u_r]$ and $[u]_1 = [u_c]$. The membership function of the triangular fuzzy number u is given by

$$u(x) = \begin{cases} \dfrac{x - u_l}{u_c - u_l}, & u_l \leq x \leq u_c \\ 1 & x = u_c \\ \dfrac{u_r - x}{u_r - u_c}, & u_c \leq x \leq u_r \end{cases}$$

and we will have . . . (2.4)

i). $u > 0$ if $u_l > 0$
ii). $u \geq 0$ if $u_l \geq 0$
iii). $u < 0$ if $u_c < 0$
iv). $u \leq 0$ if $u_c \leq 0$

Let I be a real interval. A mapping y : I → E as called a fuzzy process and its α - level set is denoted by

$$[y(t)]_\alpha = \left[\underline{y}(t, y), \overline{y}(t, y)\right], \; t \in I,\; 0 < \alpha \leq I, \quad [z(t)]_\alpha = \left[\underline{z}(t, z), \overline{z}(t, z)\right], \; t \in I,\; 0 < \alpha \leq I.$$

The Seikkala derivative y(t) of a fuzzy process is defined by $\left[y^1(t)\right]_\alpha = \left[\underline{y}^1(t, y), \overline{y}^1(t, y)\right]$, $t \in I$, $0 < \alpha \leq I$ provided the equation defines fuzzy number as in [11]. Similarly, let I be a real interval. A mapping z: I→E is called a fuzzy process and its α - level set is denoted by $[z(t)]_\alpha = \left[\underline{z}(t, y, z), \overline{z}(t, y, z)\right]$, $t \in I$, $0 < \alpha \leq I$. For $u, v \in E$ and $\lambda \in \Re$, the $u + v$ and the product λu can be defined by $[u + v]_\alpha = [u]_\alpha + [v]_\alpha$ and $[\lambda u]_\alpha = \lambda[u]_\alpha$, where $\alpha \in [0, 1]$ and $[u]_\alpha + [v]_\alpha$ means the addition of two intervals of $\Re$ and $[u]_\alpha$ means the product between a scalar and a subset of $\Re$.

Arithmetic operation of arbitrary fuzzy numbers $u = \left(\underline{u}(r), \overline{u}(r)\right)$ and $v = \left(\underline{v}(r), \overline{v}(r)\right)$ and $\lambda \in \Re$ can be defined as

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i). u = v if $\underline{u}(r) = \underline{v}(r)$ and $\overline{u}(r) = \overline{v}(r)$

ii). u + v = $\left(\underline{u}(r) + \underline{v}(r), \overline{u}(r) + \overline{v}(r)\right)$

iii). u - v = $\left(\underline{u}(r) - \overline{v}(r), \overline{u}(r) - \underline{v}(r)\right)$

iv). λu = $\left(\lambda \underline{u}(r), \lambda \overline{u}(r)\right)$ if $\lambda \geq 0$

= $\left(\lambda \overline{u}(r), \lambda \underline{u}(r)\right)$ if $\lambda < 0$

III. A FUZZY CAUCHY PROBLEM

Consider the first order simultaneous differential equation

$\frac{dy}{dt} = f(t, y, z)$ & $\frac{dz}{dt} = g(t, y, z)$, $t_0 \leq t \leq b$, initial conditions $y(t_0) = y_0$, $z(t_0) = z_0$

Let the function f be a continuous mapping from R x R → R and $y_0 \in E$ with r-level sets $[y_0]_r = \left[\underline{y}(0:r), \overline{y}(0:r)\right]$, r ∈ [0, 1] and the function g be a continuous mapping from R x R → R and $z_0 \in E$ with r-level sets $[z_0]_r = \left[\underline{z}(0:r), \overline{z}(0:r)\right]$, r ∈ [0, 1]. The extension principle of Zadeh [4] leads to the definition of f(t, y, z) and g(t, y, z), y = y(t), z = z(t) are the fuzzy numbers.

