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International Journal For Research in  
Applied Science and Engineering Technology



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# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

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**Volume: 7      Issue: IV      Month of publication: April 2019**

**DOI: <https://doi.org/10.22214/ijraset.2019.4489>**

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# Bianchi Type-VI<sub>0</sub> String Cosmological Model For Barotropic Fluid Distribution with Cosmological Term $\Lambda$

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**Abstract:** Bianchi type-VI<sub>0</sub> string cosmological model for barotropic fluid distribution with cosmological term  $\Lambda$  is investigated. To obtain the explicit solution of the model, we suppose expansion  $\theta$  is proportional to the shear  $\sigma$  and  $\Lambda$  is proportional to  $R^{-3}$ , where  $R$  is scale factor. We have also discussed physical and geometrical characteristics of the cosmological model.

**Keywords:** Bianchi type-VI<sub>0</sub>, barotropic fluid, cosmic string, cosmological term, perfect fluid.

## I. INTRODUCTION

In recent years, the study of cosmological models with dark energy create much more interest due to the fact that our universe is undergoing an accelerated expansion which is driven by an exotic energy with large negative pressure named dark energy. Cosmologists have proposed many candidates for this dark energy, however, cosmological constant  $\Lambda$  is the simplest and most hypothetically attractive candidate of it with equation of state  $\omega = -1$ . According to recent observations,  $\Lambda \approx 10^{-56} \text{cm}^{-2}$  while particle physics predicts that  $\Lambda$  is much greater than this value by factor of order  $10^{120}$ . This inconsistency is called cosmological constant problem and the simplest way to resolve this problem is to consider cosmological term  $\Lambda$ . Bhojar et al. [3], Pradhan et al. [10], Tyagi et al. [14] have studied cosmological models with cosmological term ( $\Lambda$ ) in various contexts.

Due to the large scale distribution of galaxies in our cosmos, the matter distribution can be suitably described by perfect fluid. Bianchi Type-VI<sub>0</sub> cosmological models in different context have studied by different researchers viz. Amirhaschi [1], Deo et al. [4], Khadekar et al. [6, 7], Pradhan et al. [8, 9], Tyagi et al. [11, 13], Verma et al. [15]. String dust cosmological model for barotropic fluid distribution with vacuum energy density in FRW space-time is investigated by Bali and Bola [2]. Dewri [5] has studied magnetized anisotropic Bianchi type-VI cosmological model containing dark energy. Trivedi and Shrimali [12] have investigated barotropic Bianchi type-VI<sub>0</sub> cosmological model in general relativity.

In this paper, we have investigated Bianchi type-VI<sub>0</sub> string cosmological model for barotropic fluid distribution and cosmological term  $\Lambda$ . To obtain the explicit solution of the model, we suppose expansion  $\theta$  is proportional to the shear  $\sigma$  and  $\Lambda$  is proportional to  $R^{-3}$ , where  $R$  is scale factor. We have also discussed physical and geometrical characteristics of the cosmological model.

## II. METRIC AND FIELD EQUATION

The line element for Bianchi type-VI<sub>0</sub> space-time is considered as

$$ds^2 = -dt^2 + A^2 dx^2 + e^{-2x} B^2 dy^2 + e^{2x} C^2 dz^2 \quad (1)$$

where  $A$ ,  $B$  and  $C$  depends on cosmic time  $t$  only.

The Einstein's field equation in the geometrized unit ( $c=8\pi G=1$ ) is given by

$$R_i^j - \frac{1}{2} R g_i^j + \Lambda g_i^j = -T_i^j \quad (2)$$

where  $R_i^j$  is Ricci tensor and  $R = g^{ij} R_{ij}$  is Ricci scalar.

The energy momentum tensor ( $T_i^j$ ) for the cloud of strings in the presence of perfect fluid distribution is given by

$$T_i^j = (\rho + p)v_i v^j + p g_i^j - \lambda x_i x^j \quad (3)$$

Here  $\rho$  is proper energy density,  $p$  is pressure and  $\lambda$  is string tension density. Also  $x^i$ , the unit space like vector specifying the direction of strings and  $v^i$ , the unit time like vector satisfying the following conditions:

$$v_i v^i = -1 = -x_i x^i \text{ and } v^i x_i = 0 \tag{4}$$

In a co-moving coordinate system, we have

$$v^i = (0,0,0,1); \quad x^i = \left(\frac{1}{A}, 0,0,0\right) \tag{5}$$

The Einstein's field equation (2) for metric (1) together with (3) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \frac{1}{A^2} + \Lambda = \lambda - p \tag{6}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} + \Lambda = -p \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{1}{A^2} + \Lambda = -p \tag{8}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} + \Lambda = \rho \tag{9}$$

