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Application of Mathematics in Cosmology

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Abstract: This paper reviews three important mathematical concepts in the field of cosmology such as density of spherical objects, escape speed and black holes and also contains problems based on those concepts.
Keywords: Black holes, Density, Escape speed, Velocity and mass.

I. INTRODUCTION

One of the important contributions that the science of astronomy has make to human progress is an understanding of the distance and size of the celestial objects. After millennia of using our eyes and about four centuries of using telescope, we now have a very good idea of where we are in the universe and how our planets fits in among the other bodies in our solar system, the milky way galaxy, and the universe. Several of the techniques astronomers use to estimate distance and size are based on angles, and the purpose of this paper is to make sure you to understand the mathematical foundation of these techniques. Specifically, the concepts of parallax method and angular size, angular resolutions of astronomical instruments are discussed.

This document contains the basic definitions and problems based on black holes, escape speed and density of spherical objects. Cosmology is the study of the large scale structure and behaviour of the universe. Concepts of density, black holes, escape speed, etc. play a major role in cosmology.

II. BASIC CONCEPTS

A. Density

Density is a measurement of how closely packed together the matter in an object is. Thus density relates to how heavy an object is relative to its size . If an object is “light” (small mass) and fluffy (large volume),it has a low density .If it is “heavy” (high mass) and compact (small volume),it has high density. Thus density depends on both the mass and the volume of the object, as you can see in the following definition:

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

This equation makes it clear that the dimensions of density are the dimensions of mass divided by the dimensions of volume which means the SI units of density are kilograms over cubic meters (kg/m^3)

B. Density Of Spherical Objects

Most of the astronomical objects that you are likely to encounter in density problems are roughly spherical in shape: planets, large moon, and stars are a few examples. Since the volume of a sphere is V= $\frac{4}{3}\pi R^3$ where R is the sphere’s radius, the density of a sphere of mass m is

$$\text{Density} = \frac{m}{\frac{4}{3}\pi R^3}$$

C. Escape Speed

It is the speed required for one object to “escape” from another object despite its gravitational pull.

D. Calculating Escape Speed

To calculate the value of v_{esc} at a given distance from the center of a given mass, use the equation

$$v_{esc} = \sqrt{\frac{2Gm}{R}}$$

In which G is the universal gravitational constant, m is the mass of the object from which the other object is escaping, and R is the distance between the centers of the object.

E. Black Holes

By definition, a black hole exists at any location at which the density of matter is so great that the escape speed from the vicinity of that location equals or exceeds the speed of light.

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III.PROBLEMS

A. Calculate the Densities of the Following Objects

1) A white dwarf (same mass at Sun; same radius as Earth) ,
2) A neutron star (three times Sun's mass,1/1000th of Earth's radius) and
3) A black hole (same mass as the neutron star, zero radius).

a) Given

m=same as sun=$2x10^{30}$ kg

r=same as earth=$6.4x10^{6}$ m

Formula:

$$\text{Density} = \frac{m}{\frac{4}{3}\pi R^3}$$

Solution:

$$\text{density} = \frac{2x10^{30}}{4/3 \text{ x } 3.14x(6.4x10^{6})^3}$$

$$= \frac{2x10^{30}x10^{-18}}{1.33 \text{ x } 3.14x(262.144)}$$

$$= \frac{2x10^{12}}{1094.765}$$

$$=0.00182687x10^{12}$$

Density of white dwarf =$1.82687x10^{9}$ kg/m^3

b) Given

m=3x mass of sun

$$R= \frac{1}{1000} \text{ x } 6.4x10^{6}$$

$=6.4x10^{6}x10^{-3}$

R=$6.4x10^{3}$ m

Formula:

$$\text{Density} = \frac{m}{\frac{4}{3}\pi R^3}$$

Solution

$$\text{Density}= \frac{6x10^{30}}{1.33 \text{ x } 3.14x(6.4x10^{3})^3}$$

$$= \frac{6x10^{30}x10^{-9}}{1.33x3.14x262.144}$$

$$= \frac{6x10^{21}}{1094.965}$$

$$= 0.00548062x10^{21}$$

Density of neutron star =$54.8062x10^{17}$

c) Given

m=same mass of neutron star=$1.9891x10^{30}$ kg

R=0

Formula $\text{Density} = \frac{m}{\frac{4}{3}\pi R^3}$

Solution

$$= \frac{1.9891x10^{30}}{1.33 \text{ x}3.14x(0)^3}$$

$$= \frac{1.9891x10^{30}}{0}$$

Density of black hole = ∞

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B. Find The Surface Escape Speed Of White Dwarf

Given:

(i)G= 6.67x10-11

R=7000 km or $7x10^3$ km (radius of white dwarf)