$[f(t,y,z)]_r = \left[\underline{f}(t,y,z:r), \overline{f}(t,y,z:r)\right]$, r∈[0, 1], It follows that

$$\underline{f}(t,y,z:r) = \min\left\{f(t,u,v) \,\middle|\, u \in \left[\underline{y}(r), \overline{y}(r)\right], v \in \left[\underline{z}(r), \overline{z}(r)\right]\right\}$$
$$\overline{f}(t,y,z:r) = \max\left\{f(t,u,v) \,\middle|\, u \in \left[\underline{y}(r), \overline{y}(r)\right], v \in \left[\underline{z}(r), \overline{z}(r)\right]\right\} \quad \ldots (3.1)$$

and

$$\underline{g}(t,y,z:r) = \min\left\{g(t,u,v) \,\middle|\, u \in \left[\underline{y}(r), \overline{y}(r)\right], v \in \left[\underline{z}(r), \overline{z}(r)\right]\right\}$$
$$\overline{g}(t,y,z:r) = \max\left\{g(t,u,v) \,\middle|\, u \in \left[\underline{y}(r), \overline{y}(r)\right], v \in \left[\underline{z}(r), \overline{z}(r)\right]\right\} \quad \ldots (3.2)$$

A. *Theorem*

Let f satisfy $\left|f(t,v) - f(t,\overline{v})\right| \leq g\left(t, \left|v - \overline{v}\right|\right)$, $t \geq 0$ and v, $\overline{v}$ ∈ R, ... (3.3)

where g : $R_+ \times R_+ \to R_+$ is a continuous mapping such that r → g(t, r) is non-decreasing an initial value problem $u^1(t) = g(t, u(t))$, $u(0) = u_0$ ... (3.3) has a solution on R_+ or $u_0 > 0$ and that $u(t) \equiv 0$ is the only solution of (3.3) for $u_0 = 0$ then the fuzzy initial value problem has a unique fuzzy solution.

Proof : See [16].

IV. FUZZY SIXTH ORDER RUNGE–KUTTA METHOD

Let the exact solution of the given equation $[Y(t)]_r = \left[\underline{Y}(t:r), \overline{Y}(t:r)\right]$ is approximated by some solution $[y(t)]_r = \left[\underline{y}(t:r), \overline{y}(t:r)\right]$ and the exact solution of the given equation $[Z(t)]_r = \left[\underline{Z}(t:r), \overline{Z}(t:r)\right]$ is approximated by some solution $[z(t)]_r = \left[\underline{z}(t:r), \overline{z}(t:r)\right]$ also we define

$$\underline{y}(t_{n+1}:r) - \underline{y}(t_n:r) = \sum_{i=1}^{7} w_i \underline{k}_i \quad \overline{y}(t_{n+1}:r) - \overline{y}(t_n:r) = \sum_{i=1}^{7} w_i \overline{k}_i$$ where w_i's are constant

$[k_i(t, y(t,r))]_r = \left[\underline{k}_i(t, y(t,r)), \overline{k}_i(t, y(t,r))\right]$, i = 1, 2, 3, 4, 5, 6 and 7; where

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$$\underline{k_1}(t, y(t:r)) = hf\left(t_n, \underline{y}(t_n:r)\right) \qquad \overline{k_1}(t, y(t:r)) = hf\left(t_n, \overline{y}(t_n:r)\right)$$

$$\underline{k_2}(t, y(t:r)) = hf\left(t_n + \frac{h}{2}, \underline{y}(t_n:r) + v\underline{k_1}\right) \qquad \overline{k_2}(t, y(t:r)) = hf\left(t_n + \frac{h}{2}, \overline{y}(t_n:r) + v\overline{k_1}\right)$$

$$\underline{k_3}(t, y(t:r)) = hf\left(t_n + \frac{h}{2}, \underline{y}(t_n:r) + \left((4v-1)\underline{k_1} + \underline{k_2}\right)/8v\right)$$

$$\overline{k_3}(t, y(t:r)) = hf\left(t_n + \frac{h}{2}, \overline{y}(t_n:r) + \left((4v-1)\overline{k_1} + \overline{k_2}\right)/8v\right)$$

$$\underline{k_4}(t, y(t:r)) = hf\left(t_n + \frac{2h}{3}, \underline{y}(t_n:r) + \left((10v-2)\underline{k_1} + 2\underline{k_2} + 8v\underline{k_3}\right)/27v\right)$$

$$\overline{k_4}(t, y(t:r)) = hf\left(t_n + \frac{2h}{3}, \overline{y}(t_n:r) + \left((10v-2)\overline{k_1} + 2\overline{k_2} + 8v\overline{k_3}\right)/27v\right)$$

