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On the Negative Pell Equation $y^2 = 48x^2 - 23$

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***Abstract:** The binary quadratic equation represents by negative Pellian* $y^2 = 48x^2 - 23$ *is analyzed for its distinct integer solutions. A few interesting relations among the solution are given. Further, employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas.*
***Keywords:** Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.*
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I. INTRODUCTION

The binary quadratic Diophantine equations (both homogeneous and non-homogeneous) are rich in variety. In [1-17] the binary quadratic non-homogeneous equations representing hyperbolas respectively are studied for their non-zero integral solutions. This communication concerns with yet another binary quadratic equation given by $y^2 = 48x^2 - 23$.The recurrence relations satisfied by the solutions x and y are given. Also a few interesting properties among the solutions are exhibited.

II. METHOD OF ANALYSIS

The negative Pell equation representing hyperbola under consideration is

$$y^2 = 48x^2 - 23 \quad (1)$$

whose smallest positive integer solution is

$$x_0 = 1, y_0 = 5$$

To obtain the other solutions of (1), consider the Pell equation

$$y^2 = 48x^2 + 1$$

whose smallest positive integer solution is

$$\tilde{x}_0 = 1, \tilde{y}_0 = 7$$

whose general solution is given by

$$\tilde{x}_n = \frac{1}{2\sqrt{48}} g_n , \tilde{y}_n = \frac{1}{2} f_n$$

where

$$f_n = \left(7+\sqrt{48}\right)^{n+1} + \left(7-\sqrt{48}\right)^{n+1}$$

$$g_n = \left(7+\sqrt{48}\right)^{n+1} - \left(7-\sqrt{48}\right)^{n+1}$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the other integer solutions of (1) are given by

$$x_{n+1} = \frac{1}{2} f_n + \frac{5}{8\sqrt{3}} g_n$$

$$y_{n+1} = \frac{5}{2} f_n + \frac{6}{\sqrt{3}} g_n$$

The recurrence relations satisfied by x and y are given by

$$x_{n+3} - 14x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 14y_{n+2} + y_{n+1} = 0$$

Some numerical examples of x and y satisfying (1) are given in the Table: 1 below:

Table: 1 Numerical Examples

n	x_{n+1}	y_{n+1}
-1	1	5
0	12	83
1	167	1157
2	2326	16115
3	32397	224453
4	451232	3126227

From the above table, we observe some interesting relations among the solutions which are presented below:

