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On Binary Diophantine Equation $3x^2 - 5y^2 = 12$

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Abstract:** Non-homogeneous binary quadratic equation representing hyperbola given by* $3x^2 - 5y^2 = 12$ ***is analyzed for its non-zero distinct integer solutions. A few interesting relations among its solutions are presented. Also, knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbola and parabola are presented.
***Keywords:** Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation. AMS Mathematics subject Classification (2010):11D0*

I. INTRODUCTION

The binary quadratic Diophantine equations of the form $ax^2 - by^2 = N, (a, b, c \neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of a, b and N. In this context, one may refer [1-18].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation given by, $3x^2 - 5y^2 = 12$ representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented.

II. METHOD OF ANALYSIS

The Diophantine equation representing the binary quadratic equation to be solved for its non-zero distinct integral solution is

$$3x^2 - 5y^2 = 12 \qquad (1)$$

Introduce the linear transformation

$$x = X + 5T, y = X + 3T \qquad (2)$$

From (1) & (2) we have,

$$X^2 = 15T^2 - 6 \qquad (3)$$

whose smallest positive integer solution is

$$X_0 = 3, T_0 = 1$$

To obtain the other solutions of (3), consider the Pell equation

$$X^2 = 15T^2 + 1 \qquad (4)$$

whose smallest positive integer solution is

$$\tilde{X}_0 = 4, \tilde{T}_0 = 1$$

whose general solution is given by

$$\tilde{T}_n = \frac{1}{2\sqrt{15}} g_n, \tilde{X}_n = \frac{1}{2} f_n$$

where

$$f_n = \left(4 + \sqrt{15}\right)^{n+1} + \left(4 - \sqrt{15}\right)^{n+1}$$
$$g_n = \left(4 + \sqrt{15}\right)^{n+1} - \left(4 - \sqrt{15}\right)^{n+1}$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the other integer solutions of (1) are given by

$$\sqrt{15}x_{n+1} = 4\sqrt{15} f_n + 15 g_n$$
$$\sqrt{15}y_{n+1} = 3\sqrt{15} f_n + 12 g_n$$

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The recurrence relations satisfied by x and y are given by

$$x_{n+3} - 8x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 8y_{n+2} + y_{n+1} = 0$$

Some numerical examples of x and y satisfying (1) are given in the Table: 1 below:

Table: 1 Numerical examples

n	x_{n+1}	y_{n+1}
-1	8	6
0	62	48
1	488	378
2	3842	2976
3	30248	23430
4	288142	184464

From the above table, we observe some interesting relations among the solutions which are presented below:

A. x_{n+1} & y_{n+1} values are always even .

B. Relations among the solutions

1) $x_{n+3} - 8x_{n+2} + x_{n+1} = 0$
2) $5y_{n+1} - x_{n+2} + 4x_{n+1} = 0$
3) $15y_{n+3} - 93x_{n+2} + 12x_{n+1} = 0$
4) $40y_{n+3} + x_{n+1} - 3x_{n+3} = 0$
5) $9x_{n+2} - 36x_{n+1} - 45y_{n+1} = 0$
6) $x_{n+3} - 31x_{n+1} - 40y_{n+1} = 0$
7) $y_{n+3} - 24x_{n+1} - 31y_{n+1} = 0$
8) $x_{n+3} - x_{n+1} - 10y_{n+2} = 0$
9) $4y_{n+1} + 3x_{n+1} - y_{n+2} = 0$
10) $3x_{n+1} - 24x_{n+2} + 3x_{n+3} = 0$
11) $5y_{n+1} + 31x_{n+2} - 4x_{n+3} = 0$
12) $5y_{n+2} + 4x_{n+2} - x_{n+3} = 0$
13) $y_{n+3} - 6x_{n+2} - y_{n+1} = 0$
14) $x_{n+1} - 4x_{n+2} + 5y_{n+2} = 0$
15) $y_{n+1} + 3x_{n+2} - 4y_{n+2} = 0$
16) $4x_{n+1} - 31x_{n+2} + 5y_{n+3} = 0$
17) $4y_{n+2} + 3x_{n+2} - y_{n+3} = 0$
18) $4y_{n+1} + 3x_{n+3} - 31y_{n+2} = 0$
19) $x_{n+1} - 31x_{n+3} + 40y_{n+3} = 0$
20) $x_{n+2} - 4x_{n+3} + 5y_{n+3} = 0$