$$\left[\frac{B_4}{B} - \frac{C_4}{C}\right] = 0 \tag{10}$$

### III. SOLUTION OF FIELD EQUATIONS

The field equations (6) - (10) are the organization of five equations with seven undetermined factors A, B, C, λ, ρ, p and Λ, so to obtain an exact solution of the present model we suppose expansion θ is proportional to shear σ, which leads to

$$A = B^n \tag{11}$$

and Λ is inversely proportional to R<sup>3</sup>, which leads to

$$\Lambda = \frac{\alpha}{R^3} = \frac{\alpha}{ABC} \tag{12}$$

where α is constant of proportion.

Equation (10) leads to

$$\frac{B_4}{B} = \frac{C_4}{C} \tag{13}$$

On integrating equation (13), we get

$$B = Ck \tag{14}$$

where k is the integrating constant.

Now, without loss of generality, we suppose k=1, then equation (14) becomes

$$B = C \tag{15}$$

To get the deterministic solution, we also use the barotropic fluid condition,

$$\text{i.e. } p = \gamma\rho \tag{16}$$

Now, using conditions (11), (12), (15) and (16) in field equations (6) – (9), we obtain

$$\frac{(n+1)B_{44}}{B} + \frac{[n^2 + (2n+1)\gamma]B_4^2}{B^2} - \frac{(\gamma+1)}{B^{2n}} + \frac{\alpha(\gamma+1)}{B^{n+2}} = 0 \tag{17}$$

Now we consider the following cases:

#### A. Dust Fluid Model (γ = 0)

On putting γ = 0 in equation (17), we get

$$\frac{(n+1)B_{44}}{B} + \frac{n^2 B_4^2}{B^2} - \frac{1}{B^{2n}} + \frac{\alpha}{B^{n+2}} = 0 \tag{18}$$

Equation (18) leads to

$$2B_{44} + \frac{2n^2 B_4^2}{(n+1)B} = \frac{2}{(n+1)B^{2n-1}} - \frac{2\alpha}{(n+1)B^{n+1}} \tag{19}$$

On putting  $B_4 = f(B)$  and  $B_{44} = ff'$  in equation (19), we get

$$\frac{df^2}{dB} + \frac{2n^2 f^2}{(n+1)B} = \frac{2}{(n+1)B^{2n-1}} - \frac{2\alpha}{(n+1)B^{n+1}} \tag{20}$$

On integrating equation (20), we get

$$f^2 = \frac{1}{B^{2(n-1)}} - \frac{2\alpha}{n(n-1)B^n} + \frac{l}{B^{n+1}} \tag{21}$$

where  $l$  is constant of integration.

From equation (21), we have

$$\int \frac{dB}{\sqrt{\frac{1}{B^{2(n-1)}} - \frac{2\alpha}{n(n-1)B^n} + \frac{l}{B^{n+1}}}} = \int dt + M = t + M \tag{22}$$

where  $M$  is the integrating constant. Value of  $B$  can be obtained from equation (22).

Hence, by appropriate transformation of coordinates i.e.  $B=T, x=X, y=Y$  and  $z=Z$ , metric (1) becomes

$$ds^2 = - \left[ \frac{1}{T^{2(n-1)}} - \frac{2\alpha}{n(n-1)T^n} + \frac{l}{T^{n+1}} \right] dT^2 + T^{2n} dX^2 + e^{-2X} T^2 dY^2 + e^{2X} T^2 dZ^2 \tag{23}$$

### B. Stiff Fluid Model ( $\gamma = 1$ )

On putting  $\gamma = 1$  in equation (17), we get

$$\frac{(n+1)B_{44}}{B} + \frac{[n^2 + 2n + 1]B_4^2}{B^2} - \frac{2}{B^{2n}} + \frac{2\alpha}{B^{n+2}} = 0 \tag{24}$$

Equation (24) leads to

$$2B_{44} + \frac{2(n+1)B_4^2}{B} = \frac{4}{(n+1)B^{2n-1}} - \frac{4\alpha}{(n+1)B^{n+1}} \tag{25}$$

On putting  $B_4 = f(B)$  and  $B_{44} = ff'$  in equation (25), we get

$$\frac{df^2}{dB} + \frac{2(n+1)f^2}{B} = \frac{4}{(n+1)B^{2n-1}} - \frac{4\alpha}{(n+1)B^{n+1}} \tag{26}$$

On integrating equation (26), we get

$$f^2 = \frac{1}{(n+1)B^{2(n-1)}} - \frac{4\alpha}{(n+1)(n+2)B^n} + \frac{L}{B^{2(n+1)}} \tag{27}$$

where  $L$  is constant of integration.