$$\underline{k_5}(t, y(t:r)) = hf\left(t_n + (7+4.582576)\frac{h}{14}, \underline{y}(t_n:r) + \begin{pmatrix} -((77v-56)+(17v-8)4.582576)\underline{k_1} \\ -8(7+4.582576)\underline{k_2} + 48(7+4.582576)v\underline{k_3} \\ -3(21+4.582576)v\underline{k_4} \end{pmatrix}/392v\right)$$

$$\overline{k_5}(t, y(t:r)) = hf\left(t_n + (7+4.582576)\frac{h}{14}, \overline{y}(t_n:r) + \begin{pmatrix} -((77v-56)+(17v-8)4.582576)\overline{k_1} \\ -8(7+4.582576)\overline{k_2} + 48(7+4.582576)v\overline{k_3} \\ -3(21+4.582576)v\overline{k_4} \end{pmatrix}/392v\right)$$

$$\underline{k_6}(t, y(t:r)) = hf\left(t_n + (7-4.582576)\frac{h}{14}, \underline{y}(t_n:r) + \begin{pmatrix} -5((287v-56)-(59v-8)4.582576)\underline{k_1} \\ -40(7-4.582576)\underline{k_2} + 320(4.582576)v\underline{k_3} \\ +3(21-121(4.582576))v\underline{k_4} + 392(6-(4.582576))v\underline{k_5} \end{pmatrix}/1960v\right)$$

$$\overline{k_6}(t, y(t:r)) = hf\left(t_n + (7-4.582576)\frac{h}{14}, \overline{y}(t_n:r) + \begin{pmatrix} -5((287v-56)-(59v-8)4.582576)\overline{k_1} \\ -40(7-4.582576)\overline{k_2} + 320(4.582576)v\overline{k_3} \\ +3(21-121(4.582576))v\overline{k_4} + 392(6-(4.582576))v\overline{k_5} \end{pmatrix}/1960v\right)$$

$$\underline{k_7}(t, y(t:r)) = hf\left(t_n + h, \underline{y}(t_n:r) + \begin{pmatrix} 15((30v-8)7v(4.582576))\underline{k_1} + 120\underline{k_2} - 40(5+7(4.582576))v\underline{k_3} \\ +63(2+3(4.582576))v\underline{k_4} - 14(49-9(4.582576))v\underline{k_5} \\ +70(7+(4.582576))v\underline{k_6} \end{pmatrix}/180v\right)$$

$$\overline{k_7}(t, y(t:r)) = hf\left(t_n + h, \overline{y}(t_n:r) + \begin{pmatrix} 15((30v-8)-7v(4.582576))\overline{k_1} + 120\overline{k_2} - 40(5+7(4.582576))v\overline{k_3} \\ +63(2+3(4.582576))v\overline{k_4} - 14(49-9(4.582576))v\overline{k_5} \\ +70(7+(4.582576))v\overline{k_6} \end{pmatrix}/180v\right) \quad \ldots (4.1)$$

also define

$\underline{z}(t_{n+1}:r) - \underline{z}(t_n:r) = \sum_{i=1}^{7} w_i \underline{l_i}$ $\quad \overline{z}(t_{n+1}:r) - \overline{z}(t_n:r) = \sum_{i=1}^{7} w_i \overline{l_i}$ where l_i's are constant

$[l_i(t, z(t,r))]_r = \left[\underline{l_i}(t, z(t,r)), \overline{l_i}(t, z(t,r))\right]$, i = 1, 2, 3, 4, 5, 6 and 7

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where

$$\underline{l_1}(t,z(t:r)) = hg(t_n, \underline{z}(t_n:r)) \qquad \overline{l_1}(t,z(t:r)) = hg(t_n, \overline{z}(t_n:r))$$

$$\underline{l_2}(t,z(t:r)) = hg\left(t_n + \frac{h}{2}, \underline{z}(t_n:r) + v\underline{l_1}\right) \qquad \overline{l_2}(t,z(t:r)) = hg\left(t_n + \frac{h}{2}, \overline{z}(t_n:r) + v\overline{l_1}\right)$$

$$\underline{l_3}(t,z(t:r)) = hg\left(t_n + \frac{h}{2}, \underline{z}(t_n:r) + \left((4v-1)\underline{l_1} + \underline{l_2}\right)/8v\right)$$

$$\overline{l_3}(t,z(t:r)) = hg\left(t_n + \frac{h}{2}, \overline{z}(t_n:r) + \left((4v-1)\overline{l_1} + \overline{l_2}\right)/8v\right)$$