$M=2x10^{30}$ kg

Formula:

$$\text{Density} = \frac{m}{\frac{4}{3}\pi R^3}$$

Solution:

$$= \frac{7x10^3}{26.68x10^{-14}x10^{30}}$$

$$= \frac{7}{3.8114x10^{16}}$$

$=3.8114 \text{ x } (10^8)^2$

Surface escape speed of white dwarf = $1.95228x10^8$ m/s

C. A spherical asteroid has a density of 2 g/cm^3 and a mass of 3x10^{19} kg. What is its radius?

Given:

density=2 g/cm^3 =2000kg/m^3

$m=3x10^{19}kg$

Formula:

$$\text{Density} = \frac{m}{\frac{4}{3}\pi R^3}$$

Solution:

$$2000= \frac{3x10^{19}}{1.33x3.14xR^3}$$

$$R^3= \frac{3x10^{19}}{1.33x3.14x2x10^3}$$

$$R^3= \frac{3x10^{16}}{1.33x3.14x2}$$

R^3=0.35917

R=0.7108369 km

Radius of the spherical asteroid=0.7108369 km

D. Find The Density Of Saturn Planet

Given:

$m=5.683 \text{ x}10^{26}kg$

R=58,232 km (or) $58232x10^3$ m

Formula:

$$\text{Density} = \frac{m}{\frac{4}{3}\pi R^3}$$

Solution:

$$= \frac{5.683x10^{26}}{1.33x3.14x(58232) \text{ x}(10^3)^3}$$

$$= \frac{5.683x10^{26}x10^{-9}}{4.1762x1.9746x10^{14}}$$

$= 0.68915551x10^{26-23}$

$= 0.68915551x10^3$

Density of Saturn planet =689.1551 kg/m^3

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IV. CONCLUSIONS

Mathematics is an incomparable branch of study which is applied in every disciplines of universe. Every object in the universe needs a calculation. Hence Mathematics is applied in cosmology to calculate the density speed, velocity etc.

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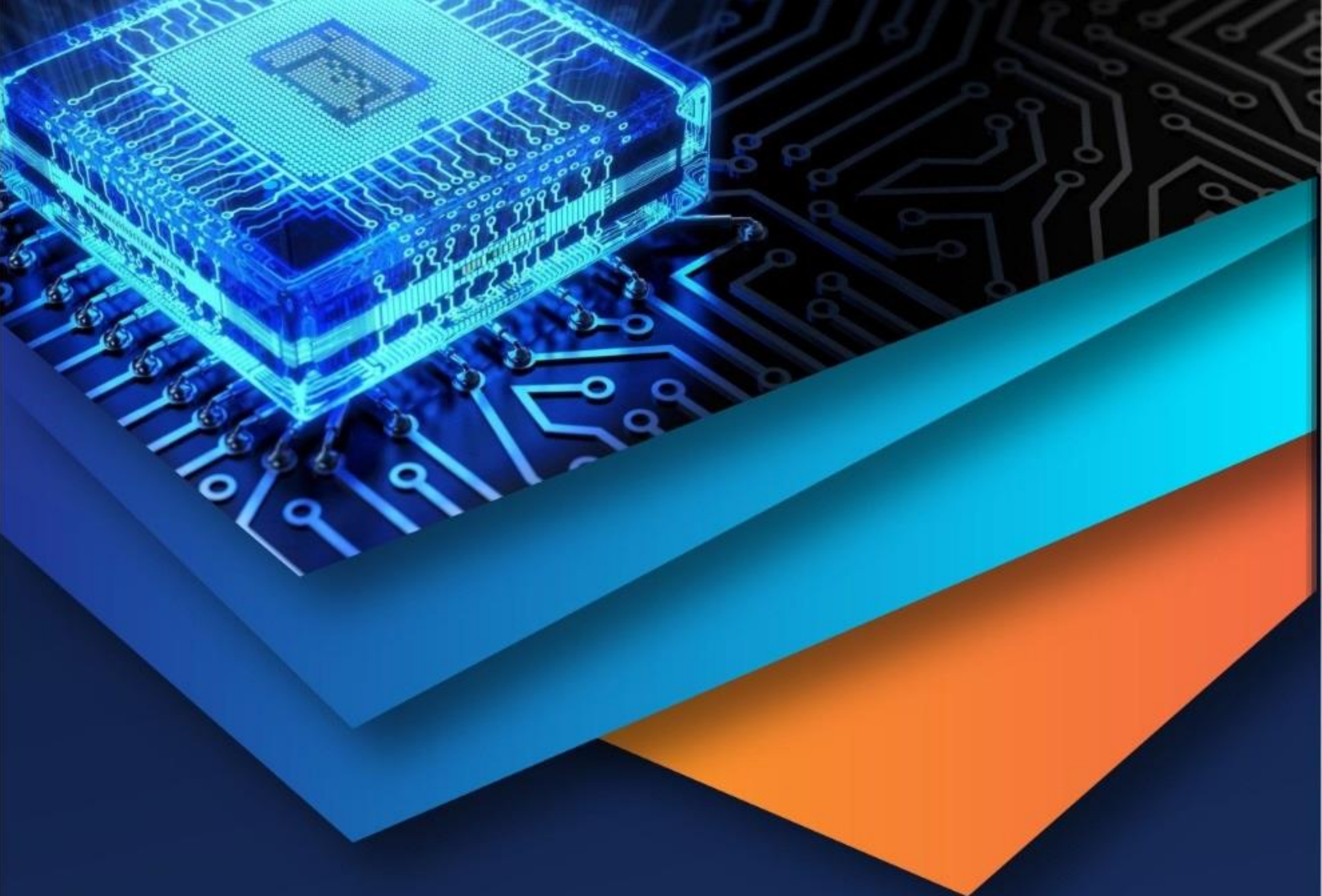

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