From equation (27), we have

$$\int \frac{dB}{\sqrt{\frac{1}{(n+1)B^{2(n-1)}} - \frac{4\alpha}{(n+1)(n+2)B^n} + \frac{L}{B^{2(n+1)}}}} = \int dt + M' = t + M' \tag{28}$$

where  $M'$  is the integrating constant. Value of  $B$  can be obtained from equation (28).

Hence, by appropriate transformation of coordinates i.e.  $B=T, x=X, y=Y$  and  $z=Z$ , metric (1) becomes

$$ds^2 = - \left[ \frac{1}{(n+1)T^{2(n-1)}} - \frac{4\alpha}{(n+1)(n+2)T^n} + \frac{L}{T^{2(n+1)}} \right] dT^2 + T^{2n} dX^2 + e^{-2X} T^2 dY^2 + e^{2X} T^2 dZ^2 \tag{29}$$

### C. Radiation Dominated Model ( $\gamma = 1/3$ )

On putting  $\gamma = 1/3$  in equation (17), we get

$$\frac{(n+1)B_{44}}{B} + \frac{[3n^2 + 2n + 1]B_4^2}{3B^2} - \frac{4}{3B^{2n}} + \frac{4\alpha}{3B^{n+2}} = 0 \tag{30}$$

Equation (30) leads to

$$2B_{44} + \frac{2[3n^2 + 2n + 1]B_4^2}{3(n+1)B} = \frac{8}{3(n+1)B^{2n-1}} - \frac{8\alpha}{3(n+1)B^{n+1}} \tag{31}$$

On putting  $B_4 = f(B)$  and  $B_{44} = ff'$  in equation (31), we get

$$\frac{df^2}{dB} + \frac{2[3n^2 + 2n + 1]f^2}{3(n+1)B} = \frac{8}{3(n+1)B^{2n-1}} - \frac{8\alpha}{3(n+1)B^{n+1}} \tag{32}$$

On integrating equation (32), we get

$$f^2 = \frac{2}{(n+2)B^{2(n-1)}} - \frac{8\alpha}{[3n^2 + n + 2]B^n} + \frac{N}{B^{\frac{2(3n^2+2n+1)}{3(n+1)}}} \tag{33}$$

where  $N$  is constant of integration.

From equation (33), we have

$$\int \frac{dB}{\sqrt{\frac{2}{(n+2)B^{2(n-1)}} - \frac{8\alpha}{[3n^2 + n + 2]B^n} + \frac{N}{B^{\frac{2(3n^2+2n+1)}{3(n+1)}}}}} = \int dt + P = t + P \tag{34}$$

where  $P$  is the integrating constant. Value of  $B$  can be obtained from equation (34).

Hence, by appropriate transformation of coordinates i.e.  $B=T, x=X, y=Y$  and  $z=Z$ , metric (1) becomes

$$ds^2 = - \left[ \frac{dT^2}{\frac{2}{(n+2)T^{2(n-1)}} - \frac{8\alpha}{[3n^2 + n + 2]T^n} + \frac{N}{B^{\frac{2(3n^2+2n+1)}{3(n+1)}}}} \right] + T^{2n}dX^2 + e^{-2X}T^2dY^2 + e^{2X}T^2dZ^2 \tag{35}$$

#### IV. PHYSICAL AND GEOMETRICAL CHARACTERISTICS

For the model (23), energy density ( $\rho$ ), string tension density ( $\lambda$ ), pressure ( $p$ ), expansion  $\theta$ , and shear  $\sigma$  are given by

$$\rho = \frac{2n}{T^{2n}} + \frac{\alpha(n^2 - 5n - 2)}{n(n-1)T^{n+1}} + \frac{l(2n+1)}{T^{\frac{2n^2+2n+1}{n+1}}} \tag{36}$$

$$\lambda = \frac{(n+2)\alpha}{nT^{n+2}} - \frac{2(n-2)}{T^{2n}} + \frac{l(n+1-2n^2)}{(n+1)T^{\frac{2n^2+2n+1}{n+1}}} \tag{37}$$

$$p = 0 \tag{38}$$

$$\theta = (n+2) \left[ \frac{1}{T^{2n}} - \frac{2\alpha}{n(n-1)T^{n+2}} + \frac{l}{T^{\frac{2n^2+2n+1}{n+1}}} \right]^{\frac{1}{2}} \tag{39}$$

$$\sigma = \frac{(n-1)}{\sqrt{3}} \left[ \frac{1}{T^{2n}} - \frac{2\alpha}{n(n-1)T^{n+2}} + \frac{l}{T^{\frac{2n^2+2n+1}{n+1}}} \right]^{\frac{1}{2}} \tag{40}$$