$$\underline{l_4}(t,z(t:r)) = hg\left(t_n + \frac{2h}{3}, \underline{z}(t_n:r) + \left((10v-2)\underline{l_1} + 2\underline{l_2} + 8v\underline{l_3}\right)/27v\right)$$

$$\overline{l_4}(t,z(t:r)) = h\left(t_n + \frac{2h}{3}, \overline{z}(t_n:r) + \left((10v-2)\overline{l_1} + 2\overline{l_2} + 8v\overline{l_3}\right)/27v\right)$$

$$\underline{l_5}(t,z(t:r)) = hg\left(t_n + (7+4.582576)\frac{h}{14}, \underline{z}(t_n:r) + \begin{pmatrix} -((77v-56)+(17v-8)4.582576)\underline{l_1} \\ -8(7+4.582576)\underline{l_2} + 48(7+4.582576)v\underline{l_3} \\ -3(21+4.582576)v\underline{l_4} \end{pmatrix} /392v\right)$$

$$\overline{l_5}(t,z(t:r)) = hg\left(t_n + (7+4.582576)\frac{h}{14}, \overline{z}(t_n:r) + \begin{pmatrix} -((77v-56)+(17v-8)4.582576)\overline{l_1} \\ -8(7+4.582576)\overline{l_2} + 48(7+4.582576)v\overline{l_3} \\ -3(21+4.582576)v\overline{l_4} \end{pmatrix} /392v\right)$$

$$\underline{l_6}(t,z(t:r)) = hg\left(t_n + (7-4.582576)\frac{h}{14}, \underline{z}(t_n:r) + \begin{pmatrix} -5((287v-56)-(59v-8)4.582576)\underline{l_1} \\ -40(7-4.582576)\underline{l_2} + 320(4.582576)v\underline{l_3} \\ +3(21-121(4.582576))v\underline{l_4} + 392(6-(4.582576))v\underline{l_5} \end{pmatrix} /1960v\right)$$

$$\overline{l_6}(t,z(t:r)) = hg\left(t_n + (7-4.582576)\frac{h}{14}, \overline{z}(t_n:r) + \begin{pmatrix} -5((287v-56)-(59v-8)4.582576)\overline{l_1} \\ -40(7-4.582576)\overline{l_2} + 320(4.582576)v\overline{l_3} \\ +3(21-121(4.582576))v\overline{l_4} + 392(6-(4.582576))v\overline{l_5} \end{pmatrix} /1960v\right)$$

$$\underline{l_7}(t,z(t:r)) = hg\left(t_n + h, \underline{z}(t_n:r) + \begin{pmatrix} 15((30v-8)7v(4.582576))\underline{l_1} + 120\underline{l_2} - 40(5+7(4.582576))v\underline{l_3} \\ +63(2+3(4.582576))v\underline{l_4} - 14(49-9(4.582576))v\underline{l_5} \\ +70(7+(4.582576))v\underline{l_6} \end{pmatrix} /180v\right)$$

$$\overline{l_7}(t,z(t:r)) = hg\left(t_n + h, \overline{z}(t_n:r) + \begin{pmatrix} 15((30v-8)-7v(4.582576))\overline{l_1} + 120\overline{l_2} - 40(5+7(4.582576))v\overline{l_3} \\ +63(2+3(4.582576))v\overline{l_4} - 14(49-9(4.582576))v\overline{l_5} \\ +70(7+(4.582576))v\overline{l_6} \end{pmatrix} /180v\right)$$

. . . (4.2)

$$F(t,y(t:r)) = \frac{9\underline{k_1}(t,y(t:r)) + 64\underline{k_3}(t,y(t:r)) + 49\underline{k_5}(t,y(t:r)) + 49\underline{k_6}(t,y(t:r)) + 9\underline{k_7}(t,y(t:r))}{180}$$

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$$G(t,y(t:r))=\frac{9\overline{k}_1(t,y(t:r))+64\overline{k}_3(t,y(t:r))+49\overline{k}_5(t,y(t:r))+49\overline{k}_6(t,y(t:r))+9\overline{k}_7(t,y(t:r))}{180}$$

and

$$P(t,z(t:r))=\frac{9\underline{l}_1(t,z(t:r))+64\underline{l}_3(t,z(t:r))+49\underline{l}_5(t,z(t:r))+49\underline{l}_6(t,z(t:r))+9\underline{l}_7(t,z(t:r))}{180}$$

$$Q(t,z(t:r))=\frac{9\overline{l}_1(t,z(t:r))+64\overline{l}_3(t,z(t:r))+49\overline{l}_5(t,z(t:r))+49\overline{l}_6(t,z(t:r))+9\overline{l}_7(t,z(t:r))}{180}$$

