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On Q* - Homeomorphisms in Topological Spaces

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Abstract: In this paper , we first introduce a new class of closed map called Q* - closed map . Moreover, we introduce a new class of homeomorphism called Q* - homeomorphism, which are weaker than homeomorphism. We also introduce $Q^\star$ - homeomorphisms and prove that the set of all $Q^\star$-homeomorphisms forms a group under the operation of composition of maps. We prove that Q* - homeomorphism and $Q^\star$- homeomorphism are independent.
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I. INTRODUCTION

The notion homeomorphism plays a very important role in topology. By definition, a homeomorphism between two topological spaces X and Y is a bijective map f : X → Y when both f and $f^{-1}$are continuous. It is well known that as Janich [[8], p.13] says correctly: homeomorphisms play the same role in topology that linear isomorphism's play in linear algebra, or that biholomorphic maps play in function theory, or group isomorphism's in group theory, or isometrics in Riemannian geometry . In 1995, Maki, Devi and Balachandran [4] introduced the concepts of semi - generalized homeomorphisms and generalized semi-homeomorphisms and studied some semi topological properties. Devi and Balachandran [3] introduced a generalization of α-homeomorphism in 2001. Recently, Devi, Vigneshwaran, Vadivel and Vairamanickam [5,23] introduced g*αc-homeomorphisms and rgα- homeomorphisms and obtained some topological properties. In this paper , I first introduce a new class of closed map called Q* - closed map . Moreover, we introduce a new class of homeomorphism called Q* - homeomorphism, which are weaker than homeomorphism. We also introduce $Q^\star$ - homeomorphisms and prove that the set of all $Q^\star$-homeomorphisms forms a group under the operation of composition of maps . We prove that Q* - homeomorphism and $Q^\star$- homeomorphism are independent.

II. PRELIMINARIES

Throughout this paper (X , τ) and (Y , σ) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X , τ) , cl (A) , int (A) and A^c denote the closure of A , the interior of A and the complement of A in X , respectively. We recall the following definitions and some results, which are used in the sequel.
Definition 2.1 A set A is said to be semi open if there exists an open set G such that G ⊂ A ⊂ cl G or A⊂ cl (int (A)) . The set A is semi closed if int(cl (A)) .
Definition 2.2 The function f : X → Y is said to be irresolute if f^{-1} (A) is a semi open set whenever A is a semi open set in Y .
Definition 2.3 A subset A of a space (X, τ) is called pre-closed if cl(int(A)) ⊂A.
Definition 2.4 A space X is said to Q*- regular if for every Q* - closed set F and a point x ∉F, there exist disjoint Q* - open sets U and V such that x ∈U and F ⊆V.
Definition 2.5[15] A subset A of a topological space (X , τ) is called a Q^* - closed if int (A) = ϕ and A is closed

III. Q* HOMEOMORPHISMS

Q* - closed sets in topological spaces were introduced by M. Murugalingam and N. Laliltha in 2010 .In this section we study the properties of Q* - homeomorphisms, Q* compact spaces are discussed.

Definition 3.1A function f : (X , τ) → (Y , σ) is *pre Q* - closed* if f (U) is Q* closed in X for every Q* closed set in Y .
Definition 3.2 A bijection f : (X , τ) → (Y , σ) is a *Q* - homeomorphisms* if f is Q* irresolute and pre Q* - open .
Example 3.1 Let X = Y = { a , b , c } , τ = { ϕ , X , { b } ,{ b , c }} and σ = { ϕ , Y , { b , c } }. Let f : X → Y be the identity map . Hence f is Q* - homeomorphism .
Proposition 3.1 Every Q* - closed set is α-closed and hence pre-closed.

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Definition 3.3 A space X is said to be *Q* - compact* if every cover of X by Q* - open sets has a finite sub cover .

Definition 3.4 A map f : X → Y is called *Q^* - closed map* if the image of each Q^* - closed in(X , τ) is closed in (Y , σ) .
Example 3.2 (1) Let X = Y = { a , b , c } , τ = { ϕ , X , { b } } } and σ = { ϕ , Y , { b } , { b , c } }. Then ϕ , { a , c } are Q^* - closed in X . Let f : X → Y be the identity map . Then f (ϕ) = ϕ , f ({ a , c }) = { a , c } . Since ϕ , { a , c } are closed in Y . Therefore , f is - Q^* closed map .
Let X = Y = { a , b , c }, τ = { ϕ , X , { a , b } } and σ = { ϕ , Y , { a } , { b } } . Let f : (X , τ) → (Y , σ) be the identity map. Then f is not a Q* - closed map .

