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Estimation Algorithm to Define Paris Law Constants using Extended Kalman Filter (EKF) for Crack Length Evolution in Aircraft Fuselage Panels

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Abstract: Metal fatigue in aircraft fuselage structure is one of the leading causes in structural defects leading to accidents that include stall, mid-air disintegration and crashes. Several theories and laws have been devised to study the crack propagation properly, the most accurate of which is the Paris Law defined by Paris and Erdogan (1963). But the uncertainties present in crack growth model and measured crack size have to be determined systematically to ensure accurate predictions. This paper aims to estimate the two Paris Law constants m and C using an estimation algorithm based on the Extended Kalman Filter (EKF). Previous numerical experiments indicate the EKF indicated proper crack length and defined the unknown parameters accurately.
Keywords: Extended Kalman Filter (EKF), crack length evolution

I. INTRODUCTION

Several crack propagation theories have been studied beforehand. In addition, recently done research by Apte et al. [1] and Wang et al. [2] have been able to determine flaw size in fuselage panels using the Extended Kalman Filters (EKF) and the Unscented Kalman Filter respectively. However, the theory [1] has not been verified on actual data and subsequently proved. This paper aims to prove it by generating the Jacobian matrix of the augmented system matrix involving the variables of Paris Law.

Estimation in all non-linear systems is extremely important. The constants in the Paris Law are also non-linear [3]. In order to detect faults within the system and predict damage in the aerospace structure before a disaster, there is an urgent need to accurately estimate the state of these systems. The optimal (Bayesian) solution to the problem requires the propagation of the description of the full probability density function (PDF) [4]. This solution is extremely general and incorporates aspects such as asymmetries and discontinuities. However, because the form of the PDF is not restricted, it cannot, in general, be described using a finite number of parameters. Therefore, any practical estimator must use an approximation of some kind. The Extended Kalman Filter aims to linearise all the non-linear models so that the Kalman Filter can be applied.

II. PARIS LAW

It was proved previously that speed of crack propagation was dependent on time. But it could not be understood as how. It was Paris et al.[7][8] who suggested using the stress intensity factor range, there are three fracture regimes with respect to variation of crack growth rate per loading cycle due to fatigue. Paris Law analyses the fatigue crack growth mechanism when $10^{-8} \le (da/dN) \le 10^{-5}$ m/cycle. For a given load ration $R = \sigma_{min}/\sigma_{max}$, there exists a linear relationship between log (da/dN) and log ΔK_t where ΔK_t is the range of the stress intensity factor.

$$\begin{aligned} \mathrm{Log}_{10}(da/dN) &= \log_{10}A + m\log_{10}(\Delta K_t) \\ &= \log_{10}A + \log_{10}\{(\Delta K_t)^m\} \\ &= \log_{10}\{A(\Delta K_t)^m\} \end{aligned}$$

Removing log, we obtain:

$$\frac{da}{dN} = A(\Delta K_t^m)$$

Here A and m are constants that depend on the material, environment and stress ratio.

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The Paris Law was different from S-N curve as it took considerably different components than the latter. This caused slower damage propagation in uncracked surfaces which was expected. The Paris Law's long crack propagation theory further implied that the dependence on the initial size of the crack was different from that calculated by crack propagation threshold and toughness value. The law often gives fairly accurate results and which may be termed as 'beautiful'. But when it does not work, it gives a serious limitation of having stress intensity factor on the x-axis. If the factor range cannot correctly predict the crack, we are left with a multivariate function with sparse data points making it more difficult for analysis.

Although there has been significant development and modifications regarding Paris Law, most design processes use the empirical approaches of the Pre-Paris Law era for high cycle fatigue. Since cracks develop slowly and are undetectable until the final stages, it is very difficult to process a damage-tolerant approach. Crack propagation starts from the initiation phase, continuing with the propagation phase (where the Paris Law is supposed to be applied) and then fast crack propagation that leads to failure of the structure.