A. x_{n+1} values are alternatively odd and even according as n values & y_{n+1} values are alternatively odd.

B. Relations Among The Solutions

1) $x_{n+3} - 14x_{n+2} + x_{n+1} = 0$

2) $7x_{n+1} - x_{n+2} + y_{n+1} = 0$

3) $x_{n+1} - 7x_{n+2} + y_{n+2} = 0$

4) $7x_{n+1} - 97x_{n+2} + y_{n+3} = 0$

5) $97x_{n+1} - x_{n+3} + 14y_{n+1} = 0$

6) $x_{n+1} - x_{n+3} + 2y_{n+2} = 0$

7) $x_{n+1} - 97x_{n+3} + 14y_{n+3} = 0$

8) $y_{n+2} - 7y_{n+1} - 48x_{n+1} = 0$

9) $y_{n+3} - 97y_{n+1} - 672x_{n+1} = 0$

10) $7y_{n+3} - 97y_{n+2} - 48x_{n+1} = 0$

11) $97x_{n+2} - 7x_{n+3} + y_{n+1} = 0$

12) $7x_{n+2} - x_{n+3} + y_{n+2} = 0$

13) $x_{n+2} - 7x_{n+3} + y_{n+3} = 0$

14) $7y_{n+3} - 7y_{n+1} - 672x_{n+2} = 0$

15) $48x_{n+2} + y_{n+1} - 7y_{n+2} = 0$

16) $y_{n+3} - 7y_{n+2} - 48x_{n+2} = 0$

17) $97y_{n+2} - 7y_{n+1} - 48x_{n+3} = 0$

18) $97y_{n+3} - y_{n+1} - 672x_{n+3} = 0$

19) $7y_{n+3} - y_{n+2} - 48x_{n+3} = 0$

20) $y_{n+1} - 14y_{n+2} + y_{n+3} = 0$

C. Each Of The Following Expressions Represents A Nasty Number

1) $\frac{1}{23}\left(996x_{2n+2} - 60x_{2n+3} + 276\right)$

2) $\frac{1}{161}\left(6942x_{2n+2} - 30x_{2n+4} + 1932\right)$

3) $\frac{1}{23}\left(576x_{2n+2} - 60y_{2n+2} + 276\right)$

4) $\frac{1}{161}\left(6912x_{2n+2} - 60y_{2n+3} + 1932\right)$

5) $\frac{1}{2231}\left(96192x_{2n+2} - 60y_{2n+4} + 26772\right)$

6) $\frac{1}{23}\left(13884x_{2n+3} - 996x_{2n+4} + 276\right)$

7) $\frac{1}{161}\left(576x_{2n+3} - 996y_{2n+2} + 1932\right)$

8) $\frac{1}{23}\left(6912x_{2n+3} - 996y_{2n+3} + 276\right)$

9) $\frac{1}{161}\left(96192x_{2n+3} - 996y_{2n+4} + 1932\right)$

10) $\frac{1}{2231}\left(576x_{2n+4} - 13884y_{2n+2} + 26772\right)$

11) $\frac{1}{161}\left(6912x_{2n+4} - 13884y_{2n+3} + 1932\right)$

12) $\frac{1}{23}\left(96192x_{2n+4} - 13884y_{2n+4} + 276\right)$

13) $\frac{1}{138}\left(72y_{2n+3} - 864y_{2n+2} + 1656\right)$

14) $\frac{1}{161}\left(6y_{2n+4} - 1002y_{2n+2} + 1932\right)$

15) $\frac{1}{69}\left(432y_{2n+4} - 6012y_{2n+3} + 828\right)$

D. Each Of The Following Expressions Represents A Cubical Integer

1) $\frac{1}{23}\left[166x_{3n+3} - 10x_{3n+4} + 498x_{n+1} - 30x_{n+2}\right]$

2) $\frac{1}{161}\left[1157x_{3n+3} - 5x_{3n+5} + 3471x_{n+1} - 15x_{n+3}\right]$

3) $\frac{1}{23}\left[96x_{3n+3} - 10y_{3n+3} + 228x_{n+1} - 30y_{n+1}\right]$

4) $\frac{1}{161}\left[1152x_{3n+3} - 10y_{3n+4} + 3456x_{n+1} - 30y_{n+2}\right]$

5) $\frac{1}{2231}\left[16032x_{3n+3} - 10y_{3n+5} + 48096x_{n+1} - 30y_{n+3}\right]$

6) $\frac{1}{23}\left[2314x_{3n+4} - 166x_{3n+5} + 6942x_{n+2} - 498x_{n+3}\right]$

7) $\frac{1}{161}\left[96x_{3n+4} - 116y_{3n+3} + 288x_{n+2} - 498y_{n+1}\right]$

8) $\frac{1}{23}\left[1152x_{3n+4} - 166y_{3n+4} + 3456x_{n+2} - 498y_{n+2}\right]$

9) $\frac{1}{161}\left[16032x_{3n+4} - 166y_{3n+5} + 48096x_{n+2} - 498y_{n+3}\right]$

10) $\frac{1}{2231}\left[96x_{3n+5} - 2314y_{3n+3} + 288x_{n+3} - 6942y_{n+1}\right]$

11) $\frac{1}{161}\left[1152x_{3n+5} - 2314y_{3n+4} + 3456x_{n+3} - 6942y_{n+2}\right]$