The next theorem shows that normality is preserved under continuous
Definition 3.5 Let x be a point of (X , τ) and V be a subset of X . Then V is called a *Q* -neighborhood* of x in (X , τ) if there exists a Q* - open set U of (X, τ) such that x ∈U ⊂V .
Theorem3.1 If f : (X , τ) → (Y, σ) is a Q* - continuous , Q* - closed map from a Q* -normal space (X , τ) onto a space (Y , σ) then (Y , σ) is Q* - normal.
Proof. Since every Q* -closed map is Q* -closed.
Analogous to a Q* -closed map, we define a Q* -open map as follows:
Definition 3.6 A map f : (X, τ) → (Y, σ) is said to a *Q*-open map* if theimage f (A) is Q* -open in (X, τ) for each open set A in (Y, σ).
Proposition 3.2 For any bijection f : (X, τ) → (Y, σ), the followingstatements are equivalent:
(i) f^{-1} : (Y, σ) → (X, τ) is Q* -continuous,
(ii) f is a Q* -open map and
(iii) f is a Q* -closed map.
Proof.
Step 1 :(a) ⇒(b)
Let V be a Q* - open set in X . Then X – V is Q* - closed in X . Since f^{-1} is Q* - continuous , $(f^{-1})^{-1}$(X – V) = f (X – V) = Y – f (V) is closed in Y. Then f (V) is open in Y. Hence f is a Q* - open map.
Step 2 : (b) ⇒(c) .
Let f be a Q* - open map . Let U be a Q* - closed set in X . Then X – U is Q* - open in X . Since f is Q* - open , f (X – U) = Y – f (U) is open in Y. Then f (U) is closed in Y. Hence f is a Q* - closed .
Step 3 :(c) ⇒(a) .
Let V be Q* - closed set in X . Since f : X→Y is Q* - closed , f (V) is closed in Y. That is $(f^{-1})^{-1}$(V) is closed in Y . Hence f^{-1} is Q* - continuous. ■
Proposition 3.3 Let f: X→Y be a bijective and Q* - continuous map. Then the following statements are equivalent.
(a) f is a Q* - open map.
(b) f is a Q* - homeomorphism.
(c) f is a Q* - closed map.
Proof .
Step 1 :(a) ⇒(b) .
Given f is bijective , Q* - continuous map and Q* - open map . Hence f is Q* - homeomorphism.
Step 2 :(b) ⇒(c).
Suppose (b) holds. Let F be a closed set in (X , τ) . By (b) , f^{-1} is Q* - continuous and so $(f^{-1})^{-1}$(F) = f (F) is Q*-closed in (Y , σ). This proves (c).
Step 3 : (c) ⇒(a)

Follows from Proposition 3.2 .

Definition 3.7 Let A be a subset of X. A mapping r : X → A is called a *Q* - continuous retraction* if r is Q* - continuous and the restriction τ_A is the identity mapping on A .
Definition 3.8 A topological space (X , τ) is called a *Q* - Hausdorff* if for each pair x , y of distinct points of X , there exists Q* - neighborhoods U_1 and U_2 of x and y, respectively, that are disjoint .
Lemma 3.1 Let A be a subset of X. Then p ∈Q* - cl (A) if and only if for any Q* - neighborhood N of p in X , A ∩ N ≠ϕ
Theorem 3.1 Let A be a subset of X and r : X → A be a Q* - continuous retraction . If X is Q* -Hausdorff , then A is a Q* - closed set of X.

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Proof. Suppose that A is not Q* - closed . Then there exists a point x in X such that $x \in$ Q* -cl(A) but $x \notin A$. It follows that $r(x) \neq x$ becauser is Q* -continuous retraction. Since X is Q* -Hausdorff, there exists disjoint Q* -open sets U and V in X such that $x \in U$ and $r(x) \in V$. Now let W bean arbitrary Q* -neighborhood of x. Then $W \cap U$ is a Q* -neighborhoodof x.

Since $x \in$ Q* -cl(A), by Lemma 3.1, we have $(W \cap U) \cap A \neq \phi$. Thereforethere exists a point y in $W \cap U \cap A$. Since $y \in A$, we have $r(y) = y \in U$and hence $r(y) \notin V$. This implies that $r(W) \not\subset V$ because $y \in W$. This iscontrary to the Q* -continuity of r. Consequently, A is a Q* -closed set of X.■

Theorem 3.1 Let $f : X \to Y$ be a bijection. If f is pre Q* - open then $f^{-1} : X \to Y$ is irresolute.

