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To Study Eigen Value and Eigen Vector of a Square Matrix

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Abstract: In matrix theory the term eigen values and eigen vectors are very important. The prefix Eigen is taken from the German word Eigen for "Characteristics". Originally, it is used to study Principal axes of the Rotational motion of the rigid bodies. Eigen values and Eigen vectors have various applications, for example in Stability Analysis, Atomic orbitals, Matrix Diagonalization. This paper is a brief introduction to eigen values and eigen vectors. In this we also discuss some interesting properties of eigen values and eigen vectors.
Keywords: Eigen value, Eigen vector, Characteristic Equation, Cayley Hamilton theorem.

I. INTRODUCTION

1) *Characteristic Matrix:* If A be $n \times n$ square matrix, λ be any scalar in the field F, then $(A - \lambda I)$ is said to Characteristic matrix.

a) *Example:* If $A = \begin{bmatrix} 3 & 2 \\ 0 & 2 \end{bmatrix}$ then its Characteristic Matrix is

$$A - \lambda I = \begin{bmatrix} 3 & 2 \\ 0 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-\lambda & 2 \\ 0 & 2-\lambda \end{bmatrix}$$

2) *Characteristic Polynomial:* If A be $n \times n$ square matrix, λ be any scalar in the field F, then determinant of $(A - \lambda I)$ is said to Characteristic Polynomial.

a) *Example:* If $A = \begin{bmatrix} 3 & 2 \\ 0 & 2 \end{bmatrix}$ then its Characteristic Polynomial is

$$|A - \lambda I| = \begin{vmatrix} 3-\lambda & 2 \\ 0 & 2-\lambda \end{vmatrix}$$

$$= (3-\lambda)(2-\lambda)$$

$$= \lambda^2 - 5\lambda + 6$$

3) *Characteristic Equation:* If A be $n \times n$ square matrix, λ be any scalar in the field F, then $|A - \lambda I| = 0$ is said to Characteristic Polynomial.

a) *Example:* If $A = \begin{bmatrix} 3 & 2 \\ 0 & 2 \end{bmatrix}$ then its Characteristic Equation is

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 3-\lambda & 2 \\ 0 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (3-\lambda)(2-\lambda) = 0$$

$$\Rightarrow \lambda^2 - 5\lambda + 6 = 0$$

4) *Characteristic Roots:* If A be $n \times n$ square matrix, λ be any scalar in the field F,
Then roots of Characteristic Equation are said to Characteristic root of Matrix A. They are also Called Eigen value and Latent root of Matrix A.

a) *Example:* If $A = \begin{bmatrix} 3 & 2 \\ 0 & 2 \end{bmatrix}$ then its Characteristic roots is obtained as

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 3-\lambda & 2 \\ 0 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (3-\lambda)(2-\lambda) = 0$$
$$\Rightarrow \lambda^2 - 5\lambda + 6 = 0$$
$$\Rightarrow \lambda = 3,2$$

5) *Characteristic Vector:* If A be $n \times n$ square matrix, λ be any scalar in the field F such that

$$AX = \lambda X, \quad X \neq 0$$

Then X is said to Characteristic Vector of Matrix A corresponding to Characteristic rootλ. They are also Called Eigen vector and Latent Vector of Matrix A.

a) *Example:* If $A = \begin{bmatrix} 3 & 2 \\ 0 & 2 \end{bmatrix}$ then its Characteristic Vector is obtained as

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 3-\lambda & 2 \\ 0 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (3-\lambda)(2-\lambda) = 0$$
$$\Rightarrow \lambda^2 - 5\lambda + 6 = 0$$
$$\Rightarrow \lambda = 3,2$$

Eigen vector Corresponding to $\lambda = 3$

$$AX = 3X$$
$$\Rightarrow (A - 3I)X = 0$$
$$\Rightarrow \begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\Rightarrow x_2 = 0$$

So, Eigen vector Corresponding to $\lambda = 3$ is $X = k \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Eigen vector Corresponding to $\lambda = 2$

$$AX = 2X$$
$$\Rightarrow (A - 2I)X = 0$$
$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\Rightarrow x_1 + 2x_2 = 0$$
$$\Rightarrow x_1 = -2x_2$$

So, Eigen vector Corresponding to $\lambda = 2$ is $X = k \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

II. PROPERTIES OF EIGEN VALUE AND EIGEN VECTOR

1) A square matrix A of order $n \times n$ satisfies its own Characteristic equation, this is called Cayley- Hamilton theorem.
2) Zero is Eigen value of Square matrix A ⇔ A is singular Matrix.
3) Square matrix A and A^T have same eigenvalues.
4) If A be $n \times n$ Triangular matrix, Scalar matrix then the elements of Principal diagonal are eigen value of A.
5) The sum of the eigen values of square matrix A is equal to trace of matrix A.
6) The Product of the eigen values of square matrix A is equal to determinant of matrix A.
7) If ∈ $C^{n\times n}$, then the eigen values of A^* are complex conjugates of the eigenvalues of A, counting with multiplicities.
8) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$,a,b,c,d ∈ F then its characteristic equation is $x^2 + (Trace\ of\ A)x + \det A = 0$
   i.e. $x^2 + (a+d)x + (ad - bc) = 0$
9) If A be 3×3 square matrix the its characteristic equation is

$$x^3 - (Trace\ A)x^2 + Trace(adj\ A)x - \det A = 0$$

10) If λ is an eigen value of square matrix A (A is non singular matrix), then λ^{-1}
    Is an eigen value ofA^{-1}.
11) If λ is an eigen value of square matrix A, then $\frac{\det(A)}{\lambda}$, $\lambda \neq 0$ is an eigen value of adj (A).