Scientists have therefore modified the Paris Law and published the variations. These variations include only a single factor removal or departure from the ideal conditions. Modifications worth mentioning in this paper include crack closure [9] and short cracks [10][11][12][13]. Short cracks; however don't have a single type of deviation and thus some authors have suggested a classification of short cracks:

1) Microscopic short crack where micro structural fracture mechanics is applied as in case of Hobsen et al [14] and Navarro and de los Rios [15].
2) Physically small crack for which Elastic-Plastic Fracture Mechanics (EPFM) is needed where a general relation between *da/dN* and crack tip decohesion is established under high strain fatigue.
3) Macroscopic long crack described by Linear Elastic Fracture Mechanics (LEFM)

The Paris Law as modified by Paris and Erdogan in 1963 gives the advancement of fatigue crack per unit cycle as a function of the stress intensity factor [7]:

$$v^a = \frac{da}{dN} = C\Delta K^m, \Delta K_{th} < \Delta K < K_{tc}$$

Where K_{th} = fatigue threshold and K_{tc} = fracture toughness with C and a being constants

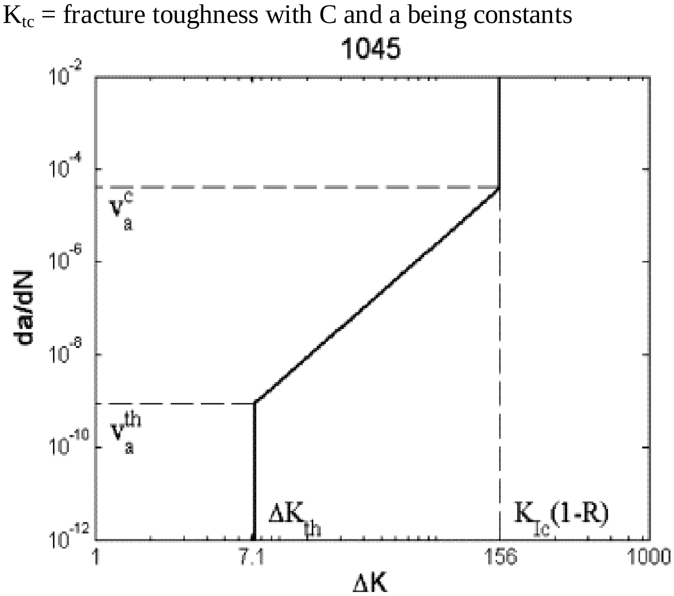

Figure 1- Paris Law for 1045 Steel

As observed from the graph in the figure given above, the law seems to be mostly valid in the range of 10^{-8} – 10^{-6} mm/cycle approximately. ΔK_{th} is at $v_a^{th}=10^{-9}$ mm/cycle where v_a^{th} is conventional velocity at the threshold. K_{tc} is at $v_a^c=10^{-5}$ mm/cycle where v_a^c is defined as the velocity at the critical conditions. This means that the constant C is not arbitrary as previously thought of.

Observing the linearity of the graph between certain conditions, Fleck et al.[16] devised a formula to calculate m:

$$mLog\ F_k = \log\frac{v_a^c}{v_a^{th}}$$

III. EXTENDED KALMAN FILTER ALGORITHM

We will start the estimation using the extended Kalman filter algorithm [5]. We will be using the following additional variable subscripts and superscripts in our algorithm.

Variable	Significance
^	Estimate
k	Time step
+	Prior estimate
-	Posterior estimate

For example, $\hat{x}^{+}_{aug,k}$ denotes the posterior estimate of augmented state vector at time step k.

A. Initialisation

The process covariance matrix Q_{aug}, measurement noise covariance matrix R_{aug}, estimated initial state value $\hat{x}_{aug,0}$ and initial state error covariance matrix P_0 should be initialized. Q_{aug} is a 3-by-3 matrix with only Q_k as the sole non-zero element. R_{aug} is a 3-by-3 diagonal matrix. R_a, R_m , R_C are taken as percentages of the orders of magnitude of m and C. The estimated initial state value $\hat{x}_{aug,0}$ is a 3-by-1 vector containing $\hat{a}_{aug,0}$, $\hat{m}_{aug,0}$ and $\hat{C}_{aug,0}$ which are the initial estimate for the crack length and other parameters generated from a normal distribution. P_0 or the confidence in the initial estimate for state is taken small in this paper.