Proof. Suppose that the bijection f is pre Q* - open then f^{-1} is a function from Y to X .Let U be a Q* - open set in X .Since every Q* - open set is semi open we have U is semi open in X .Then f (U) is a semi open set in Y .But f (U) = $(f^{-1})^{-1}$. Hence $(f^{-1})^{-1}$ (U) is a semi open set in Y .Put $g = f^{-1}$ .We have g^{-1}(U) is semi open. Consequently , g is irresolute .That is ,f^{-1} is irresolute. ■

Theorem 3.2 Let $f : X \to Y$ be a bijection. Then f is pre Q* - open if and only if $f^{-1} : Y \to X$ is Q* - irresolute.

Proof.

Step 1 : Suppose that the bijection f is pre Q* - open then $f^{-1}$is a function from Y to X .Let U be a Q* - open set in X .Then f (U) is a Q* - open set in Y .But $f(U) = (f^{-1})^{-1}(U)$. Hence $(f^{-1})^{-1}(U)$ is a Q* - open set in Y .Put $g = f^{-1}$.We have g^{-1}(U) is Q* open.Consequently , g is Q* - irresolute .That is ,$f^{-1}$is Q* - irresolute.

Step 2 : Suppose that f is Q* - irresolute .Let U be a Q* - open set in X .Then$(f^{-1})^{-1}$(U) is a Q* - open set in Y.But $(f^{-1})^{-1}$(U) = f (U) . Therefore f(U) is a Q* - open set in Y.Hence f is pre Q* open. ■

Theorem 3.3 If $f : X \to Y$andg : $Y \to Z$ are both Q* - irresolute, than their composition $g \circ f : X \to Z$ is an Q* - irresolute map.

Proof. Let V be Q* - open set in Z .Then$(g \circ f)^{-1}(V) = (f^{-1} \circ g^{-1}(V)) = (f^{-1}(g^{-1}(V)))$.Since g is Q* - irresolute , it follows that $g^{-1}(V)$ is a Q* - open set .Since f is Q* - irresolute , it follows that $(f^{-1}(g^{-1}(V))$ is a Q* - open set .Thus for each Q* - open set V in Z , $(g \circ f)^{-1}(V)$ is Q* - open in X .Therefore , $g \circ f$ is an Q* - irresolute function. ■

Theorem 3.4 Q* -homeomorphism is an equivalence relation. We write X ~ Y whenever two spaces X,Y are Q* - homeomorphic .

Proof .

Step 1 : Let $i : X \to X$ be the identity map on X .Then it is bijective and Q* - irresolute . Also $(i)^{-i}$ is a pre Q* - open map .Hence i is a Q* - homeomorphism .Accordingly X ~ X .The relation is reflexive.

Step 2 : Suppose that X ~ Y. Then there exists a Q* - homeomorphism $h : X \to Y$.But then h is bijective .Accordingly h^{-1}:$Y \to X$is bijective .Also h is Q* - irresolute .Hence $h^{-1}$is a pre Q* - open map .Hence Y ~ X.

Step 3: Suppose that X ~ Y and Y ~ Z .Then there is a Q* - homeomorphism . $f : X \to Y$ and there is a Q* -homeomorphism g from Y to Z .But then f and g are bijective.Accordingly , $g \circ f$ is bijective and pre Q* - open.Thus , $g \circ f$ is Q* - homeomorphism . Therefore, X ~ Z. Hence ~ is transitive .From step (1) , (2) and (3) , Q* - homeomorphism is an equivalence relation . ■

Theorem 3.5 Every Q* - compact subset of a Hausdorff space is semiclosed.

Proof. Suppose that A be a Q* compact subset of a Hausdorff space X . Sinceevery Q* compact space is semi compact we have A is semi compact .Let $x \in X - A$ .Then there exists 2 disjoint semi open sets U_x and V_x such that $x \in U_x$ and

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$A \subset V_x$. But then $X \in U_x \subset X - V_x \subset X - A$. Therefore , X – A is semi open .Hence A is semi - closed in Y. ■

Theorem 3.6 EveryQ* - compact subset of a Q* - Hausdorff space is Q*closed.

Proof. Suppose that A be a Q* - compact subset of a Q* - Hausdorff space X .Let $x \in X - A$ .Then there are 2 disjoint Q* - open sets U_x and V_x such that $x \in U_x$ and $A \subset V_x$. But then $X \in U_x \subset X - V_x \subset X - A$. Therefore , X – A is Q* - open .Hence A is Q*- closed in Y. ■

Theorem 3.7 Let X beQ* compact and set Y be a Hausdorff space. If $f : X \to Y$ is Q* - continuous , Q* - irresolute and bijective, then f is a Q* -homeomorphism.

