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# Skewness and Kurtosis on Edge Coloring of Certain Graphs

Stanis Arul Mary A

Assistant Professor, Department of Mathematics, Nirmala College for women, Coimbatore, Tamilnadu, India.

**Abstract:** In this paper we found minimum edge colouring sum based on a minimum proper colouring of a given graph  $G$  and we compute the statistical measures Mean, Variance, Median, Standard deviation, Skewness and Kurtosis.

**Keywords:** Graph Colouring; colouring sum of graphs; colouring mean; colouring variance; colouring median; colouring standard deviation; colouring skewness; colouring kurtosis;  $\chi'$ -chromatic.

## I. INTRODUCTION

The edge colouring or simply a colouring of a graph is an assignment of colours or labels to the edges of a graph subject to certain conditions. In a proper colouring of a graph, its edges are coloured in such a way that no two adjacent edges in that graph have the same color.

We extend the concepts of mean, median, variance, standard deviation, skewness and kurtosis are important statistical measures, to the theory of graph colouring and determine the values of these parameters for a number of standard graphs.

### A. Preliminaries

1) Let  $C = \{c_1, c_2, \dots, c_k\}$  be a particular type of proper  $k$ -colouring of a given graph  $G$  and  $\theta(c_i)$  denotes the number of times a particular color  $c_i$  is assigned to the edge of  $G$ . Then, the edge colouring sum of a colouring  $C$  of a given graph  $G$  denoted by  $\omega_C(G)$  is defined to be [9,10,11,12,13,14,15,16,17],

$$\omega_C(G) = \sum_{i=1}^k i\theta(c_i)$$

- 2) A graph in which one edge connecting every two consecutive edges and there are no other edges, is called a path [4][6].
- 3) A simple graph in which every edge must be connected to all other edges of the graph, is called as a complete graph [4][6].
- 4) A simple graph with  $n$  edges ( $n \geq 3$ ) and  $n$  edges is called a cycle graph if the degree of each edge in the graph is two and it is denoted by  $C_n$  [4][6].
- 5) The ladder graph  $L_n$  is defined as  $P_n \circ P_2$  [4][6]

### B. Colouring Statistical Parameters of Graphs

We can identify the colouring of the edges of a given graph  $G$  with a random experiment. Let  $C = \{c_1, c_2, c_3, \dots, c_k\}$  be a proper  $k$ -colouring of  $G$  and let  $X$  be the random variable (r.v) which denotes the number of edges in  $G$  having a particular color. Since the

sum of all weights of colors of  $G$  is the order of  $G$ , the real valued function  $f(i)$  is defined by,  $f(i) = \left\{ \begin{array}{l} \frac{\theta(c_i)}{|E(G)|}; i = 1, 2, \dots, k \\ 0 \end{array} \right\}$  is

the probability mass function (p.m.f) of the r.v  $X$ . If the context is clear, we can also say that  $f(i)$  is the p.m.f of the graph  $G$  with respect to the given colouring  $C$ . Hence, analogous to the definitions of the mean, median, variance, standard deviation, skewness and kurtosis of random variables, those statistical parameters of a graph  $G$ , with respect to a general colouring of  $G$  can be defined as follows.

1) *Definition .1.* Let  $C = \{c_1, c_2, c_3, \dots, c_k\}$  be a certain type of proper  $k$ -colouring of a given graph  $G$  and  $\theta(c_i)$  denotes the number of times a particular color  $c_i$  is assigned to edges of  $G$ . Then, the colouring mean of a colouring  $C$  of a given graph  $G$ , denoted

$$\text{by } \mu_C(G) \text{ is given by, } \mu_C(G) = \frac{\sum_{i=1}^k i\theta(c_i)}{\sum_{i=1}^k \theta(c_i)}$$