For the model (29), energy density ( $\rho$ ), string tension density ( $\lambda$ ), pressure ( $p$ ), expansion  $\theta$ , and shear  $\sigma$  are given by

$$\rho = p = \frac{n}{(n+1)T^{2n}} + \frac{\alpha(n^2 - 5n - 2)}{(n+1)(n+2)T^{n+2}} + \frac{L(2n+1)}{T^{2(n+2)}} \tag{41}$$

$$\lambda = \frac{2(n-1)\alpha}{(n+1)T^{n+2}} + \frac{4}{(n+1)T^{2n}} \tag{42}$$

$$\theta = (n + 2) \left[ \frac{1}{(n + 1)T^{2n}} - \frac{4\alpha}{(n + 1)(n + 2)T^{n+2}} + \frac{L}{T^{2n+4}} \right]^{\frac{1}{2}} \tag{43}$$

$$\sigma = \frac{(n - 1)}{\sqrt{3}} \left[ \frac{1}{(n + 1)T^{2n}} - \frac{4\alpha}{(n + 1)(n + 2)T^{n+2}} + \frac{L}{T^{2n+4}} \right]^{\frac{1}{2}} \tag{44}$$

For the model (35), energy density ( $\rho$ ), string tension density ( $\lambda$ ), pressure (p), expansion  $\theta$ , and shear  $\sigma$  are given by

$$\rho = \frac{3n}{(n + 2)T^{2n}} + \frac{3\alpha(n^2 - 5n - 2)}{(3n^2 + n + 2)T^{n+2}} + \frac{N(2n + 1)}{T^{\frac{2(3n^2+5n+4)}{3(n+1)}}} \tag{45}$$

$$\lambda = \frac{4(n + 2)(n - 1)\alpha}{(3n^2 + n + 2)T^{n+2}} - \frac{2(n - 4)}{(n + 2)T^{2n}} - \frac{2(n - 1)(2n + 1)N}{3(n + 1)T^{\frac{2(3n^2+5n+4)}{3(n+1)}}} \tag{46}$$

$$p = \frac{n}{(n + 2)T^{2n}} + \frac{\alpha(n^2 - 5n - 2)}{(3n^2 + n + 2)T^{n+2}} + \frac{N(2n + 1)}{3T^{\frac{2(3n^2+5n+4)}{3(n+1)}}} \tag{47}$$

$$\theta = (n + 2) \left[ \frac{2}{(n + 2)T^{2n}} - \frac{8\alpha}{(3n^2 + n + 2)T^{n+2}} + \frac{N}{T^{\frac{2(3n^2+5n+4)}{3(n+1)}}} \right]^{\frac{1}{2}} \tag{48}$$

$$\sigma = \frac{(n - 1)}{\sqrt{3}} \left[ \frac{2}{(n + 2)T^{2n}} - \frac{8\alpha}{(3n^2 + n + 2)T^{n+2}} + \frac{N}{T^{\frac{2(3n^2+5n+4)}{3(n+1)}}} \right]^{\frac{1}{2}} \tag{49}$$

Also, cosmological term  $\Lambda$  for the models (23), (29) and (35) is given by

$$\Lambda = \frac{\alpha}{T^{n+2}} \tag{50}$$

The magnitude of rotation  $\omega$  is zero, i.e.

$$\omega = 0 \tag{51}$$

### V. CONCLUSION

The models (23), (29) and (35) start expanding with big bang at  $T = 0$ . The expansion  $\theta$  is decreasing function of cosmic time  $T$  for  $n > 0$  and approaches to zero as  $T \rightarrow \infty$  and also stops at  $n = -2$ . Since  $T \rightarrow \infty, \frac{\sigma}{\theta} = \frac{n-1}{\sqrt{3}(n+2)} \neq 0$ , therefore the model does not approach isotropy for large values of  $T$ , however it is isotropized for  $n = 1$ .