12) $\frac{1}{23}\left[16032x_{3n+5} - 2314y_{3n+5} + 48096x_{n+3} - 6942y_{n+3}\right]$

13) $\frac{1}{138}\left[12y_{3n+4} - 144y_{3n+3} + 36y_{n+2} - 432y_{n+1}\right]$

14) $\frac{1}{161}\left[y_{3n+5} - 167y_{3n+3} + 3y_{n+3} - 501y_{n+1}\right]$

15) $\frac{1}{69}\left[72y_{3n+5} - 1002y_{3n+4} + 216y_{n+3} - 3006y_{n+2}\right]$

E. Each Of The Following Expressions Represents A Bi-Quadratic Integer

1) $\frac{1}{23}\left[166x_{4n+4} - 10x_{4n+5} + 664x_{2n+2} - 40x_{2n+3} + 138\right]$

2) $\frac{1}{161}\left[1157x_{4n+4} - 5x_{4n+6} + 4628x_{2n+2} - 20x_{2n+4} + 966\right]$

3) $\frac{1}{23}\left[96x_{4n+4} - 10y_{4n+4} + 384x_{2n+2} - 40y_{2n+2} + 138\right]$

4) $\frac{1}{161}\left[1152x_{4n+4} - 10y_{4n+5} + 4608x_{2n+2} - 40y_{2n+3} + 966\right]$

5) $\frac{1}{2231}\left[16032x_{4n+4} - 10y_{4n+6} + 64128x_{2n+2} - 40y_{2n+4} + 13386\right]$

6) $\frac{1}{23}\left[2314x_{4n+5} - 166x_{4n+6} + 9256x_{2n+3} - 664x_{2n+4} + 138\right]$

7) $\frac{1}{161}\left[96x_{4n+5} - 166y_{4n+4} + 384x_{2n+3} - 664y_{2n+2} + 966\right]$

8) $\frac{1}{23}\left[1152x_{4n+5} - 166y_{4n+5} + 4608x_{2n+3} - 664y_{2n+3} + 138\right]$

9) $\frac{1}{161}\left[16032x_{4n+5} - 166y_{4n+6} + 64128x_{2n+3} - 664y_{2n+4} + 966\right]$

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10) $\frac{1}{2231}\left[96x_{4n+6} - 2314y_{4n+4} + 384x_{2n+4} - 9256y_{2n+2} + 13386\right]$

11) $\frac{1}{161}\left[1152x_{4n+6} - 2314y_{4n+5} + 4608x_{3n+5} - 9256y_{3n+4} + 966\right]$

12) $\frac{1}{23}\left[16032x_{4n+6} - 2314y_{4n+6} + 64128x_{2n+4} - 9256y_{2n+4} + 138\right]$

13) $\frac{1}{138}\left[12y_{4n+5} - 144y_{4n+4} + 48y_{2n+3} - 576y_{2n+2} + 828\right]$

14) $\frac{1}{161}\left[y_{4n+6} - 167y_{4n+4} + 4y_{2n+4} - 668y_{2n+2} + 966\right]$

15) $\frac{1}{69}\left[72y_{4n+6} - 1002y_{4n+5} + 288y_{2n+4} - 4008y_{2n+3} + 414\right]$

F. Each Of The Following Expressions Represents A Quintic Integer

1) $\frac{1}{23}\left[166x_{5n+5} - 10x_{5n+6} + 830x_{3n+3} - 50x_{3n+4} + 1660x_{n+1} - 100x_{n+2}\right]$

2) $\frac{1}{161}\left[1157x_{5n+5} - 5x_{5n+7} + 5785x_{3n+3} - 25x_{3n+5} + 11570x_{n+1} - 50x_{n+3}\right]$

3) $\frac{1}{23}\left[96x_{5n+5} - 10y_{5n+5} + 480x_{3n+3} - 50y_{3n+3} + 960x_{n+1} - 100y_{n+1}\right]$