The exact and the approximate solution of the differential equation at t_n, $0 \le n \le N$ are denoted by

$[Y(t_n)]_r = \left[\underline{Y}(t_n:r),\overline{Y}(t_n:r)\right]$, $[y(t_n)]_r = \left[\underline{y}(t_n:r),\overline{y}(t_n:r)\right]$ and

$[Z(t_n)]_r = \left[\underline{Z}(t_n:r),\overline{Z}(t_n:r)\right]$, $[z(t_n)]_r = \left[\underline{z}(t_n:r),\overline{z}(t_n:r)\right]$ respectively.

Therefore we have

$$\underline{Y}(t_{n+1}:r)=\underline{Y}(t_n:r)+F[t_n,\underline{Y}(t_n:r)] \qquad \overline{Y}(t_{n+1}:r)=\overline{Y}(t_n:r)+G[t_n,\overline{Y}(t_n:r)]$$
$$\underline{Z}(t_{n+1}:r)=\underline{Z}(t_n:r)+P[t_n,\underline{Z}(t_n:r)] \qquad \overline{Z}(t_{n+1}:r)=\overline{Z}(t_n:r)+Q[t_n,\overline{Z}(t_n:r)]$$
$$\underline{y}(t_{n+1}:r)=\underline{y}(t_n:r)+F[t_n,\underline{y}(t_n:r)] \qquad \overline{y}(t_{n+1}:r)=\overline{y}(t_n:r)+G[t_n,\overline{y}(t_n:r)]$$
$$\underline{z}(t_{n+1}:r)=\underline{z}(t_n:r)+P[t_n,\underline{z}(t_n:r)] \qquad \overline{z}(t_{n+1}:r)=\overline{z}(t_n:r)+Q[t_n,\overline{z}(t_n:r)] \quad ..(4.3)$$

To show the convergence of these approximation of y is

ie) $\lim_{h\to 0}\underline{y}(t:r)=\underline{Y}(t:r)$ and

$\lim_{h\to 0}\overline{y}(t:r)=\overline{Y}(t:r)$

Similarly, To show the convergence of these approximation of y is

ie) $\lim_{h\to 0}\underline{z}(t:r)=\underline{Z}(t:r)$ and

$\lim_{h\to 0}\overline{z}(t:r)=\overline{Z}(t:r)$

A. Lemma

Let a sequence of numbers $\{W\}_{n=0}^{N}$ satisfy

$|W_{n+1}| \le A|W_n| + B$, $0 \le n \le N-1$ or some given positive constants A and B then

$|W_n| \le A^n|W_0| + B\dfrac{A^n-1}{A-1}$, $0 \le n \le N-1$

Proof : See [13]

B. Lemma

Let a sequence of numbers $\{W\}_{n=0}^{N}$ and $\{V\}_{n=0}^{N}$ satisfy the condition

$|W_{n+1}| \le |W_n| + A\max\{|W_n|,|V_n|\}+B$ and

$|V_{n+1}| \le |V_n| + A\max\{|W_n|,|V_n|\}+B$ for some given positive constants A & B and denote

$U_n = |W_n| + |V_n|$, $0 \le n \le N$ where $\overline{A} = 1 + 2A$ and $\overline{B}=2B$

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Proof : See [13]

C. Theorem

Let F(t, u, v) and G(t, u, v) belongs to $C^4(K)$ and let the partial derivatives of F and G be bounded over K, then for arbitrary fixed value r, $0 \le r \le 1$ are approximate solutions converge to the exact solutions of $\underline{Y}(t_n : r)$ and $\overline{Y}(t_n : r)$ uniformly in t.

Proof : See [13]

V. NUMERICAL EXAMPLE

Consider the first order simultaneous differential equation

$$\frac{dy}{dt} = -3y + 4z \text{ and}$$

$$\frac{dz}{dt} = -2y + 3z \text{ Fuzzy initial conditions are}$$

y(0) = (3.8 + 0.2r, 4.25-0.25r) and z(0) = (2.75 + 0.25r, 3.2-0.2r); $0 \le r \le 1$.