Cosmological term  $\Lambda$  for these models are found to be decreasing function of time  $T$  for  $n > -2$  and approaches to zero at late time, which is in agreement with present astronomical observations. A point type singularity is observed as  $T \rightarrow 0, g_{11} \rightarrow 0, g_{22} \rightarrow 0, g_{33} \rightarrow 0$  for  $n > 0$ .

Also, we can observe that energy density ( $\rho$ ), string tension density ( $\lambda$ ) and pressure (p) for these models are decreasing function of time  $T$  for  $n > 0$  and approaches to zero at  $T \rightarrow \infty$ .

Hence, in general, the models represent expanding, shearing and non-rotating universe.

### REFERENCES

- [1] Amirhashchi, H., "String Cosmology in Bianchi Type-VI<sub>0</sub> Dusty Universe with Electromagnetic Field", PRAMANA Journal of Physics, 80, 723-738, 2013.
- [2] Bali, R. and Bola, S.C., "String Dust Cosmological Model for Barotropic Fluid Distribution with Vacuum Energy Density in FRW Space-Time", Journal of Rajasthan Academy of Physical Sciences, 14, 199-208, 2015.
- [3] Bhoyar, S.R. and Chirde, V.R., "Magnetized Anti- Stiff Fluid Cosmological Models with Variable Cosmological Constant. International Journal of Scientific Research in Mathematical and Statistical Sciences, 5, 11-18, 2018.
- [4] Deo, S.D. Punwatar, G.S. and Patil, U.M., "Bulk Viscous Fluid in Bianchi Type-VI<sub>0</sub> Universe with Electromagnetic Field", IOSR Journal of Applied Physics, 8, 42-45, 2016.
- [5] Dewri, M., "Magnetized Anisotropic Bianchi Type-VI Cosmological Model containing Dark Energy", IOSR Journal of Applied Physics, 10, 31-35, 2018.
- [6] Khadekar, G.S., Samdurkar, S. and Sen, S., "Bianchi Type-VI Cosmological Model with Quadratic Form of Time Dependent  $\Lambda$  Term in General Relativity", Global Journal of Science Frontier Research: A Physics and Space Science, 16, 35-47, 2016.



- [7] Kandalkar, S.P., Khade, P.P. and Gawande, S.P., "Bianchi Type-VI Bulk Viscous Fluid String Cosmological Model in General Relativity", Bulg. Journal of Physics, 38, 145-154, 2011.
- [8] Pradhan, A. and Lata, S., "Magnetized Bianchi Type-VI<sub>0</sub> Bulk Viscous Barotropic Massive String Universe with Decaying Vacuum Energy Density  $\Lambda$ ", Electronic Journal of Theoretical Physics, 8, 153-168, 2011.
- [9] Pradhan, A. and Bali, R., "Magnetized Bianchi Type-VI<sub>0</sub> Barotropic Massive String Universe with Decaying Vacuum Energy Density  $\Lambda$ ", Electronic Journal of Theoretical Physics, 5, 91-104, 2008.
- [10] Pradhan, A., Jotania, K. and Singh, A., "Magnetized String Cosmological Model in Cylindrically Symmetric Inhomogeneous Universe with Time Dependent Cosmological Term  $\Lambda$ ", Brazilian Journal of Physics, 38, 167-177, 2008.
- [11] Sharma, A., Tyagi, A. and Chhajed, D., "Inhomogeneous Bianchi Type-VI<sub>0</sub> String Cosmological Model for Stiff Perfect Fluid Distribution in General Relativity", Prespacetime Journal, 7, 615-622, 2016.
- [12] Trivedi, T. and Shrimali, S., "Barotropic Bianchi Type-VI<sub>0</sub> Cosmological Model in General Relativity", International Journal of Engineering Science Invention, 7, 51-55, 2018.
- [13] Tyagi, A., Sharma, A. and Chhajed, D., "Inhomogeneous Bianchi Type-VI<sub>0</sub> String Dust Cosmological Model of Perfect Fluid Distribution in General Relativity", Journal of Rajasthan Academy of Physical Sciences, 14, 15-23, 2015.
- [14] Tyagi, A., Parikh, S. and Tripathi, B.R., "LRS Bianchi Type-II Homogeneous Cosmological Model for Perfect Fluid with Electromagnetic Field and Variable  $\Lambda$ ", Journal of Rajasthan Academy of Physical Sciences, 16, 1-8, 2017.
- [15] Verma, M.K. and Ram, S., "Bianchi Type-VI<sub>0</sub> Bulk Viscous Fluid Models with Variable Gravitational and Cosmological Constant", Applied Mathematics, 2, 348-354, 2011.



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