B. Algorithm Definition

$$\hat{x}^{-}_{aug,k} = f_{aug}(\hat{x}^{+}_{aug,k-1})$$

Expressing this in matrix form,

$$\begin{bmatrix} \hat{a}^{-}_{k} \\ \hat{m}^{-}_{k} \\ \hat{C}^{-}_{k} \end{bmatrix} = \begin{bmatrix} f(\hat{a}^{+}_{k-1}) \\ \hat{m}^{+}_{k-1} \\ \hat{C}^{+}_{k-1} \end{bmatrix}$$

The error covariance is given by:

$$P^{-}_{k} = \varphi_{k-1} P^{+}_{k-1} \varphi^{T}_{k-1} + Q_{aug,k-1}$$

The function Φ represents the Jacobian matrix of the augmented system matrix given by:

$$\varphi_{k-1} = \frac{\partial f_{aug}(\hat{x}^{-}_{aug,k-1})}{\partial x_{aug}}$$

$$= \begin{bmatrix} 1 + C\frac{m}{2}\pi^{\frac{m}{2}}(\frac{pr}{t})^{m} a^{\frac{m}{2}-1} & C(\frac{pr}{t}\sqrt{\pi a})^{m}\ln(\frac{pr}{t}\sqrt{\pi a}) & (\frac{pr}{t}\sqrt{\pi a})^{m} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{a=\hat{a}^{-}_{k-1}, m=\hat{m}^{-}_{k-1}, c=\hat{C}^{-}_{k-1}}$$

The Kalman gain K_k is computed as a 3-by-3 matrix given as:

$$K_k = P^{-}_{k} H_k [H_k P^{-}_{k} H^{T}_{k} + R_{aug,k}]^{-1}$$

Where H_k is the Jacobian matrix of augmented measurement matrix given as:

$$H_k = \frac{\partial h_{aug}(\hat{x}^{-}_{aug,k})}{\partial x_{aug}}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The estimated measurement can be calculated by:

$$\hat{z}_{aug,k} = h_{aug}(\hat{x}^{-}_{aug,k})$$

The posterior state estimate is expressed in matrix form:

$$\hat{x}^{+}_{aug,k} = \hat{x}^{-}_{aug,k} + K_k(z_{aug,k} - \hat{z}_{aug,k})$$

$$\begin{bmatrix} \hat{a}_k^+ \\ \hat{m}_k^+ \\ \hat{c}_k^+ \end{bmatrix} = \begin{bmatrix} \hat{a}_k^- \\ \hat{a}_k^- \\ \hat{a}_k^- \end{bmatrix} + K_k \left(\begin{bmatrix} z_{a,k} \\ z_{m,k} \\ z_{c,k} \end{bmatrix} + \begin{bmatrix} \hat{z}_{a,k} \\ \hat{z}_{m,k} \\ \hat{z}_{c,k} \end{bmatrix} \right)$$

The error covariance matrix is given as:

$$P_k^+ = [I - K_k H_k] P_k^-$$

IV. METHODOLOGY FOR ALGORITHM

Pseudo-Code for Algorithm

1) Initialize $\hat{x}_{aug,0}, P_0$

For each time step: k=1, 2, 3...

2) Compute system equation Jacobian matrix

$$\varphi_{k-1} = \frac{\partial f_{aug}(\hat{x}_{aug,k-1}^-)}{\partial x_{aug}}$$

3) Perform the extrapolation process of the state and error covariance as follows:

$$\hat{x}_{aug,k}^- = f_{aug}(\hat{x}_{aug,k-1}^+)$$

$$P_k^- = \varphi_{k-1} P_{k-1}^+ \varphi_{k-1}^T + Q_{aug,k-1}$$

4) Compute the measurement equation Jacobian matrix

$$H_k = \frac{\partial h_{aug}(\hat{x}_{aug,k}^-)}{\partial x_{aug}}$$

5) Calculate the Kalman Gain

$$K_k = P_k^- H_k [H_k P_k^- H_k^T + R_{aug,k}]^{-1}$$

6) Perform measurement update of state and covariance as follows:

$$\hat{x}_{aug,k}^+ = \hat{x}_{aug,k}^- + K_k(z_{aug,k} - \hat{z}_{aug,k})$$

$$P_k^+ = [I - K_k H_k] P_k^-$$

V. PROGRAMMING FOR ALGORITHM

```
Function [X] = flawsize( K,T,C,t,f)
%FLAWSIZE Summary of this function goes here
%   Detailed explanation goes here
%   We will be investigating and predicting the flaw size of steel plates in an aircraft with a given lifespan
%   Here K is fracture toughness
%   T is the tensile load while C is compressive load
%   t is the time limit of the aircraft(in years)
%   f is the frequency at which stress is applied per minutes
%   Initial test values of  K=80Mpa,T=500Mpa, C=60Mpa, f=1/5 cycles/minute and
%   t=10years
%   Using Paris Law
A = ((K/(1*T))^2)/pi ;
%   Where Y is a geometry constant normally taken as 1
%   Calculating the number of cycles per year (N)
N = f*60*24*365*t;
%   Applying modified equation of Paris Law
a = (exp((log((((N*(-1.2)*1.62*(10^-12)*(T^3.2)*(pi^1.6))/2)- (A^(-0.6)))*(-1)))/(-0.6)))*(10^3);
%   Where n and C are constant with value n=3.2 and C= 1.62*(10^-12)
X= ['The initial flaw size in mm will thus be ',num2str(a)];
   disp(X);
   end
```

VI. RESULTS AND DISCUSSION

Extended Kalman Filter technique has been applied to determine the constants *m* and *C* and true crack size *a* based on noisy measurements and analytical data of crack size in an aircraft fuselage panel.

Table: 1 – Calculated values

No. Of cycles	Parameter	$\hat{\bar{\theta}}_k$	$\theta_{error}(\%)$	$M\hat{S}E$	MSE
100 cycles	m	3.80627	1.70E-01	1.2206E-02	1.2246E-02
	C	1.494287E-11	3.80E-01	2.1554E-23	2.1887E-23
1000 cycles	m	3.793298	1.80E-01	1.7620E-03	1.8080E-03
	C	1.496148E-11	2.60E-01	3.2441E-24	3.3952E-24
10000 cycles	m	3.801161	3.10E-02	2.3331E-04	2.3469E-04
	C	1.499979E-11	1.10E-03	2.9494E-25	2.9494E-25
30000 cycles	m	3.799159	2.20E-02	5.3543E-05	5.4260E-05
	C	1.499838E-11	1.1E-02	1.1830E-25	1.1857E-25
60000 cycles	m	3.79968	8.40E-03	2.5125E-05	2.5227E-05
	C	1.500024E-11	1.80E-3	5.2145E-26	5.2152E-26

The total number of cycles has been set to 60000 cycles, which is the typical lifetime of short-range commercial aircrafts like an Airbus A380 or a Boeing 747. The measurement noise covariance matrix consists of the crack size and the material properties. 50 repetitions of the EKF estimation process have been carried out.

Here, $\hat{\bar{\theta}}_k$ means average value of 50 samples of the *k* th flight cycle where n_s is the number of repetitions of the solution, 50.θ_{error} is the absolute relative error on $\hat{\bar{\theta}}_k$.

$$\hat{C}_k = \frac{1}{n_s}\sum_{j=1}^{n_s} \hat{\theta}_{k,j}$$

$$\theta_{error} = \frac{|\hat{\bar{\theta}}_k - \theta|}{\theta} \times 100\%$$

The estimated variance of the flight cycle for the true values, MSE_k is determined by the equation below. $M\hat{S}E_k$ is the estimated variance in absence of the true values.

$$MSE_k = \frac{1}{n_s - 1}\sum_{j=1}^{n_s} (\hat{\theta}_{k,j} - \theta)^2$$

$$M\hat{S}E_k = \frac{1}{n_s - 1}\sum_{j=1}^{n_s} (\hat{\theta}_{k,j} - \theta)^2$$

Despite the similarity in the above equations, *MSE* is used to indicate the distance range the collections of estimates are far from the true values while $M\hat{S}E$ is used to indicate the distance range of the collection of estimates from the averages of the estimate values. These factors are used to determine the convergence process of both the constants.