Proof. Let A be a Q* - closed subset of the Q* - compact space X .Then A is Q*-compact .But f is Q* - irresolute .Hence f (A) is Q* - compact .Take $g = f^{-1}$. Then $g^{-1}(A)$ is Q* -closed , by theorem 3.4 . Consequently g is an Q* - irresolute map .That is, f^{-1} is Q* - irresolute .Therefore , f is a Q* - homeomorphism . ■

Proposition 3.4 If f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, γ) are Q*- homeomorphisms, then their composition g ∘ f : (X, τ) → (Z, γ) is also Q* - homeomorphism.
Proof. Let U be a Q* - open set in (Z , γ) .Now, $(g \circ f)^{-1}(U) = f^{-1}(g^{-1} (U)) = f^{-1} (V)$, where $V = g^{-1} (U)$. By hypothesis , V is Q* - open in (Y, σ) and so again by hypothesis ,f^{-1} (V) is Q* - open in (X, τ).Therefore, g ∘ f is Q* - irresolute.Also for a Q* - open set G in (X , τ) , we have (g ∘ f) (G) = g (f (G)) = g (W) , where W = f (G) . By hypothesis f (G) is Q* - open in (Y, σ) and so again by hypothesis , g (f (G)) is Q* - open in (Z , γ) .i.e., (g ∘ f) (G) is Q*- open in (Z , γ) and therefore $(g \circ f)^{-1}$ is Q* - irresolute. Hence g ∘ f is aQ*-homeomorphism. ■

Example 3.3 Let X = Y = Z = { a , b , c } , τ = { ϕ , X , { b } , { a , b } , { b , c } } and σ = { ϕ , Y , { b } , { b , c } } , η = { ϕ , Z , { b , c } } . Then ϕ , { a } , { a , c } are Q^*- closed in Y . Let f : X → Y be the identity map . Then f and g are Q* - homeomorphism .Here g ∘ f is Q* - continuous , since { b , c } is Q* - open in Z and $(g \circ f)^{-1}$({ b , c }) = { b , c } is Q* - open in X . Hence g ∘ f is Q* homeomorphism .

Remark 3.1. We denote the family of all Q*homeomorphisms from (X , τ) onto itself by - Q* - h (X , τ) .

Theorem 3.8 The set Q* - h(X , τ) is a group under the composition of maps .
Proof. Define a binary operation * : Q* - h (X , τ) × Q* - h (X , τ) → Q* - h (X , τ) by f ∗g = g ∘ f for all f, g ∈ Q* - h (X ,τ) and ∘ is the usual operation of composition of maps . Then by Proposition 3.4, g ∘ f ∈Q*-h(X, τ).We know that the composition of maps is associative and the identity mapI : (X, τ) → (X, τ) belonging to Q*-h(X, τ) servers as the identity element.If f ∈Q*-h(X, τ), then f^{-1}∈Q*-h(X, τ) such that $f \circ f^{-1} = f^{-1} \circ f = I$ andso inverse exists for each element of Q*-h(X, τ).Therefore, (Q*-h(X, τ), ∘)is a group under the operation of composition of maps . ■

Theorem 3.9Let f : (X, τ) → (Y, σ) be a Q*-homeomorphism. Thenf induces an isomorphism from the group Q*-h(X, τ) onto the group Q*-h(Y, σ).
Proof. Using the map f, we define a map θ_f : Q*-h(X, τ) → Q*-h(Y, σ)by $\theta_f (h) = f \circ h \circ f^{-1}$ for every h ∈Q*-h(X, τ). Then θ_f is a bijection.Further, for all h_1, h_2∈Q*-h(X, τ), $\theta_f (h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = \theta_f (h_1) \circ \theta_f (h_2)$.Therefore, θ_f is a homeomorphismand so it is an isomorphism induced by f . ■

Proposition 3.4 For any two subsets A and B of (X , τ) ,
(i) If A ⊂B, then Q* - cl(A)) ⊂ Q* - cl(B)),
(ii) Q*- cl (A ∩ B)) ⊂ Q*- cl (A)) ∩ Q* - cl (B)) . ■