21) $y_{n+2} + 3x_{n+3} - 4y_{n+3} = 0$

22) $24x_{n+3} + y_{n+1} - 31y_{n+3} = 0$

23) $8y_{n+2} - y_{n+1} - y_{n+3} = 0$

24) $3x_{n+1} + 31y_{n+2} - 4y_{n+3} = 0$

C. Each of the following expressions represents a nasty number

1) $(48x_{2n+2} - 6x_{2n+3} + 12)$

2) $\frac{1}{8}(378x_{2n+2} - 6x_{2n+4} + 96)$

3) $(24x_{2n+2} - 30y_{2n+2} + 12)$

4) $\frac{1}{12}(558x_{2n+2} - 90y_{2n+3} + 144)$

5) $\frac{1}{31}(1464x_{2n+2} - 30y_{2n+4} + 372)$

6) $\frac{1}{3}(1134x_{2n+3} - 144x_{2n+4} + 36)$

7) $(6x_{2n+3} - 60y_{2n+2} + 12)$

8) $(186x_{2n+3} - 240y_{2n+3} + 12)$

9) $(366x_{2n+3} - 60y_{2n+4} + 12)$

10) $\frac{1}{31}(24x_{2n+4} - 1890y_{2n+2} + 372)$

11) $\frac{1}{4}(186x_{2n+4} - 1890y_{2n+3} + 48)$

12) $(1464x_{2n+4} - 1890y_{2n+4} + 12)$

13) $\frac{1}{3}(24y_{2n+3} - 186y_{2n+2} + 36)$

14) $\frac{1}{6}(6y_{2n+4} - 366y_{2n+2} + 72)$

15) $\frac{1}{3}(186y_{2n+4} - 1464y_{2n+3} + 36)$

D. Each of the following expressions represents a cubical integer

1) $[8x_{3n+3} - x_{3n+4} + 24x_{n+1} - 3x_{n+2}]$

2) $\frac{1}{8}[63x_{3n+3} - x_{3n+5} + 189x_{n+1} - 3x_{n+3}]$

3) $[4x_{3n+3} - 5y_{3n+3} + 12x_{n+1} - 15y_{n+1}]$

4) $\frac{1}{4}[31x_{3n+3} - 5y_{3n+4} + 93x_{n+1} - 15y_{n+2}]$

5) $\frac{1}{31}\left[244x_{3n+3}-5y_{3n+5}+732x_{n+1}-15y_{n+3}\right]$
6) $\left[63x_{3n+4}-8x_{3n+5}+189x_{n+2}-24x_{n+3}\right]$
7) $\left[x_{3n+4}-10y_{3n+3}+3x_{n+2}-30y_{n+1}\right]$
8) $\left[31x_{3n+4}-40y_{3n+4}+93x_{n+2}-120y_{n+2}\right]$
9) $\left[61x_{3n+4}-10y_{3n+5}+183x_{n+2}-30y_{n+3}\right]$
10) $\frac{1}{31}\left[4x_{3n+5}-315y_{3n+3}+12x_{n+3}-945y_{n+1}\right]$
11) $\frac{1}{4}\left[31x_{3n+5}-315y_{3n+4}+93x_{n+3}-945y_{n+2}\right]$
12) $\left[244x_{3n+5}-315y_{3n+5}+732x_{n+3}-945y_{n+3}\right]$
13) $\frac{1}{3}\left[4y_{3n+4}-31y_{3n+3}+12y_{n+2}-93y_{n+1}\right]$
14) $\frac{1}{6}\left[y_{3n+5}-61y_{3n+3}+3y_{n+3}-183y_{n+1}\right]$
15) $\frac{1}{3}\left[31y_{3n+5}-244y_{3n+4}+93y_{n+3}-732y_{n+2}\right]$