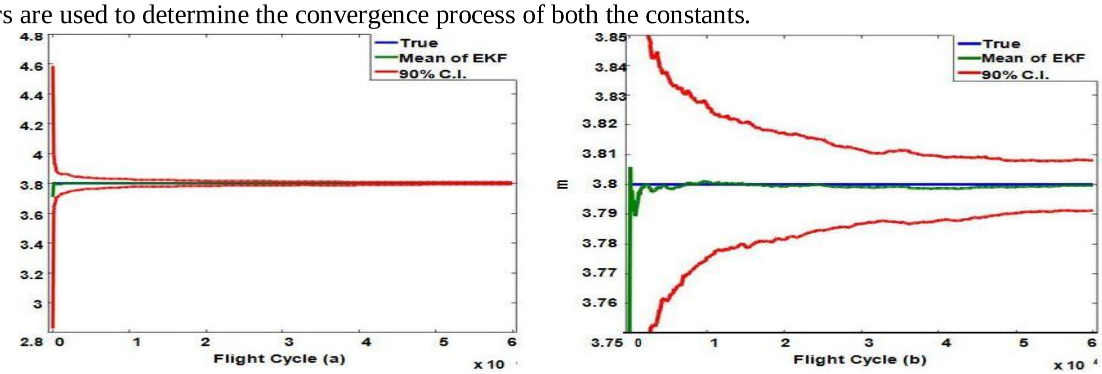

Figure 2- Convergence process of m by EKF

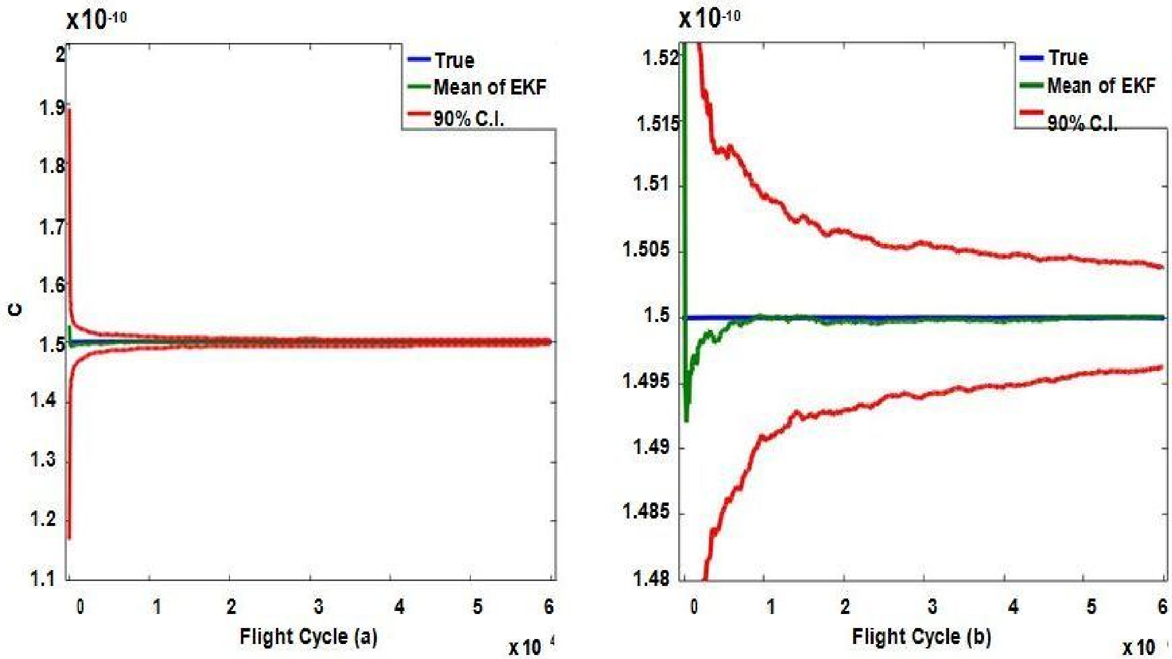

Figure 3- Convergence process of C by EKF

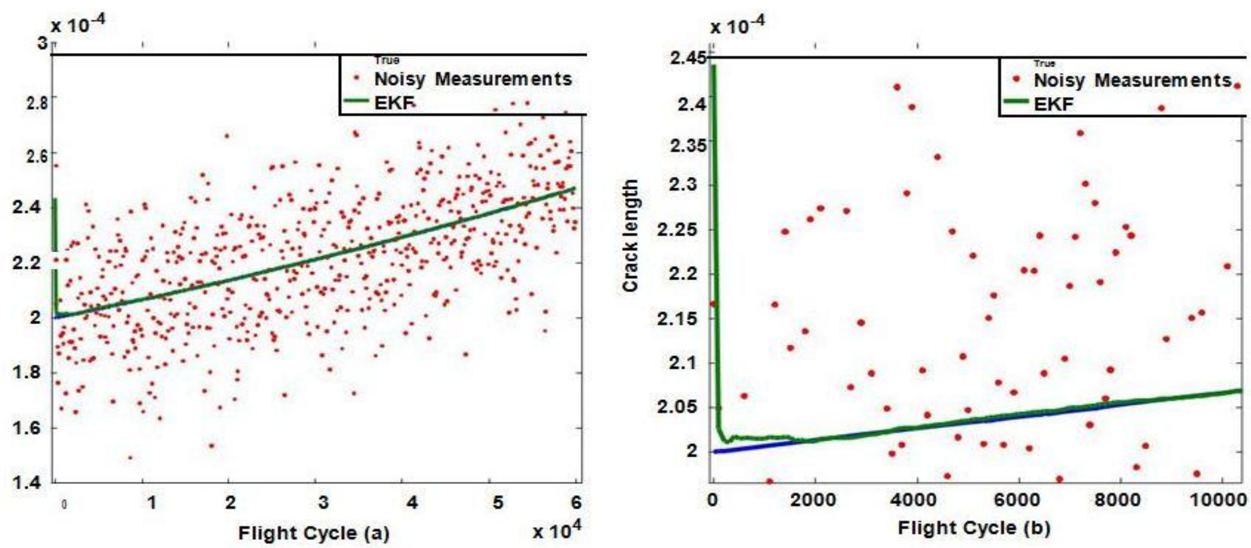

Figure 4- Crack length evolution by EKF based on one simulation, measurement points are plotted every 100 cycles

It can be observed that both *m* and *C* have been accurately identified and they rapidly converge to the true value. Although we have considered second order Taylor expression, the values are quite similar to the values defined by a first order expression with the crack length evolution identical. This proves that higher differential terms do not significantly affect the crack length evolution.

The initial values of *m* and *C* are uniformly distributed with a range of approximately 50% around the true values while the error of the EKF estimate for *m* and *C* has decreased to 0.17% after 100 cycles and 0.38% respectively. These errors fluctuate to a small extent but remain very low through the remainder of the cycles (up to 60,000 cycles): less than 0.32% for m and less than 0.53% for C. Fig 4 illustrates the crack length evaluation estimated by EKF based on one simulation out of the 50 repetitions. Each point represents a noisy measurement of the crack length used in the EKF algorithm. In order not to overload the figure, only one point in 100 cycles is represented. It can be seen that the estimated crack length (dashed line) fits very well the true one (solid line). Thus, we can state that with respect to actual measurement noise, our algorithm is relatively robust.

VII. CONCLUSION