12) If λ_1 and λ_2 be two eigen values of square matrix A and X be an eigen vector corresponding to λ_1 then X cannot be an eigen vector corresponding to λ_2.

13) The Geometric multiplicity of any eigen value λ of square matrix A cannot be greater than Algebraic multiplicity of λ.

14) If A be $n \times n$ real symmetric matrix or Hermitian matrix then its eigen values are real.

15) If A be $n \times n$ real skew symmetric matrix or skew Hermitian matrix then its eigen values are purely imignary or zero.

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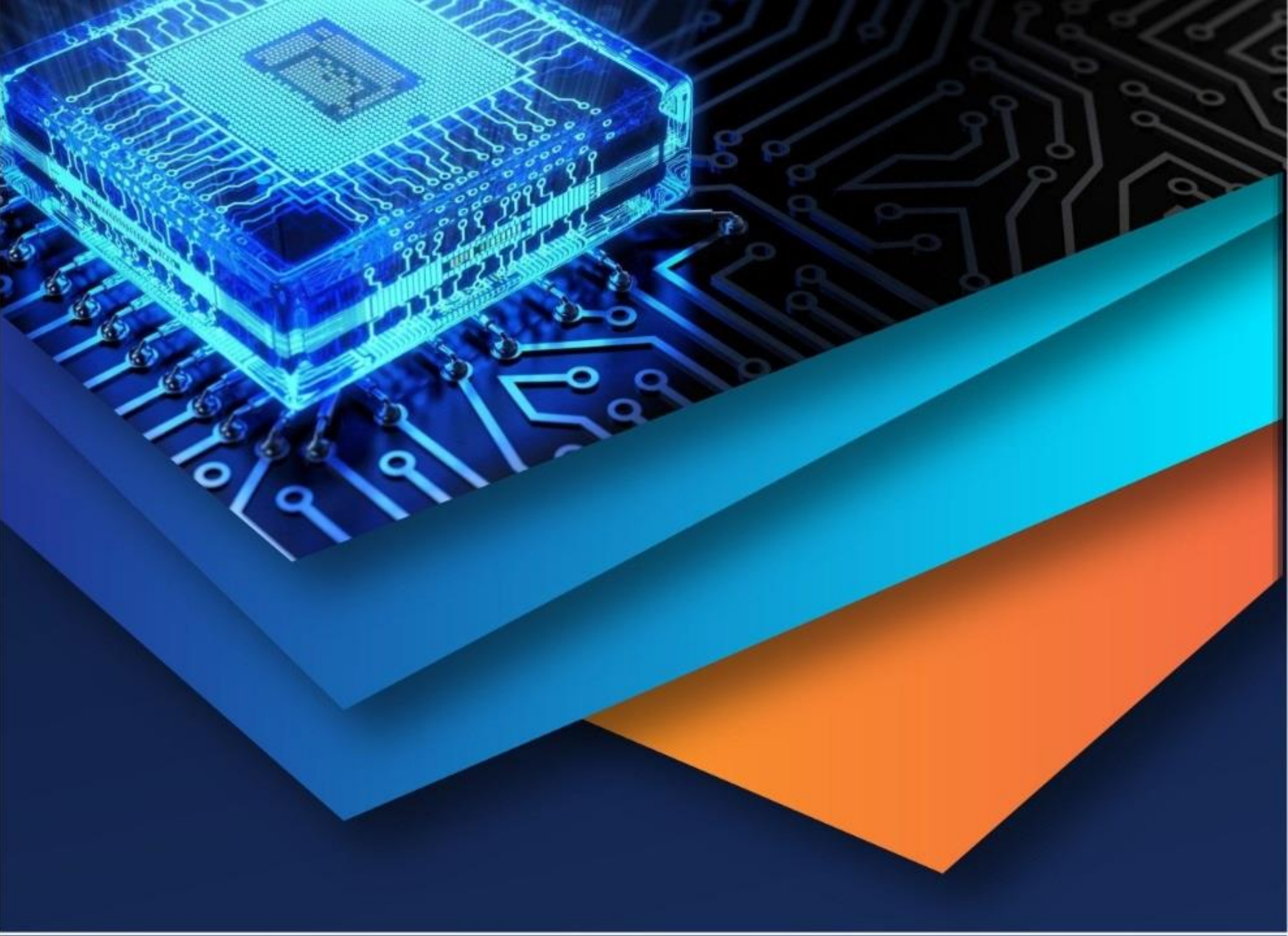

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