2) *Definition .2.* The colouring median of a colouring  $C$  of a given graph  $G$ , denoted by  $M_C(G)$  and is defined to be

$$M_C(G) = \frac{\sum_{i=1}^k \theta(c_i)}{2}$$

3) *Definition .3.* The colouring variance of a colouring  $C$  of a given graph  $G$ , denoted by  $\sigma^2_C(G)$  is given by,

$$\sigma^2_C(G) = \frac{\sum_{i=1}^k i^2\theta(c_i)}{\sum_{i=1}^k \theta(c_i)} - \left[ \frac{\sum_{i=1}^k i\theta(c_i)}{\sum_{i=1}^k \theta(c_i)} \right]^2$$

4) *Definition .4.* The colouring standard deviation of a colouring  $C$  of a given graph  $G$ , denoted by  $\sigma_C(G)$  is given by,

$$\sigma_C(G) = \sqrt{\frac{\sum_{i=1}^k i^2\theta(c_i)}{\sum_{i=1}^k \theta(c_i)} - \left[ \frac{\sum_{i=1}^k i\theta(c_i)}{\sum_{i=1}^k \theta(c_i)} \right]^2}$$

5) *Definition.5.* The colouring skewness of a colouring  $C$  of a given graph  $G$ , denoted by  $\gamma_C(G)$  and is defined to be  $\gamma_C(G) =$

$$3 \left( \frac{\text{Mean} - \text{Median}}{\text{Standard Deviation}} \right)$$

6) *Definition 6.* For a positive integer  $r$ , the  $r^{\text{th}}$  moment of the colouring  $C$  is denoted by  $\mu_{C^r}(G)$  is given by,

$$\mu_{C^r}(G) = \frac{\sum_{i=1}^k i^r\theta(c_i)}{\sum_{i=1}^k \theta(c_i)}$$

We have various moments as follows:

a) *1st Moment: 2nd Moment*

$$\mu_{C^1}(G) = \frac{\sum_{i=1}^k i\theta(c_i)}{\sum_{i=1}^k \theta(c_i)} \quad \mu_{C^2}(G) = \frac{\sum_{i=1}^k i^2\theta(c_i)}{\sum_{i=1}^k \theta(c_i)} - \left[ \frac{\sum_{i=1}^k i\theta(c_i)}{\sum_{i=1}^k \theta(c_i)} \right]^2$$

b) *3rd Moment*

$$\mu_{C^3}(G) = \frac{\sum_{i=1}^k i^3\theta(c_i)}{\sum_{i=1}^k \theta(c_i)} - 3 \left[ \frac{\sum_{i=1}^k i^2\theta(c_i)}{\sum_{i=1}^k \theta(c_i)} \right] \left[ \frac{\sum_{i=1}^k i\theta(c_i)}{\sum_{i=1}^k \theta(c_i)} \right] + 2 \left[ \frac{\sum_{i=1}^k i^3\theta(c_i)}{\sum_{i=1}^k \theta(c_i)} \right]^3$$

c) 4th Moment

$$\mu_{c^4}(G) = \frac{\sum_{i=1}^k i^4 \theta(c_i)}{\sum_{i=1}^k \theta(c_i)} - 4 \left[ \frac{\sum_{i=1}^k i^3 \theta(c_i)}{\sum_{i=1}^k \theta(c_i)} \right] \left[ \frac{\sum_{i=1}^k i \theta(c_i)}{\sum_{i=1}^k \theta(c_i)} \right] + 6 \left[ \frac{\sum_{i=1}^k i^3 \theta(c_i)}{\sum_{i=1}^k \theta(c_i)} \right] \left[ \frac{\sum_{i=1}^k i \theta(c_i)}{\sum_{i=1}^k \theta(c_i)} \right]^2 - 3 \left[ \frac{\sum_{i=1}^k i \theta(c_i)}{\sum_{i=1}^k \theta(c_i)} \right]^4$$

7) Definition 7. The colouring kurtosis of a colouring C of a given graph G, denoted by  $\beta_2(G)$  and defined by,

$$\beta_2(G) = \frac{\mu_{c^4}(G)}{(\mu_{c^2}(G))^2}$$

### $\chi'$ - Chromatic Statistical Parameters of Graphs

Colouring mean, median, variance, standard deviation, skewness and kurtosis corresponding to a particular type of minimal proper colouring of the edges of G are defined as follows:

8) Definition .8. A colouring mean of a graph G, with respect to a proper edgecolouring C is said to be a  $\chi'$  - chromatic mean of G, if C is the minimum proper colouring of G and the colouring sum  $\omega_c(G)$  is also minimum. The  $\chi'$  -chromatic mean of a graph G is denoted by  $\mu_{\chi'}(G)$ .