In this paper, estimation for Paris' law constants and crack length evolution has been formalized as a nonlinear filtering problem. Extended Kalman filter has been applied to determine the Paris' law constants and the fatigue crack length behaviour using the second order Taylor expression in the original Paris's law equation. The yielded results indicate that this method identifies Paris' law constants and estimate the crack length fairly well. The error of the estimated EKF values for *m* and *C* are 0.17% and 0.38% after 100 cycles respectively. The errors seldom fluctuate in a normal aircraft cycle of 60,000 cycles. Accordingly, we conclude that EKF is an accurate and efficient state/parameter estimation approach for fatigue crack propagation problem. Future work may include investigating the truncated terms in a more proper basis to determine their effects on the crack length evolution.

REFERENCES

[1] Dhruv Apte, Mohit Dhoriya, Prof. Nitinchandra R Patel, , , , , ."*Theoretical Approach to Determine Flaw Size in Aircraft Fuselage Panels using Extended Kalman Filters"*, Volume 7, III, International Journal for Research in Applied Science and Engineering Technology (IJRASET)
[2] Y.Wang,et al., Determination of Paris' law constants and crack length evolution via Extended and Unscented Kalman filter : An application to aircraft fuselage panels, Mech.Syst.SignalProcess.(2016), http://dx.doi.org/ 10.1016/j.ymssp.2016.04.027i
[3] P. Paris and F. Erdogan, "A Critical Analysis of Crack Propagation Laws", *J. Basic Eng* 85(4),528-533, December,1963. doi:10.1115/1.3656900
[4] H. J. Kushner, "Dynamical equations for optimum nonlinear filtering," J. Different. Equat., vol. 3, pp. 179–190, 1967.
[5] G. Welch, G. Bishop, An Introduction to the Kalman Filter, 1995.