Solution:

The exact solution of y is $\underline{Y}(t:r) = 2\underline{y_1}(t:r)e^{-t} + \underline{y_2}(t:r)e^{t}$ and the exact solution of z is

$\underline{Z}(t:r) = \underline{z_1}(t:r)e^{-t} + \underline{z_2}(t:r)e^{t}$ when t = 1 then the exact solution is given by,

$$\underline{Y}(1:r) = 2(0.85+0.15r)e^{-1} + (1.8+0.2r)e \quad \overline{Y}(1:r) = 2(1.2-0.2r)e^{-1} + (2.25-0.25r)e \quad \text{and}$$

$$\underline{Z}(1:r) = (0.75+0.25r)e^{-1} + (1.85+0.15r)e \quad \overline{Z}(1:r) = (1.25-0.25r)e^{-1} + (2.3-0.3r)e$$

The exact and approximate solutions for the first order simultaneous fuzzy differential equation obtained by the Runge-Kutta sixth order method for taking h = 0.1

Table – 5.1 Comparison between Exact and SFDRK6 Solution

r	Exact Solution with h = 0.1				Approximate Solution by SFDRK6 with h=0.1			
	Y-Lower	Y-Upper	Z-Lower	Z-Upper	y-Lower	y-Upper	z-Lower	z-Upper
0.0	5.518302341	6.999044773	5.304730964	6.711897507	5.3936266899	6.6168532372	5.0073537827	6.2305793762
0.1	5.583704361	6.916372549	5.354702177	6.621152066	5.4714970589	6.5724005699	5.0870628357	6.1879668236
0.2	5.649106381	6.833700326	5.404673390	6.530406625	5.5493655205	6.5279479027	5.1667709351	6.1453542709
0.3	5.714508401	6.751028103	5.454644604	6.439661184	5.6272358894	6.4834933281	5.2464814186	6.1027379036
0.4	5.779910420	6.668355879	5.504615817	6.348915743	5.7051038742	6.4390401840	5.3261880875	6.0601248741
0.5	5.845312440	6.585683656	5.554587031	6.258170303	5.7829737663	6.3945870399	5.4058976173	6.0175104141
0.6	5.910714460	6.503011433	5.604558244	6.167424862	5.8608441353	6.3501329422	5.4856071472	5.9748954773
0.7	5.976116480	6.420339209	5.654529458	6.076679421	5.9387121201	6.3056821823	5.5653147697	5.9322843552
0.8	6.041518500	6.337666986	5.704500671	5.985933980	6.0165834427	6.2612280846	5.6450252533	5.8896698952
0.9	6.106920519	6.254994763	5.754471885	5.895188539	6.0944519043	6.2167749405	5.7247323990	5.8470563889
1.0	6.172322539	6.172322539	5.804443098	5.804443098	6.1723237038	6.1723237038	5.8044452667	5.8044452667

Figure – 5.1 Solution Fuzzy Number y(t) and z(t)

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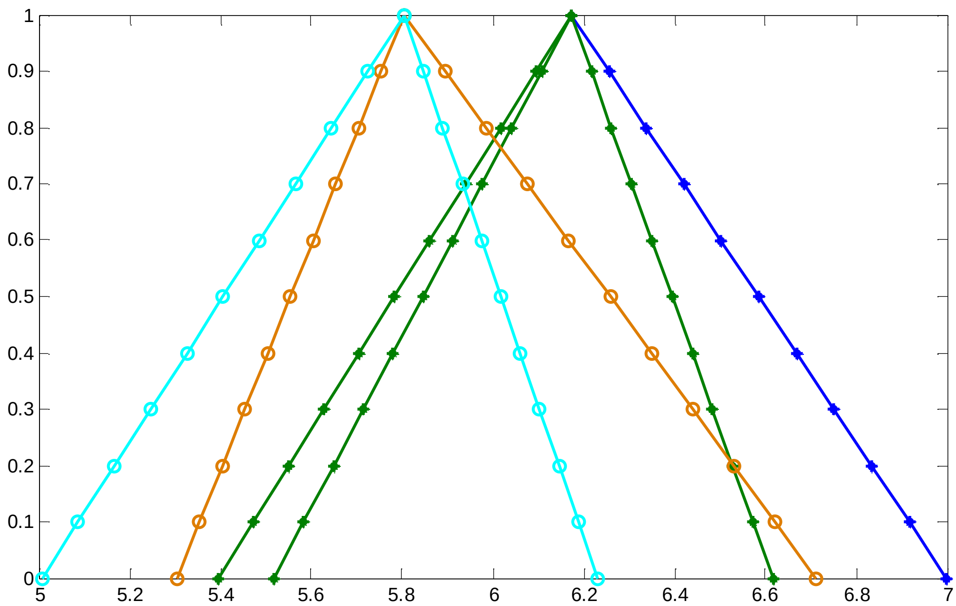