9) Definition .9. A colouring median of a graph G, with respect to a proper edge colouring C is said to be a  $\chi'$  - chromatic median of G. The  $\chi'$  -chromatic median of a graph G is denoted by  $M_{\chi'}(G)$ .

10) Definition.10. The  $\chi'$  chromatic variance of G, denoted by  $\sigma_{\chi'}^2(G)$ , is a colouring variance of G with respect to a minimal proper edge colouring of G which yields the minimum colouring sum.

11) Definition .11. The  $\chi'$  chromatic standard deviation of G, denoted by  $\sigma_{\chi'}(G)$ , is a colouring standard deviation of G with respect to a minimal proper edge colouring of G which yields the minimum colouring sum.

12) Definition.12. The  $\chi'$  chromatic skewness of G, denoted by  $\gamma_{\chi'}(G)$ , is a colouring variance of G with respect to a minimal proper edge colouring of G which yields the minimum colouring sum.

13) Definition .13. The  $\chi'$  chromatic kurtosis of G, denoted by  $\beta_{2\chi'}(G)$ , is a colouring kurtosis of G with respect to a minimal proper edge colouring of G which yields the minimum colouring sum. If  $\beta_2 = 3$ , then it is known as **MESOKURTIC** Curve, if  $\beta_2 < 3$ , then it is known as **PLATYKURTIC** Curve and if  $\beta_2 > 3$ , then it is known as **LEPTOKURTIC** Curve

Let us now determine the  $\chi'$  - chromatic mean, median, variance, standard deviation, skewness and kurtosis of certain standard graph classes. The following result discusses on Complete graph  $K_n$ .

a) Proposition1. The  $\chi'$  -Chromatic mean of a Complete graph  $K_n$  is  $\frac{n+1}{2}$ ,  $\chi'$  -chromatic variance is  $\frac{n^2-1}{12}$ ,  $\chi'$  -

chromatic median is  $\frac{n}{2}$ ,  $\chi'$  - chromatic standard deviation,  $\sqrt{\frac{n^2-1}{12}}$ ,  $\chi'$  - chromatic skewness is  $3\left(\sqrt{\frac{3}{n^2-1}}\right)$  and  $\chi'$  -

chromatic kurtosis  $\frac{9n^4 - 10n^2 + 7}{5n^4 - 10n^2 - 5}$ .

Proof: Consider a Complete graph  $K_n$  has different colours as they are adjacent to each other. (i.e)  $\theta(c_i)=1$  for the color  $c_i$ ;  $1 \leq i \leq n$

$$\text{Therefore } \mu_{\chi'}(K_n) = \frac{n+1}{2} \quad \sigma^2_{\chi'}(K_n) = \frac{n^2-1}{12} \quad M_{\chi'}(K_n) = \frac{n}{2} \quad \sigma_{\chi'}(K_n) = \sqrt{\frac{n^2-1}{12}} \quad \gamma_{\chi'}(K_n) = 3 \left( \sqrt{\frac{3}{n^2-1}} \right)$$

We can obtain kurtosis by various moments as,

$$\mu'_1 = \frac{n+1}{2}, \quad \mu'_2 = \frac{(n+1)(2n+1)}{6}, \quad \mu'_3 = \frac{n(n+1)^2}{4}, \quad \mu'_4 = \frac{(n+1)(2n+1)(3n^2+3n+1)}{30}$$

$$\mu_3 = 0 \quad \mu_4 = \frac{3n^4 - 10n^2 + 7}{240} \quad \text{And the } \chi' \text{-chromatic kurtosis of } \beta_{2\chi'}(K_n) = \frac{9n^4 - 10n^2 + 7}{5n^4 - 10n^2 - 5}$$

If  $\beta_{2\chi'}(K_n) < 3$ , then it is known as PLATYKURTIC Curve.