4) $\frac{1}{161}\left[1152x_{5n+5} - 10y_{5n+6} + 5760x_{3n+3} - 50y_{3n+4} + 11520x_{n+1} - 100y_{n+2}\right]$

5) $\frac{1}{2231}\left[16032x_{5n+5} - 10y_{5n+7} + 80160x_{3n+3} - 50y_{3n+5} + 160320x_{n+1} - 100y_{n+3}\right]$

6) $\frac{1}{23}\left[2314x_{5n+6} - 166x_{5n+7} + 11570x_{3n+4} - 830x_{3n+5} + 23140x_{n+2} - 1660x_{n+3}\right]$

7) $\frac{1}{161}\left[96x_{5n+6} - 166y_{5n+5} + 480x_{3n+4} - 830y_{3n+3} + 960x_{n+2} - 1660y_{n+1}\right]$

8) $\frac{1}{23}\left[1152x_{5n+6} - 166y_{5n+6} + 5760x_{4n+5} - 830y_{4n+5} + 11520x_{n+2} - 1660y_{n+2}\right]$

9) $\frac{1}{161}\left[16032x_{5n+6} - 166y_{5n+7} + 80160x_{3n+4} - 830y_{3n+5} + 160320x_{n+2} - 1660y_{n+3}\right]$

10) $\frac{1}{2231}\left[96x_{5n+7} - 2314y_{5n+5} + 480x_{3n+5} - 11570y_{3n+3} + 960x_{n+3} - 23140y_{n+1}\right]$

11) $\frac{1}{161}\left[1152x_{5n+7} - 2314y_{5n+6} + 5760x_{3n+5} - 11570y_{3n+4} + 11520x_{n+3} - 23140y_{n+2}\right]$

12) $\frac{1}{23}\left[16032x_{5n+7} - 2314y_{5n+7} + 80160x_{3n+5} - 11570y_{3n+5} + 160320x_{n+3} - 23140y_{n+3}\right]$

13) $\frac{1}{138}\left[12y_{5n+6} - 144y_{5n+5} + 60y_{3n+4} - 720y_{3n+3} + 120y_{n+2} - 1440y_{n+1}\right]$

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14) $\frac{1}{161}\left[y_{5n+7}-167y_{5n+5}+5y_{3n+5}-835y_{3n+3}+10y_{n+3}-1670y_{n+1}\right]$

15) $\frac{1}{69}\left[72y_{5n+7}-1002y_{5n+6}+360y_{3n+5}-5010y_{3n+4}+720y_{n+3}-10020y_{n+2}\right]$

III. REMARKABLE OBSERVATIONS

A. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbola which are presented in the Table: 2 below:

Table: 2 Hyperbolas

S. No	Hyperbola	(X,Y)
1	$X^2-48Y^2=529$	$(83x_{n+1}-5x_{n+2},\ x_{n+2}-12x_{n+1})$
2	$X^2-48Y^2=103684$	$(1157x_{n+1}-5x_{n+3},x_{n+3}-167x_{n+1})$
3	$X^2-48Y^2=529$	$(48x_{n+1}-5y_{n+1},y_{n+1}-5x_{n+1})$
4	$X^2-48Y^2=25921$	$(576x_{n+1}-5y_{n+2},y_{n+2}-83x_{n+1})$
5	$X^2-48Y^2=4977361$	$(8016x_{n+1}-5y_{n+3},y_{n+3}-1157x_{n+1})$
6	$X^2-48Y^2=529$	$(1157x_{n+2}-83x_{n+3},12x_{n+3}-167x_{n+2})$
7	$X^2-48Y^2=25921$	$(48x_{n+2}-83y_{n+1},12y_{n+1}-5x_{n+2})$
8	$X^2-48Y^2=529$	$(576x_{n+2}-83y_{n+2},12y_{n+2}-83x_{n+2})$
9	$X^2-48Y^2=25921$	$(8016x_{n+2}-83y_{n+3},12y_{n+3}-1157x_{n+2})$
10	$X^2-48Y^2=4977361$	$(48x_{n+3}-1157y_{n+1},167y_{n+1}-5x_{n+3})$
11	$X^2-48Y^2=25921$	$(576x_{n+3}-1157y_{n+2},167y_{n+2}-83x_{n+3})$
12	$X^2-48Y^2=529$	$(8016x_{n+3}-1157y_{n+3},167y_{n+3}-1157x_{n+3})$
13	$16X^2-3Y^2=76176$	$(3y_{n+2}-36y_{n+1},83y_{n+1}-5y_{n+2})$
14	$48X^2-Y^2=4976832$	$(y_{n+3}-167y_{n+1},1157y_{n+1}-5y_{n+3})$
15	$16X^2-3Y^2=76176$	$(36y_{n+3}-501y_{n+2},1157y_{n+2}-83y_{n+3})$