Table – 5.2 Solution of SFDE by SFDRK4 and Euler's Method

r	Approximate Solution by SFDRK4				Approximate Solution by Euler Method			
	Y-Lower	Y-Upper	Z-Lower	Z-Upper	Y-Lower	Y-Upper	Z-Lower	Z-Upper
0.0	5.3936238289	6.6168489456	5.0073494911	6.2305755615	5.1415872574	6.3087706566	4.7754745483	5.9426579475
0.1	5.4714927673	6.5723958015	5.0870590210	6.1879615784	5.2159128189	6.2663784027	4.8515443802	5.9020099640
0.2	5.5493617058	6.5279436111	5.1667671204	6.1453495026	5.2902379036	6.2239851952	4.9276123047	5.8613595963
0.3	5.6272330284	6.4834890366	5.2464776039	6.1027331352	5.3645634651	6.1815938950	5.0036811829	5.8207120895
0.4	5.7051010132	6.4390368462	5.3261847496	6.0601205826	5.4388890266	6.1391983032	5.0797500610	5.7800593376
0.5	5.7829713821	6.3945851326	5.4058952332	6.0175089836	5.5132141113	6.0968060493	5.1558184624	5.7394108772
0.6	5.8608412743	6.3501310349	5.4856038094	5.9748930931	5.5875396729	6.0544133186	5.2318878174	5.6987619400
0.7	5.9387102127	6.3056769371	5.5653128624	5.9322791100	5.6618642807	6.0120205879	5.3079552650	5.6581120491
0.8	6.0165801048	6.2612237930	5.6450209618	5.8896646500	5.7361907959	5.9696283340	5.3840255737	5.6174635887
0.9	6.0944485664	6.2167716026	5.7247295380	5.8470520973	5.8105154037	5.9272346497	5.4600939751	5.5768136978
1.0	6.1723184586	6.1723184586	5.8044385910	5.8044385910	5.8848414421	5.8848414421	5.5361633301	5.5361633301

Table – 5.3 Complete Error Analysis

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r	Error in SFDRK6 Method				Error in Euler Method				Error in SFDRK4 Method			
	Y-Lower	Y-Upper	y-Lower	Z-Upper	y-Lower	y-Upper	z-Lower	z-Upper	y-Lower	y-Upper	z-Lower	z-Upper
0.0	0.124676	0.382192	0.297377	0.481318	0.376715	0.690274	0.529256	0.769240	0.124679	0.382196	0.297381	0.481322
0.1	0.112207	0.343972	0.267639	0.433185	0.367792	0.649994	0.503158	0.719142	0.112212	0.343977	0.267643	0.433190
0.2	0.099741	0.305752	0.237902	0.385052	0.358868	0.609715	0.477061	0.669047	0.099745	0.305757	0.237906	0.385057
0.3	0.087273	0.267535	0.208163	0.336923	0.349945	0.569434	0.450963	0.618949	0.087275	0.267539	0.208167	0.336928
0.4	0.074807	0.229316	0.178428	0.288791	0.341021	0.529158	0.424866	0.568856	0.074809	0.229319	0.178431	0.288795
0.5	0.062339	0.191097	0.148689	0.240660	0.332098	0.488878	0.398769	0.518759	0.062341	0.191099	0.148692	0.240661
0.6	0.049870	0.152878	0.118951	0.192529	0.323175	0.448598	0.372670	0.468663	0.049873	0.152880	0.118954	0.192532
0.7	0.037404	0.114657	0.089215	0.144395	0.314252	0.408319	0.346574	0.418567	0.037406	0.114662	0.089217	0.144400
0.8	0.024935	0.076439	0.059475	0.096264	0.305328	0.368039	0.320475	0.368470	0.024938	0.076443	0.059480	0.096269
0.9	0.012469	0.038220	0.029739	0.048132	0.296405	0.327760	0.294378	0.318375	0.012472	0.038223	0.029742	0.048136
1.0	0.000001	0.000001	0.000002	0.000002	0.287481	0.287481	0.268280	0.268280	0.000004	0.000004	0.000005	0.000005

VI. CONCLUSION

In this work, we have used the proposed fuzzy sixth order Runge-Kutta method to find the numerical solution of simultaneous fuzzy differential equations. Taking into account the convergence order of the Euler method is O(h), a higher order of convergence $O(h^3)$ is obtained by the proposed method and by the method proposed in [14]. Comparison of the solutions of example 5 shows that the proposed method gives a better solution than the Euler method and by the Runge-Kutta fourth order method.