- i) *Theorem 1:* Any proper colouring of a complete graph  $K_n$  has the discrete uniform distribution on  $\{1,2,\dots,k\}$  (D(U(k)) that is discrete uniform distribution.

Proof: Let X be the r.v representing the number of colors in a proper k-colouring of a complete graph  $K_n$ . For any proper k-

colouring C of the complete graph  $K_n$ ,  $\theta(c_i)=1$  and  $k=n$ . Hence, the corresponding p.m.f is  $f(i) = \begin{cases} \frac{1}{n}; & n=1,2,\dots,n \\ 0 & ; \text{elsewhere} \end{cases}$  which is

that of the discrete uniform distribution on  $\{1,2,\dots,k\}$ . Hence, X follows DU (k)

- b) *Proposition 2:* The  $\chi'$ -chromatic mean of a path  $P_n$  is

$$\mu_{\chi'}(P_n) = \begin{cases} \frac{3}{2}; & \text{if } n \text{ is even} \\ \frac{3n-4}{2n-2}; & \text{if } n \text{ is odd} \end{cases}$$

the  $\chi'$ -chromatic variance of a path  $P_n$  is

$$\sigma^2_{\chi'}(P_n) = \begin{cases} \frac{1}{4}; & \text{if } n \text{ is even} \\ \frac{n^2-2n}{4n^2-8n+4}; & \text{if } n \text{ is odd} \end{cases}$$

the  $\chi'$ -chromatic median of a path  $P_n$  is  $\frac{n-1}{2}$ ,

the  $\chi'$ -chromatic standard deviation of path  $P_n$  is

$$\sigma_{\chi'}(P_n) = \begin{cases} \frac{1}{2}; & \text{if } n \text{ is even} \\ \sqrt{\frac{n^2-2n}{4n^2-8n+4}}; & \text{if } n \text{ is odd} \end{cases}$$



the  $\chi'$ -chromatic skewness of  $P_n$  is

$$\gamma_{\chi'}(P_n) = \begin{cases} 3(4 - n); & \text{if } n \text{ is even} \\ 3\left(\frac{-n^2 - 5n - 5}{\sqrt{n^2 - 2n}}\right); & \text{if } n \text{ is odd} \end{cases}$$

the  $\chi$ -chromatic kurtosis of a path  $P_n$  is,

$$\beta_{2\chi}(P_n) = \begin{cases} 1; & \text{if } n \text{ is even} \\ \frac{272n^5 - 1735n^4 + 4340n^3 - 5328n^2 + 3216n - 768}{n^4 - 4n^3 + 4n^2}; & \text{if } n \text{ is odd} \end{cases}$$

Proof:. Consider a path  $P_n$  on  $n$  edges. Then, we have

the following cases.

(1) If  $n$  is even and is 2-colourable then  $P_n$  has exactly  $\frac{n-1}{2}$  edges having colour  $c_1$  and  $c_2$  each. Then,

the  $\chi'$ -chromatic mean of  $P_n$  is  $\mu_{\chi'}(P_n) = \frac{3}{2}$ , the  $\chi'$ -chromatic variance of  $P_n$  is  $\sigma_{\chi'}^2(P_n) = \frac{1}{4}$ , the  $\chi'$ -chromatic median of  $P_n$  is

$\frac{n-1}{2}$ , the  $\chi'$ -chromatic standard deviation of  $P_n$  is  $\sigma_{\chi'}(P_n) = \frac{1}{2}$ , the  $\chi'$ -chromatic skewness of  $P_n$  is  $\gamma_{\chi'}(P_n) = 3(4-n)$  we

can obtain kurtosis by various moments  $\mu_1' = \frac{3}{2}, \mu_2' = \frac{5}{2}, \mu_3' = \frac{9}{2}, \mu_4' = \frac{17}{2}, \mu_3 = 0, \mu_4 = \frac{1}{16}$ . And the  $\chi$ -chromatic kurtosis of

$\beta_{2\chi'}(P_n) = 1$  If  $\beta_{2\chi'}(P_n) < 3$  then it is known as PLATYKURTIC Curve.