Theorem 3.10. If f : (X, τ) → (Y, σ) is a Q*-homeomorphism, then Q* cl(f^{-1}(B)) = f^{-1}(Q* -cl(B)) for all B ⊂Y .
Proof. Since f is a Q*-homeomorphism, f is Q* -irresolute. Since Q* -cl(f (B))is a Q* -closed set in (Y, σ), f^{-1}(Q* -cl(f(B))) is Q* -closed in (X, τ). Now,f^{-1}(B) ⊂f^{-1}(Q*-cl(B)) and so by Proposition 3.4, Q* -cl(f^{-1}(B)) ⊂f^{-1}(Q*-cl(B)).Again since f is a Q*-homeomorphism, f^{-1} is Q* -irresolute. Since Q*-cl(f^{-1}(B))is Q* -closed in (X, τ), $(f^{-1})^{-1}$(Q* -cl(f^{-1}(B))) = f (Q*-cl(f^{-1}(B))) is Q*-closedin (Y, σ). Now, B ⊂$(f^{-1})^{-1}$(f^{-1}(B))) ⊂$(f^{-1})^{-1}$(Q* -cl(f^{-1}(B))) =f (Q* -cl(f^{-1}(B))) and so

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Q*-cl(B) ⊂f (Q*-cl(f^{-1}(B))). Therefore,f^{-1}(Q*-cl(B)) ⊂f^{-1}(f (Q*-cl(f^{-1}(B)))) ⊂Q* -cl(f^{-1}(B)) and hence the equalityholds. ■

Corollary 3.1 If f : (X, τ) → (Y, σ) is a Q*-homeomorphism, then Q* - cl(f (B)) = f (Q* -cl(B)) for all B ⊂X.
Proof. Since f : (X, τ) → (Y, σ) is a Q*-homeomorphism, f^{-1} : (Y, σ) →(X, τ) is also a Q*-homeomorphism. Therefore, by theorem 3.10 , Q* -cl($(f^{-1})^{-1}$(B)) = $(f^{-1})^{-1}$(Q* -cl(B)) for all B ⊂X. i.e., Q* -cl(f (B)) = f (Q* - cl(B)). ■

Corollary 3.2 If f : (X, τ) → (Y, σ) is a Q*-homeomorphism, then f (Q* - int(B)) = Q* -int(f (B)) for all B ⊂X.
Proof. For any set B ⊂X, Q* -int(B) = (Q*-cl(B^c)$)^c$. Thus, by utilizing corollary 3.2 , we obtain f (Q* -int(B)) = f ((Q* -cl(B^c)$)^c$) = (f (Q* -cl(B^c))$)^c$ =(Q*-cl(f (B^c))$)^c$ = (Q* -cl((f (B)$)^c$)$)^c$ = Q* -int(f (B)). ■

Corollary 3.3 If f : (X, τ) → (Y, σ) is a Q*-homeomorphism, thenf^{-1}(Q*-int(B)) = Q* -int(f^{-1}(B)) for all B ⊂Y .
Proof. Since f^{-1} : (Y, σ) → (X, τ) is also a Q*-homeomorphism, theproof follows from Corollary 3.2 . ■

IV. Q^* - HOMEOMORPHISM

Throughout this paper X and Y always represent nonempty topological spaces (X , τ) and (Y , σ) . The family of all closed set in (X , τ) is denoted by C (X) .

Theorem 4.1 Every Q* homeomorphism is a homeomorphism.
Proof. Let f :(X , τ) → (Y, σ) be a Q* - homeomorphism . Then f is bijective and both f and f^{-1} are Q* - continuous . Since every Q* continuous function is continuous we have f and f^{-1} are continuous . This shows that f is a homeomorphism.■
Remark 4.1 The converse of the above theorem need not be true, as shown in the following example.
Example 4.1Let X = Y = { a , b , c } , τ = { ϕ , X , { b } ,{ a } , { b , c }} and σ = { ϕ , Y , { b } , { b , c } }. Let f : X → Y be the identity map . Therefore , f is a homomorphism but not Q* homeomorphism .

Definition 4.1 For a subset A of a space (X , τ_1 , τ_2) we define the Q* kernel of A (briefly, Q* ker (A)) as follows : Q* ker (A) = ∩ { F : F ∈Q* O (X) ; A ⊂ F } . A is said to be a Q* - Λset inX if A = Q* ker (A) , or equivalently, if A is the intersection of Q* open sets. A is said to be Q* λ - closed in X if it is the intersection of a Q* - Λ set in X and clearly, Q* - Λsets and Q* closed sets are Q* - λclosed; complements of Q* - λ closed sets in X are said to be Q* - λopen in X .
Proposition 4.1For a subset A of a space X,the following are equivalent:

(i) A is Q* - λclosed in X .
(ii) A = L ∩Q* - cl (A) , where L is aQ* - Λ set in X.
(iii) A = Q* - ker (A) ∩Q* - cl (A) .■

Definition 4.2 A bijection f : X → Y is called Q*- homeomorphism , if f isQ^* irresolute and its inverse also Q^* irresolute .