E. Each of the following expressions represents a bi-quadratic integer

1) $\left[8x_{4n+4}-x_{4n+5}+32x_{2n+2}-4x_{2n+3}+6\right]$
2) $\frac{1}{8}\left[63x_{4n+4}-x_{4n+6}+252x_{2n+2}-4x_{2n+4}+48\right]$
3) $\left[4x_{4n+4}-5y_{4n+4}+16x_{2n+2}-20y_{2n+2}+6\right]$
4) $\frac{1}{4}\left[31x_{4n+4}-5y_{4n+5}+124x_{2n+2}-20y_{2n+3}+24\right]$
5) $\frac{1}{31}\left[244x_{4n+4}-5y_{4n+6}+976x_{2n+2}-20y_{2n+4}+186\right]$
6) $\left[63x_{4n+5}-8x_{4n+6}+252x_{2n+3}-32x_{2n+4}+6\right]$
7) $\left[x_{4n+5}-10y_{4n+4}+4x_{2n+3}-40y_{2n+2}+6\right]$
8) $\left[31x_{4n+5}-40y_{4n+5}+124x_{2n+3}-160y_{2n+3}+6\right]$
9) $\left[61x_{4n+5}-10y_{4n+6}+244x_{2n+3}-40y_{2n+4}+16\right]$
10) $\frac{1}{31}\left[4x_{4n+6}-315y_{4n+4}+16x_{2n+4}-1260y_{2n+2}+186\right]$
11) $\frac{1}{4}\left[31x_{4n+6}-315y_{4n+5}+124x_{2n+4}-1260y_{2n+3}+24\right]$
12) $\left[244x_{4n+6}-315y_{4n+6}+976x_{2n+4}-1260y_{2n+4}+6\right]$
13) $\frac{1}{3}\left[4y_{4n+5}-31y_{4n+4}+16y_{2n+3}-124y_{2n+2}+18\right]$

14) $\frac{1}{6}\left[y_{4n+6} - 61y_{4n+4} + 4y_{2n+4} - 244y_{2n+2} + 36\right]$

15) $\frac{1}{3}\left[31y_{4n+6} - 244y_{4n+5} + 124y_{2n+4} - 976y_{2n+3} + 18\right]$

F. Each of the following expressions represents a quintic integer

1) $\left[8x_{5n+5} - x_{5n+6} + 40x_{3n+3} - 5x_{3n+4} + 80x_{n+1} - 10x_{n+2}\right]$

2) $\frac{1}{8}\left[63x_{5n+5} - x_{5n+7} + 315x_{3n+3} - 5x_{3n+5} + 630x_{n+1} - 10x_{n+3}\right]$

3) $\left[4x_{5n+5} - 5y_{5n+5} + 20x_{3n+3} - 25y_{3n+3} + 40x_{n+1} - 50y_{n+1}\right]$

4) $\frac{1}{4}\left[31x_{5n+5} - 5y_{5n+6} + 155x_{3n+3} - 25y_{3n+4} + 310x_{n+1} - 50y_{n+2}\right]$

5) $\frac{1}{31}\left[244x_{5n+5} - 5y_{5n+7} + 1220x_{3n+3} - 25y_{3n+5} + 2440x_{n+1} - 50y_{n+3}\right]$

6) $\left[63x_{5n+6} - 8x_{5n+7} + 315x_{3n+4} - 40x_{3n+5} + 630x_{n+2} - 80x_{n+3}\right]$

7) $\left[x_{5n+6} - 10y_{5n+5} + 5x_{3n+4} - 50y_{3n+3} + 10x_{n+2} - 100y_{n+1}\right]$

8) $\left[31x_{5n+6} - 40y_{5n+6} + 155x_{3n+4} - 200y_{3n+4} + 310x_{n+2} - 400y_{n+2}\right]$

9) $\left[61x_{5n+6} - 10y_{5n+7} + 305x_{3n+4} - 50y_{3n+5} + 610x_{n+2} - 100y_{n+3}\right]$

10) $\frac{1}{31}\left[4x_{5n+7} - 315y_{5n+5} + 20x_{3n+5} - 1575y_{3n+3} + 40x_{n+3} - 3150y_{n+1}\right]$

11) $\frac{1}{4}\left[31x_{5n+7} - 315y_{5n+6} + 155x_{3n+5} - 1575y_{3n+4} + 310x_{n+3} - 3150y_{n+2}\right]$

12) $\left[244x_{5n+7} - 315y_{5n+7} + 1220x_{3n+5} - 1575y_{3n+5} + 2440x_{n+3} - 3150y_{n+3}\right]$

13) $\frac{1}{3}\left[4y_{5n+6} - 31y_{5n+5} + 20y_{3n+4} - 155y_{3n+3} + 40y_{n+2} - 310y_{n+1}\right]$

14) $\frac{1}{6}\left[y_{5n+7} - 61y_{5n+5} + 5y_{3n+5} - 305y_{3n+3} + 10y_{n+3} - 610y_{n+1}\right]$

15) $\frac{1}{31}\left[31y_{5n+7} - 244y_{5n+6} + 155y_{3n+5} - 1220y_{3n+4} + 310y_{n+3} - 2440y_{n+2}\right]$