(2) If  $n$  is odd, Then, the p:m:f of the corresponding r:v:xis given by,  $f(i) = \begin{cases} \frac{n}{2(n-1)}; & i = 1 \\ \frac{n-2}{2(n-1)}; & i = 2 \\ 0 & ; \text{elsewhere} \\ 0; & \text{otherwise} \end{cases}$  the  $\chi'$ -chromatic mean

of  $P_n$  is  $\mu_{\chi'}(P_n) = \frac{3n-4}{2n-2}$ , the  $\chi'$ -chromatic variance of  $P_n$  is  $\sigma_{\chi'}^2(P_n) = \frac{n^2-2n}{4n^2-8n+4}$ , the  $\chi'$ -chromatic median of  $P_n$  is  $\frac{n-1}{2}$ ,

the  $\chi'$ -chromatic standard deviation of  $P_n$  is  $\sigma_{\chi'}(P_n) = \sqrt{\frac{n^2-2n}{4n^2-8n+4}}$ , the  $\chi'$ -chromatic skewness of  $P_n$  is

$\gamma_{\chi'}(P_n) = 3\left(\frac{-n^2 - 5n - 5}{\sqrt{n^2 - 2n}}\right)$ , we can obtain kurtosis by various moments,

$$\mu_1' = \frac{3n-4}{2n-2}, \mu_2' = \frac{5n-8}{2n-2}, \mu_3' = \frac{9n-16}{2n-2}, \mu_4' = \frac{17n-32}{2n-2},$$

$$\mu_3 = \frac{2n^2 - 4n}{(2n-2)^3}, \mu_4 = \frac{272n^5 - 1735n^4 + 4340n^3 - 5328n^2 + 3216n - 768}{(2n-2)^4}, \text{ and}$$

$$\chi' - \text{chromatic kurtosis of } \beta_{2\chi'}(P_n) = \frac{272n^5 - 1735n^4 + 4340n^3 - 5328n^2 + 3216n - 768}{n^4 - 4n^3 + 4n^2}$$

If  $\beta_{2\chi'}(P_n) < 3$ , then it is known as PLATYKURTIC Curve.

c) *Proposition 3:* The  $\chi'$  -chromatic mean of acycle  $C_n$  is

$$\mu_{\chi'}(C_n) = \begin{cases} \frac{3}{2}; & \text{if } n \text{ is even} \\ \frac{3n+3}{2n}; & \text{if } n \text{ is odd} \end{cases}$$

And the  $\chi'$  - chromatic variance of a cycle  $C_n$  is

$$\sigma^2_{\chi'}(C_n) = \begin{cases} \frac{1}{4}; & \text{if } n \text{ is even} \\ \frac{n^2 + 8n - 9}{4n^2}; & \text{if } n \text{ is odd} \end{cases}$$

the  $\chi'$  - chromatic median of a cycle  $C_n$  is  $\frac{n}{2}$ ,

the  $\chi'$  - chromatic standard deviation of a cycle  $C_n$  is,

$$\sigma_{\chi'}(C_n) = \begin{cases} \frac{1}{4}; & \text{if } n \text{ is even} \\ \sqrt{\frac{n^2 + 8n - 9}{4n^2}}; & \text{if } n \text{ is odd} \end{cases}$$

The  $\chi'$  - chromatic skewness of a cycle  $C_n$  is

$$\gamma_{\chi'}(C_n) = \begin{cases} 3(3-n); & \text{if } n \text{ is even} \\ 3\left(\frac{-n^2 + 3n + 3}{\sqrt{n^2 + 8n - 9}}\right); & \text{if } n \text{ is odd} \end{cases}$$

The  $\chi'$  - chromatic kurtosis of  $C_n$  is,

$$\beta_{2\chi'}(C_n) = \begin{cases} 1; & \text{if } n \text{ is even} \\ \frac{n^4 + 80n^3 - 270n^2 + 432n - 243}{n^4 + 16n^3 + 46n^2 - 144n + 81}; & \text{if } n \text{ is odd} \end{cases}$$

Proof.: Consider a cycle  $C_n$  on  $n$  edges. Then, we have the following cases.