Remark 4.2 We say that spaces X and Y are Q*- homeomorphic if there exists a Q*homeomorphism from (X , τ) onto (Y, σ) .We denote the family of all Q*homeomorphisms from (X , τ) onto itself by Q*- H(X) .

Theorem 4.2Every Q*- homeomorphism is a Q^*- homeomorphism.
Proof. Let f :(X , τ) → (Y, σ) be a Q*- homeomorphism . Then f is bijective , Q^* irresolute and f^{-1} is Q^* irresolute . Since every Q^* irresolute is Q^*continuous , f and f^{-1} are Q^* continuous and so f is a Q^* homeomorphism.■
Remark 4.3 The following example shows that the converse of the above theorem need not be true.
Example 4.3Let X = Y = { a , b , c } , τ= { ϕ , X , { a , b } , { b }} and σ= { ϕ , Y ,{ b }, { c } } . Let f :X→ Y be a map defined by f (a) = f (b) = c and f (c) = b. Therefore , f is a Q* - homomorphism but not Q*- homeomorphism .

Remark 4.4 The concepts of homeomorphisms and Q* - homeomorphism are independent.

Example 4.4 Let X = Y = { a , b , c } , τ= { ϕ , X , { b } , { b , c } , { a }} and σ= { ϕ , Y , { a } , { b } } . Let f :X→ Y be a identity map . Then f is homeomorphism but not Q* homeomorphism.

Lemma 4.1A function f : X → Y is Q* - irresolute if and only if f^{-1} (V) is a Q* - closed set in X for everyQ* - closed set V in Y .
Theorem 4.3 If f : (X , τ) → (Y, σ) is a Q*- homeomorphism, then Q*- cl (f^{-1}(B)) = f^{-1} (Q*- cl(B)) for every B ⊆ Y.
Proof. Let f :(X , τ) → (Y, σ) be a Q* - homeomorphism. Then by Definition 4.2, both f and f^{-1} areQ* - irresolute and f is bijective. Let B ⊆Y. Since Q*-cl(B) is a Q*-closed set in (Y,σ) , using Lemma 4.1, f^{-1}(Q*-cl(B)) is Q*-closed in (X, τ). But

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Q*-cl(f^{-1}(B)) is the smallest Q*-closed setcontaining f^{-1}(B).

Therefore Q*-cl(f^{-1}(B)) $\subseteq f^{-1}$(Q*-cl(B)). $\rightarrow$ (1) .

Again, Q*-cl(f^{-1}(B)) is Q*-closed in (X , τ) . Since $f^{-1}$is Q*-irresolute, f (Q*-cl(f^{-1}(B))) is Q*-closed in (Y,σ). Now, B = f (f^{-1}(B)) $\subseteq$f (Q* -cl(f^{-1}(B))). Since f (Q*-cl(f^{-1}(B))) is Q*-closed and Q*-cl(B) is the smallest Q*-closed set containing B, Q*-cl(B) $\subseteq$f(Q*-cl(f^{-1}(B))) thatimplies f^{-1}(Q*-cl(B)) $\subseteq f^{-1}$(f(Q*-cl(f^{-1}(B)))) = Q*-cl(f^{-1}(B)).

That is, f^{-1}(Q*-cl(B)) Q*-cl(f^{-1}(B)) $\rightarrow$ (2)

From (1) and (2), Q*-cl(f^{-1}(B)) = f^{-1}(Q*-cl(B)).■

Corollary 4.1 If f : (X , τ) → (Y, σ) is aQ*- homeomorphism , then Q*-cl(f (B)) = f (Q* -cl(B)) for everyB $\subseteq$X.

Proof. Let f :(X , τ) → (Y, σ) be a Q*- homeomorphism . Since f is Q*- homeomorphism ,f^{-1} is also aQ*- homeomorphism . Therefore by Theorem 4.3, it follows that Q*-cl(f (B)) = f (Q*-cl(B)) forevery B $\subseteq$X.■

Corollary 4.2 If f : (X , τ) → (Y, σ) is a Q*- homeomorphism , then f (Q*- int(B)) = Q* - int(f (B)) for every B $\subseteq$ X .

Proof. Let f :(X , τ) → (Y, σ) be a Q*- homeomorphism. For any set B $\subseteq$ X , Q*- int(B) = (Q* - cl (B^c) $)^c$. f (Q*- int(B)) = f ((Q* - cl (B^c) $)^c$) = (f (Q*- cl (B^c)) $)^c$. Then using Corollary 4.2 , we see that f (Q* - int(B)) = (Q*- cl (f (B^c)) $)^c$ = Q*- int(f (B)) .■

Corollary 4.3 If f : (X , τ) → (Y, σ) is a Q*-homeomorphism, then for every B $\subseteq$Y,f^{-1}(Q*-int(B)) = Q*-int(f^{-1}(B)).