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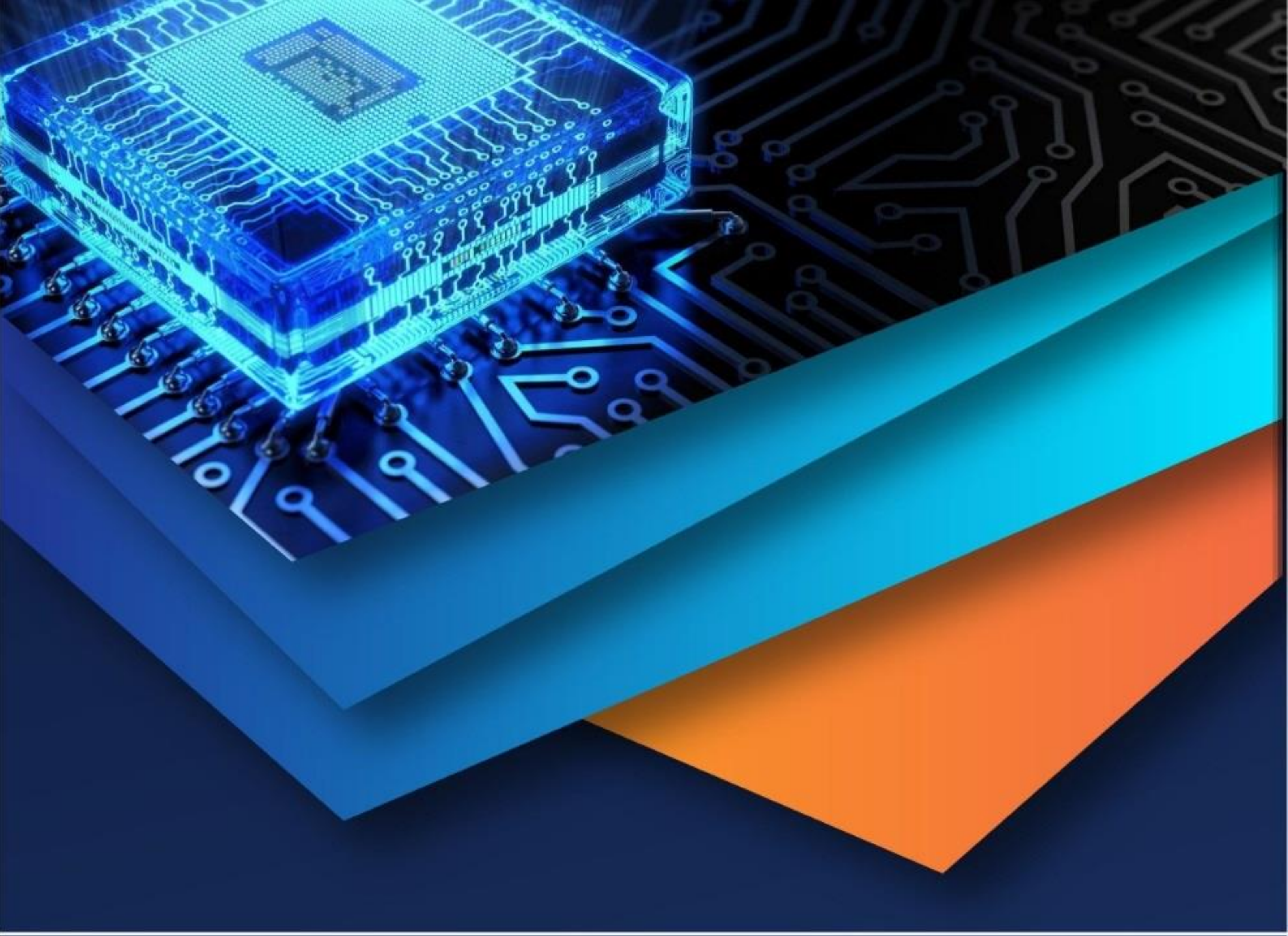

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