(1) If  $n$  is even and is 2-colourable then  $C_n$  has exactly  $\frac{n}{2}$  edges having colour  $C_1$  and  $C_2$  each. Then,

i)  $\chi'$  - chromatic mean of cycle  $C_n$  is,  $\mu_{\chi'}(C_n) = \frac{3}{2}$

ii)  $\chi'$  - chromatic variance of cycle  $C_n$  is,  $\sigma^2_{\chi'}(C_n) = \frac{1}{4}$

iii)  $\chi'$  - chromatic median of cycle  $C_n$  is  $\frac{n}{2}$

iv)  $\chi'$  - chromatic standard deviation of  $C_n$  is,  $\sigma_{\chi'}(C_n) = \frac{1}{2}$

v)  $\chi'$  - chromatic skewness of  $C_n$  is  $\gamma_{\chi'}(C_n) = 3(3-n)$

we can obtain kurtosis by various moments,

$$\mu'_1 = \frac{3}{2}, \mu'_2 = \frac{5}{2}, \mu'_3 = \frac{9}{2}, \mu'_4 = \frac{17}{2}, \mu_3 = 0, \mu_4 = \frac{1}{16},$$

And the  $\chi'$  - chromatic kurtosis of  $C_n$  is  $\beta_{2\chi'}(C_n) = 1$

If  $\beta_{2\chi'}(C_n) < 3$  then it is known as PLATYKURTIC Curve.

(2) If  $n$  is odd, then is  $C_n$  3-colourable. Let  $C = \{c_1, c_2, c_3\}$  be the minimal proper colouring of  $C_n$ . Then, the p.m.f of the

corresponding (r.v.X) is given by  $f(i) = \begin{cases} \frac{n-1}{2n}; & i = 1, 2 \\ \frac{1}{n}; & i = 3 \\ 0; & \text{elsewhere} \end{cases}$  then,

1.  $\chi'$  - chromatic mean of cycle  $C_n$  is  $\mu_{\chi'}(C_n) = \frac{3n+3}{2n}$

2.  $\chi'$  - chromatic variance of cycle  $C_n$  is,  $\sigma^2_{\chi'}(C_n) = \frac{n^2+8n-9}{4n^2}$

3.  $\chi'$  - chromatic median of cycle  $C_n$  is  $\frac{n}{2}$

4.  $\chi'$  - chromatic standard deviation of cycle  $C_n$  is,  $\sigma_{\chi'}(C_n) = \sqrt{\frac{n^2+8n-9}{4n^2}}$

5.  $\chi'$  - chromatic skewness of cycle  $C_n$  is  $\gamma_{\chi'}(C_n) = 3\left(\frac{-n^2+3n+3}{\sqrt{n^2+8n-9}}\right)$

we can obtain kurtosis by various moments,

$$\mu'_1 = \frac{3n+3}{2n}, \mu'_2 = \frac{5n+13}{2n}, \mu'_3 = \frac{9n+45}{2n}, \mu'_4 = \frac{17n+145}{2n},$$

$$\mu_3 = \frac{9n^2 - 36n + 27}{4n^3}, \mu_4 = \frac{n^4 + 80n^3 - 270n^2 + 432n - 243}{16n^4},$$

And the  $\chi'$  - chromatic kurtosis of  $\beta_{2\chi'}(C_n) = \frac{n^4 + 80n^3 - 270n^2 + 432n - 243}{n^4 + 16n^3 + 46n^2 - 144n + 81}$ ,