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III. REMARKABLE OBSERVATIONS

A. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbola which are presented in the Table: 2 below:

Table: 2 Hyperbolas

S. No	Hyperbola	(X,Y)
1	$15X^2-Y^2=60$	$(8x_{n+1}-x_{n+2},31x_{n+1}-4x_{n+2})$
2	$15X^2-16Y^2=3840$	$(63x_{n+1}-x_{n+3},61x_{n+1}-x_{n+3})$
3	$3X^2-5Y^2=12$	$(4x_{n+1}-5y_{n+1},3x_{n+1}-4y_{n+1})$
4	$X^2-240Y^2=576$	$(93x_{n+1}-15y_{n+2},6x_{n+1}-y_{n+2})$
5	$3X^2-5Y^2=11532$	$(244x_{n+1}-5y_{n+3},189x_{n+1}-4y_{n+3})$
6	$5X^2-3Y^2=180$	$(189x_{n+2}-24x_{n+3},244x_{n+2}-31x_{n+3})$
7	$48X^2-5Y^2=192$	$(x_{n+2}-10y_{n+1},3x_{n+2}-31y_{n+1})$
8	$3X^2-5Y^2=12$	$(31x_{n+2}-40y_{n+2},24x_{n+2}-31y_{n+2})$
9	$48X^2-5Y^2=192$	$(61x_{n+2}-10y_{n+3},189x_{n+2}-31y_{n+3})$
10	$3X^2-5Y^2=11532$	$(4x_{n+3}-315y_{n+1},3x_{n+3}-244y_{n+1})$
11	$3X^2-5Y^2=192$	$(31x_{n+3}-315y_{n+2},24x_{n+3}-244y_{n+2})$
12	$3X^2-5Y^2=12$	$(244x_{n+3}-315y_{n+3},189x_{n+3}-244y_{n+3})$
13	$X^2-15Y^2=36$	$(4y_{n+2}-31y_{n+1},8y_{n+1}-y_{n+2})$
14	$16X^2-15Y^2=2304$	$(y_{n+3}-61y_{n+1},63y_{n+1}-y_{n+3})$
15	$X^2-15Y^2=36$	$(31y_{n+3}-244y_{n+2},63y_{n+2}-8y_{n+3})$

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B. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of parabola which are presented in the Table: 3 below:

Table: 3 Parabolas

S. No	Parabola	(X,Y)
1	$15X - Y^2 = 30$	$(8x_{2n+2} - x_{2n+3}, 31x_{n+1} - 4x_{n+2})$
2	$15X - 2Y^2 = 240$	$(63x_{2n+2} - x_{2n+4}, 61x_{n+1} - x_{n+3})$
3	$3X - 5Y^2 = 6$	$(4x_{2n+2} - 5y_{2n+2}, 3x_{n+1} - 4y_{n+1})$
4	$X - 20Y^2 = 24$	$(93x_{2n+2} - 15y_{2n+3}, 6x_{n+1} - y_{n+2})$
5	$93X - 5Y^2 = 5766$	$(244x_{2n+2} - 5y_{2n+4}, 189x_{n+1} - 4y_{n+3})$
6	$5X - Y^2 = 30$	$(189x_{2n+3} - 24x_{2n+4}, 244x_{n+2} - 31x_{n+3})$
7	$48X - 5Y^2 = 96$	$(x_{2n+3} - 10y_{2n+2}, 3x_{n+2} - 31y_{n+1})$
8	$3X - 5Y^2 = 6$	$(31x_{2n+3} - 40y_{2n+3}, 24x_{n+2} - 31y_{n+2})$
9	$48X - 5Y^2 = 96$	$(61x_{2n+3} - 10y_{2n+4}, 189x_{n+2} - 31y_{n+3})$
10	$93X - 5Y^2 = 5766$	$(4x_{2n+4} - 315y_{2n+2}, 3x_{n+3} - 244y_{n+1})$
11	$12X - 5Y^2 = 96$	$(31x_{2n+4} - 315y_{2n+3}, 24x_{n+3} - 244y_{n+2})$
12	$3X - 5Y^2 = 6$	$(244x_{2n+4} - 315y_{2n+4}, 189x_{n+3} - 244y_{n+3})$
13	$X - 5Y^2 = 6$	$(4y_{2n+3} - 31y_{2n+2}, 8y_{n+1} - y_{n+2})$
14	$32X - 5Y^2 = 384$	$(y_{2n+4} - 61y_{2n+2}, 63y_{n+1} - y_{n+3})$
15	$X - 5Y^2 = 6$	$(31y_{2n+4} - 244y_{2n+3}, 63y_{n+2} - 8y_{n+3})$

IV. CONCLUSION

In this paper, we have presented infinitely many integer solutions for the Diophantine equation, represented by hyperbola is given by $3x^2 - 5y^2 = 12$. As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of equations and determine their integer solutions along with suitable properties.