Proof. Let f :(X , τ) → (Y, σ) be a Q*-homeomorphism. Since f is Q*-homeomorphism, f^{-1} is also aQ*-homeomorphism. Therefore by Corollary 4.2 , f^{-1}(Q*-int(B)) = Q*-int(f^{-1}(
B)) for every B$\subseteq$Y.■

Theorem 4.3 If f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, γ) are Q*- homeomorphisms, then their composition g ∘ f : (X, τ) → (Z, γ) is also Q*- homeomorphism.

Proof. Let U be a Q* - open set in (Z , γ) . Since g is Q*- homeomorphism , g is Q* - irresolute and so g^{-1} (U) is Q* - open in (Y , σ) . Since f is Q*- homeomorphism , f is Q* - irresolute and so f^{-1}(g^{-1} (U)) = (g ∘ f $)^{-1}$ (U) is Q* open in (X , τ) . This implies that g ∘ f is Q* irresolute . Again let G be Q* - open in (X, τ) . Since f is Q*- homeomorphism, f^{-1} is Q* - irresolute and so (f^{-1}) $^{-1}$ (G) = f (G) is Q* - open in (Y, σ) . Since g is Q*- homeomorphism , $g^{-1}$is Q* - irresolute and so (g^{-1}) $^{-1}$ (f (G)) = g (f (G)) = (g ∘ f) (G) = ((g ∘ f $)^{-1}$ $)^{-1}$(G) is Q*- open in (Z , γ) . This implies that (g∘ f $)^{-1}$ is Q* - irresolute . Since f and g areQ*- homeomorphism f and g are bijective and so g ∘ f is bijective . This completes the proof .■

Theorem 4.4 The set Q*H (X , τ) is a group under composition of functions.

Proof. Let f, g ∈Q*H (X , τ) . Then f ∘ g∈Q*H (X , τ) by Theorem 3.17. Since f is bijective,
f^{-1}∈Q*H (X , τ) . This completes the proof.■

Theorem 4.5 If f : (X, τ) → (Y, σ) is a Q*- homeomorphism , then f induces an isomorphism from the groupQ*H (X , τ) onto the group Q*H (Y , σ) .

Proof. Let f ∈Q*H (X , τ) . Then define a map ψ_f : Q*H (X , τ) → Q*H (Y , σ) by $\psi_f(H)$= f ∘ h ∘ $f^{-1}$for every h ∈Q*H (X , τ) . Let h_1, h_2∈Q*H (X , τ) .

Then $\psi_f(h_1\circ h_2) = f \circ(h_1\circ h_2)\circ f^{-1}$

$= f \circ(h_1\circ f^{-1}\circ f \circ h_2) \circ f^{-1}$

$= (f\circ h_1\circ f^{-1}) \circ(f \circ h_2\circ f^{-1})$

$= \psi_f(h_1) \circ f (h_2)$.

Since $\psi_f(f^{-1}\circ h\circ f) = h_1\psi_f$ is onto. Now, ψ_f(h) = I implies f ∘ h ∘ f^{-1}= I. That implies h = I.This proves that $\psi_f$is one-one. This shows that ψ_f is a isomorphism.■

Theorem 4.6 Every Q* open set is Λ_r- open .

Theorem 4.7Every Q* - homeomorphism is a Λ_r-homeomorphism.

Proof. Let f : (X, τ) → (Y, σ) be a Q* - homeomorphism. Then f is bijective and both f and f^{-1} are Q* - continuous. Since every Q* - continuous function is Λ_r-continuous, f and f^{-1} are Λ_r -continuous. Thisshows that f is a Λ_r-homeomorphism.

Remark 4.5 The converse of the above theorem need not be true, as shown in the following example.

Example 4.4 Let X = Y = {a,b,c}, τ= {X,ϕ,{a},{b},{a,b},{b,c}} and σ= {Y,ϕ,{b},{c},{b,c}}.Then Λ_rO(X, τ) = τand Λ_rO(Y, σ) = {Y, ϕ,{b},{c},{b,c},{a,c},{a,b}}.Let f : (X, τ) → (Y, σ)be defined by f (a) = c, f (b) = b and f (c) = a.Then f is Λ_r-homeomorphism. Since f ({b,c}) = {a, b} is not Q* - open in (Y,σ), $f^{-1}$is not Q* - continuousthat implies f is not a Q* - homeomorphism.