If  $\beta_{2\chi'}(C_n) < 3$  then it is known as PLATYKURTIC Curve.



d) Proposition 4: The  $\chi'$  - chromatic mean of a ladder graph  $(L_n)$  is  $\mu_{\chi'}(L_n) = \frac{6n - 6}{3n - 2}$ . Then

i)  $\chi'$  - chromatic variance of a ladder graph  $(L_n)$  is  $\sigma^2_{\chi'}(L_n) = \frac{6n^2 - 8n}{3n - 2}$

ii)  $\chi'$  - chromatic median of a ladder graph  $(L_n)$  is  $\frac{3n-2}{2}$

iii)  $\chi'$  - chromatic standard deviation of a ladder graph  $(L_n)$  is  $\sigma_{\chi'}(L_n) = \sqrt{\frac{6n^2 - 8n}{3n - 2}}$

iv)  $\chi'$  - chromaticskewness of a ladder graph  $(L_n)$  is

$$\gamma_{\chi'}(L_n) = \frac{-9n + 12(\sqrt{18n^3 - 36n^2 + 16n})}{12n^2 - 8n}$$

(v)  $\chi'$  - chromatic kurtosis of a ladder graph  $(L_n)$  is

$$\beta_{2\chi'}(L_n) = \frac{54n^4 - 162n^3 + 180n^2 - 136n}{36n^4 - 96n^3 + 64n^2}$$

Proof: In a Ladder Graph  $(L_n)$ , we have  $3n - 2$  edges and the graph is 3- colourable

Hence the corresponding p.m.f of the corresponding  $W_n$  is given by,  $f(i) = \begin{cases} \frac{n}{3n - 2}; i = 1, 2 \\ \frac{n - 2}{3n - 2}; i = 3 \\ 0; elsewhere \end{cases}$

1.  $\chi'$  - chromatic mean of  $(L_n)$  is  $\mu_{\chi'}(L_n) = \frac{6n-6}{3n-2}$

2.  $\chi'$  - chromatic variance of  $(L_n)$  is  $\sigma^2_{\chi'}(W_n) = \frac{6n^2 - 8n}{3n - 2}$

3.  $\chi'$  - chromatic median of  $(L_n)$  is  $\frac{3n - 2}{2}$

4.  $\chi'$  - chromatic standard deviation of  $(L_n)$  is  $\sigma_{\chi'}(L_n) = \sqrt{\frac{6n^2 - 8n}{3n - 2}}$

5.  $\chi'$  - chromaticskewness of is,  $\gamma_{\chi'}(L_n) = \frac{-9n + 12(\sqrt{18n^3 - 36n^2 + 16n})}{12n^2 - 8n}$

We can obtain kurtosis by various moments,

$$\mu'_1 = \frac{6n-6}{3n-2}, \mu'_2 = \frac{14n-18}{3n-2}, \mu'_3 = \frac{36n-54}{3n-2}, \mu'_4 = \frac{98n-162}{3n-2},$$

$$\mu_3 = \frac{18n^2 - 36n}{(3n-2)^3}, \mu_4 = \frac{54n^4 - 162n^3 + 180n^2 + 136n}{(3n-2)^4},$$

And the  $\chi'$  - chromatic kurtosis of  $\beta_{2\chi'}(L_n) = \frac{54n^4 - 162n^3 + 180n^2 - 136n}{36n^4 - 96n^3 + 64n^2}$

If  $\beta_{2\chi'}(L_n) < 3$  then it is known as PLATYKURTIC Curve.

## II. CONCLUSION

In this paper, we extend the concept of mean, variance, standard deviation, median, skewness and kurtosis, some important statistical parameters to various graphs based on edge coloring sum. Based on the edge coloring sum of graphs, we have investigated these statistical inferences for edge colorable graphs such as Complete graphs, Path, Cycle and ladder graphs. This concept can be extended to several other operations on graphs such as Cartesian product, total coloring graphs, Johan colouring, lexicographic product, corona product, sum and product of graphs etc.,

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