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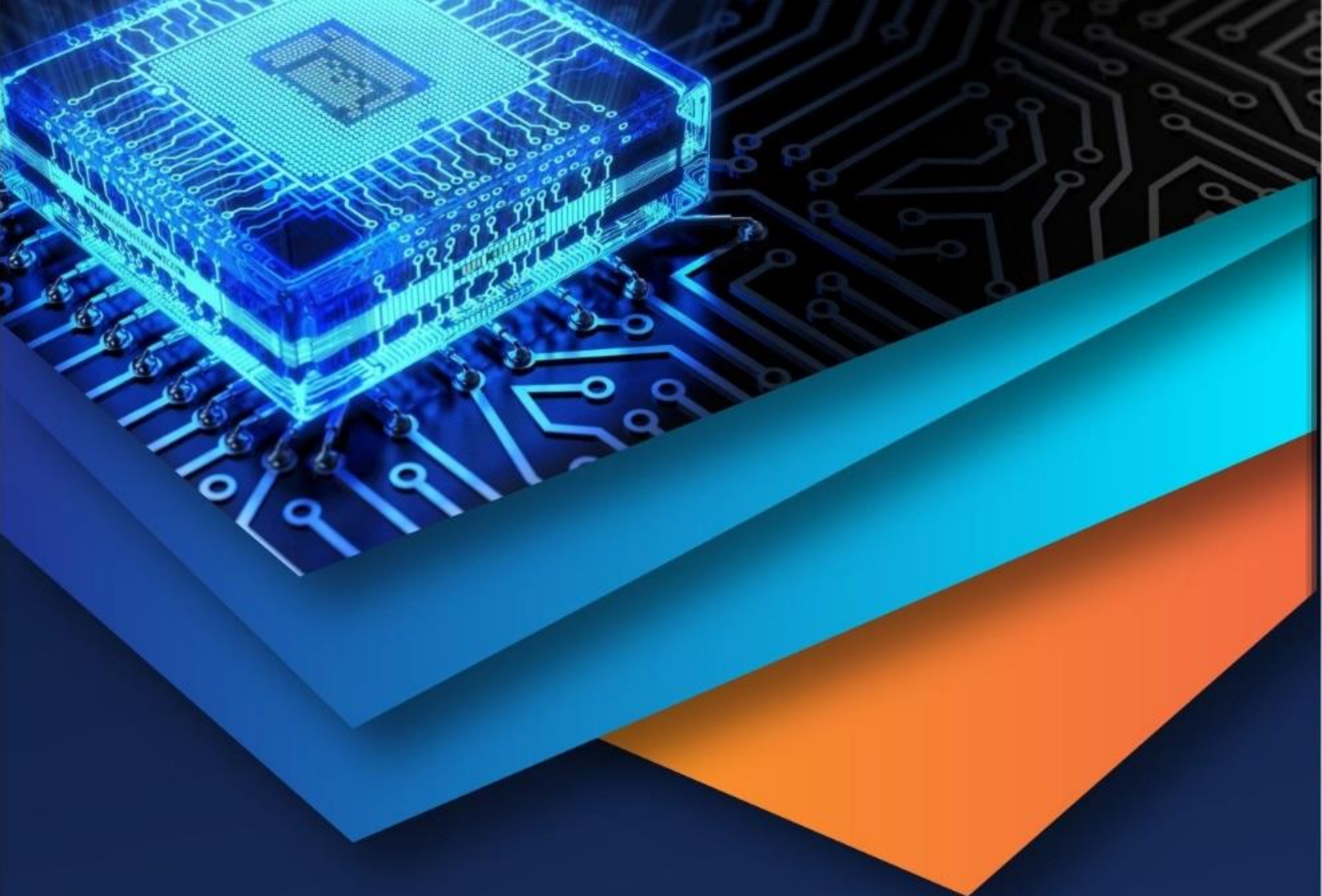

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