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B. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of parabola which are presented in the Table: 3 below:

Table: 3 Parabolas

S. No	Parabola	(X, Y)
1	$23X - 96Y^2 = 529$	$(83x_{2n+2} - 5x_{2n+3}, x_{n+2} - 12x_{n+1})$
2	$322X - 96Y^2 = 103684$	$(1157x_{2n+2} - 5x_{2n+4}, x_{n+3} - 167x_{n+1})$
3	$23X - 96Y^2 = 529$	$(48x_{2n+2} - 5y_{2n+2}, y_{n+1} - 5x_{n+1})$
4	$161X - 96Y^2 = 25921$	$(576x_{2n+2} - 5y_{2n+3}, y_{n+2} - 83x_{n+1})$
5	$2231X - 96Y^2 = 4977361$	$(8016x_{2n+2} - 5y_{2n+4}, y_{n+3} - 1157x_{n+1})$
6	$23X - 96Y^2 = 529$	$(1157x_{2n+3} - 83x_{2n+4}, 12x_{n+3} - 167x_{n+2})$
7	$161X - 96Y^2 = 25921$	$(48x_{2n+3} - 83y_{2n+2}, 12y_{n+1} - 5x_{n+2})$
8	$23X - 96Y^2 = 529$	$(576x_{2n+3} - 83y_{2n+3}, 12y_{n+2} - 83x_{n+2})$
9	$161X - 96Y^2 = 25921$	$(8016x_{2n+3} - 83y_{2n+4}, 12y_{n+3} - 1157x_{n+2})$
10	$2231X - 96Y^2 = 4977361$	$(48x_{2n+4} - 1157y_{2n+2}, 167y_{n+1} - 5x_{n+3})$
11	$161X - 96Y^2 = 25921$	$(576x_{2n+4} - 1157y_{2n+3}, 167y_{n+2} - 83x_{n+3})$
12	$23X - 96Y^2 = 529$	$(8016x_{2n+4} - 1157y_{2n+4}, 167y_{n+3} - 1157x_{n+3})$
13	$552X - 3Y^2 = 38088$	$(3y_{2n+3} - 36y_{2n+2}, 83y_{n+1} - 5y_{n+2})$
14	$23184X - 3Y^2 = 7465248$	$(y_{2n+4} - 167y_{2n+2}, 1157y_{n+1} - 5y_{n+3})$
15	$552X - 3Y^2 = 38088$	$(36y_{2n+4} - 501y_{2n+3}, 1157y_{n+2} - 83y_{n+3})$

IV. CONCLUSION

In this paper, we have presented infinitely many integer solutions for al hyperbola represented by the negative Pell Equation $y^2 = 48x^2 - 23$. As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of negative Pell Equations and determine their integer solutions along with suitable properties.