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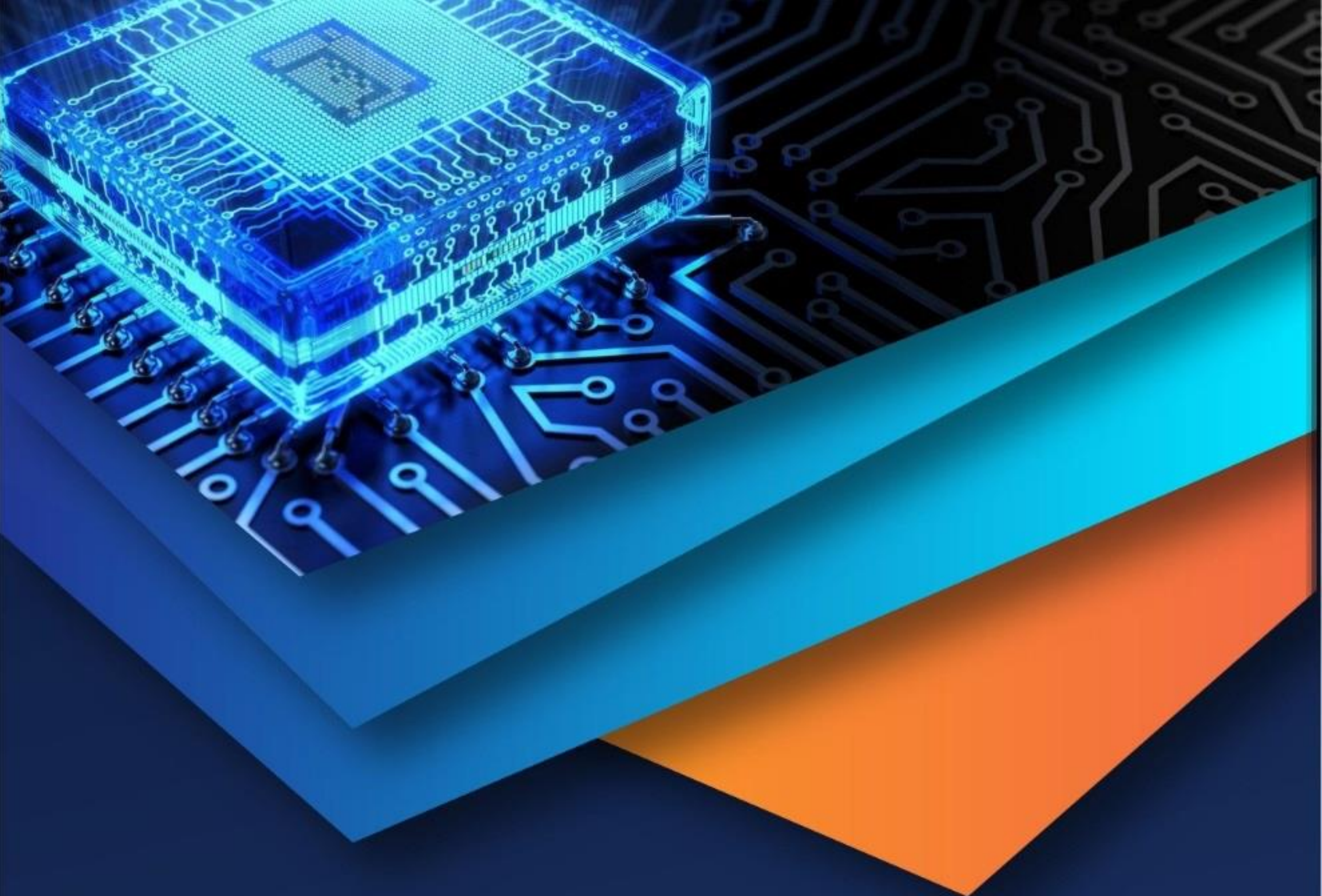

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