V. CONCLUSION

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In this paper, we introduce two classes of maps called Q* - homeomorphisms and Q⋆-homeomorphisms and study theirproperties. From all of the above statements, we have the
following diagram:

$$\begin{tikzcd}
& \Lambda_r\text{-homeomorphism} & \Lambda_r^*\text{ - homeomorphism} \arrow[l] \\
Q^*\text{ - homeomorphism} \arrow[ur] \arrow[r] & \text{homeomorphism} \arrow[u] & Q^\star\text{-homeomorphism} \arrow[l] \arrow[u] \arrow[ll, bend left]
\end{tikzcd}$$

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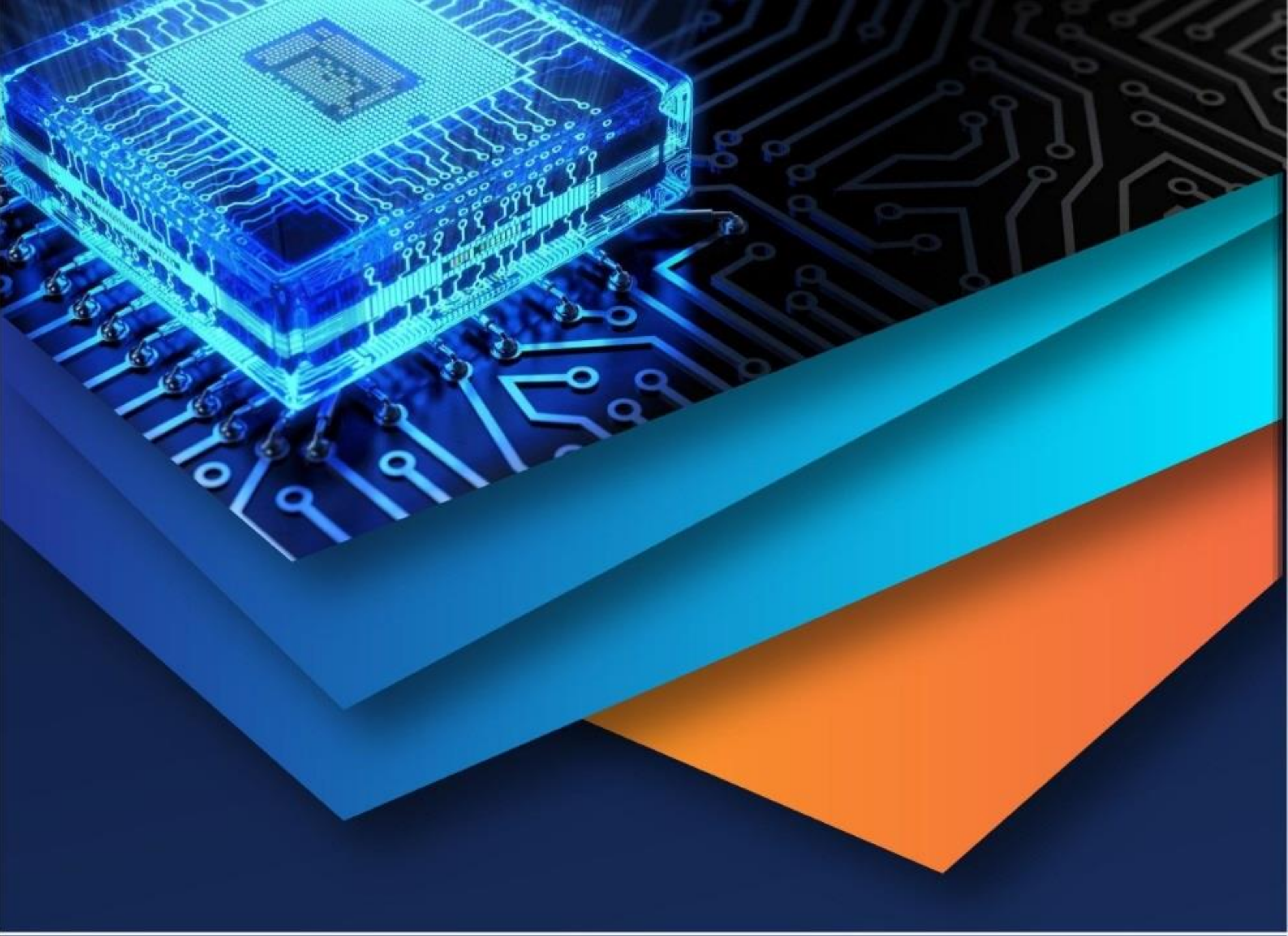

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