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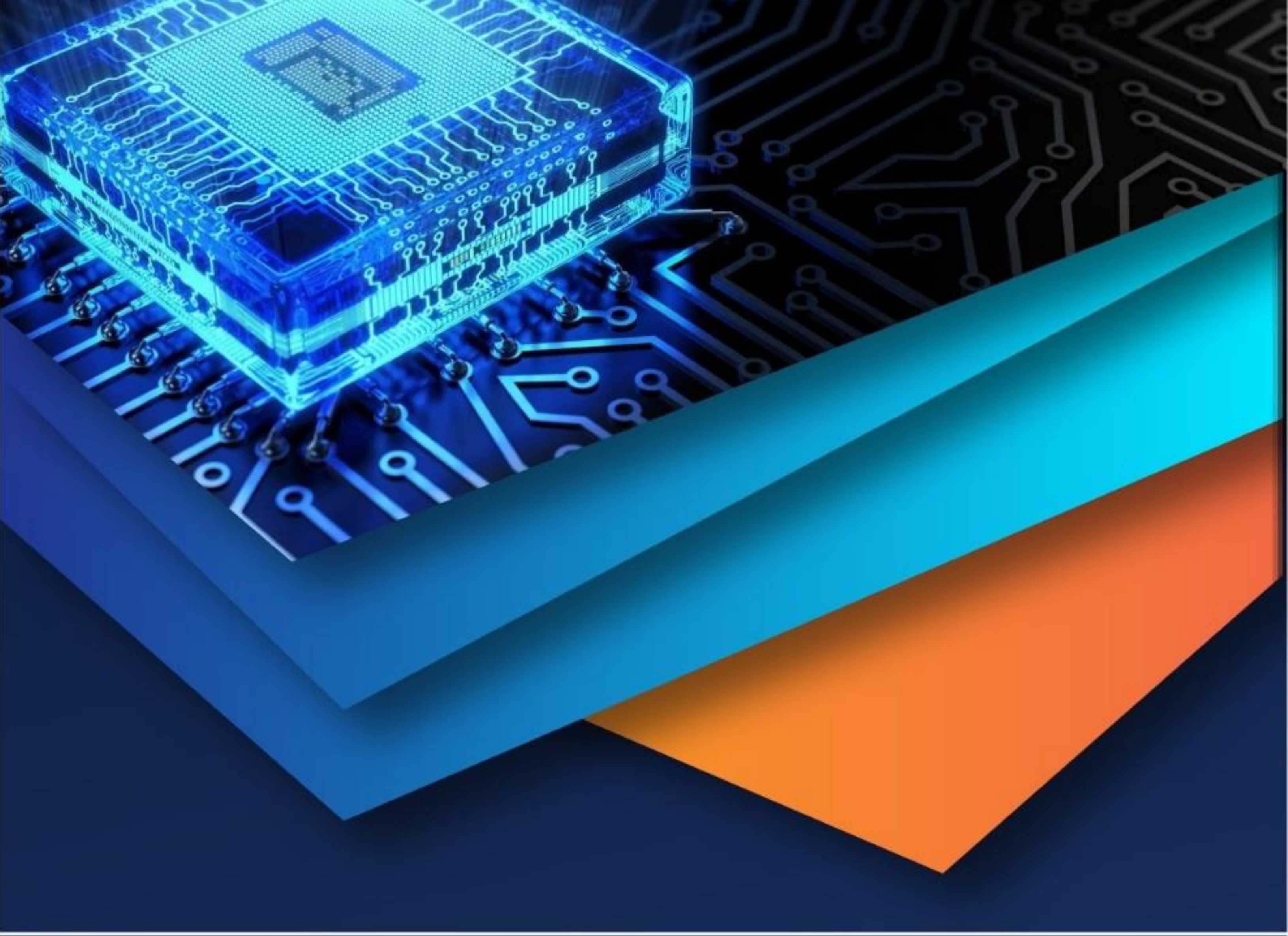

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