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Frobenius Series Method

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***Abstract:** We introduce the Frobenius series method to salve second order linear equations, and illustrate it by concrete examples.*
***keywords:** Ordinary Differential Equation, Ordinary Point, Regular Singular Point, Series Solution, Frobenius Method.*

I. INTRODUCTION

Ordinary differential equation is a differential equation that includes the derivative an ordinary differential equation, cannot be separated in the study of finding a solution of the differential equations. The Solution of differential equations order –n is a function that satisfies the differential equation. There are many ways in solving ordinary differential equations, but frequent obstacles in obtaining the solution of differential equations, especially the differential equation that has the form of a variable coefficient. Variable coefficient differential equations which have special shapes such as differential equation and Legendre can be solved by simplifying the form of the differential equation Cauchy-Euler-Lagrange differential equation and Legendre, can be solved by simplifying the form of the differential equation. However if the coefficient of variables do not have a particular form it will be difficult to obtain the differential equation solution. The connection for the completion of differential equation developed a method that can resolve the variable coefficient differential equations. That method can be used to declare the settlement in the form of power series solutions. This method is very efficient to use the variable coefficient differential equation, because the coefficients of the differential equations are not required to have a certain shape. Frobenius was very efficient method used to find the solution of differential equations with coefficient in the form of function.

Frobenius method is widely used in the search for a solution from the application of differential equation, including Bessel equation, deployment the temperature in the tube,

Laguerre equation used in quantum mechanics of the hydrogen, atom, and hyper geometric of Gauss equation.

II. REGULAR SINGULAR POINTS

A singular point $x = x_0$ is called the regular singular point of the differential equation (y” + P(x) y’ +Q(x) y =0) if both (x - x_0) P(x) and $(x - x_0)^2$ Q(x) y can be expanded is series of x - x_0 in the neighborhood of $x = x_0$ i.e. if both (x - x_0) P(x) and $(x - x_0)^2$ Q(x) are defined and so are analytic at $x = x_0$.

Example. Consider the equation

$x(x-1)^3$ y” +2 $(x-1)^3$ y’ $^+$3y = 0

Solution: Dividing by $x(x-1)^3$, the coefficient of y”, the given equation reduces to

Y” $+ \frac{2}{x} y' + \frac{3}{x(x-1)^3} y = 0$ (1)

Comparing (1) with standard equation

Y” + P(x) y’ + Q(x) y = 0 (2)

We get, P(x) = $\frac{2}{x}$ *and* $Q(x) \frac{3}{x\,(x-1)^3}$

The point x = 0. Since both P(x) and Q(x) are not defined at x =0 so they are not analytic at x =0 thus x =0 is a singular point.

Also (x -0) P(x) = 2 and $(x-0)^2$ Q (x) = $\frac{3x}{(x-1)^3}$

Since there both are defined at x =0. So are analytics at x =0 hence x =0 is a regular singular point.

III. FORMULATION OF THE METHOD

Let F(x) Y” P(x) Y’ + Q,(x) Y =0 (1)

Be a linear differential equation of second order recall that if f(0) = 0, then x = 0 is a singular point of (1) and if P(x) = (x -0), P_1 (x)/f(x)and Q(x) = $(x-0)^2$ Q_1 (x)/f(x) are finite for x = 0 then x =0 is a regular singular point of (1)in Frobenius method the series solution of (1) about the regular singular point x = 0 is taken as

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$$Y = \sum_{r=0}^{\infty} a_r x^{m+r}, \quad a_r \# 0 \qquad (2)$$

Diff (2) $$y' = \sum_{r=0}^{\infty} (m+r) a_r x^{m+r-1} \qquad (3)$$

$$Y'' = \sum_{r=0}^{\infty} (m+r)(m+r-1) a_r x^{m+r-2} \quad (4)$$

Putting from (2), (3) and (4) in (1). And simplifying it reduces to an identity in x.

Equate to zero the coefficient of the lowest power of x. thus we obtain a quadratic equation in m. this quadratic equation is called the indicial equation of differential equation (1) solve the indicial equation and determine its roots.

Which are often called the exponents of the differential equation (1). We denote the roots of indicial equation by m_1 and m_2 where $m_1 \geq m_2$.

These arise following case depending upon the nature of two roots of the indicial equation.

Case 1: The roots (m_1 , m_2) of the indicial equation unequal and differing by a quantity not an integer i. e. $m_1 - m_2 \# 0,1,2$.

Case 2: The roots of the indicial equation unequal differing by an integer mating a coefficient of y indeterminate.

Case 3: The roots of the indicial equation differing by an integer making a coefficient of y infinity.

Case 4: The roots of the indicial equation equal i.e. $m_1 - m_{2\,=}\, 0$

Now we equate to zero the coefficient of the general power term in the identity obtained in equation (1) usually we equate to zero the coefficient of the general term having the lowest general power and obtain an equation which connects a_1, a_r, or a_r, a_2 -2 etc. this equation is called as recurrence relation. If this recurrence relation connects a_r and a_r -2 then we also equate to zero the coefficient of next higher power of x (higher to the power used to obtained indicial equation).

Now replacing r by 0, 1, 2… (As needed) in the above recurrence relation we obtain the values of various.

Coefficients. Substituting in (2) the series solution of differential equation (1) is obtained.

IV. EXAMPLES

Consider the equation

$5x^2 y'' + x(1+x)y' - y = 0$, ………………..…. (1)

comparing with $y'' + P(x)y' + Q(x) = 0$

We get $P(x) = \frac{x(1+x)}{5x^2}$ *and* $Q(x) \frac{-1}{5x^2}$

1) $P(x)$ and $Q(x)$ do not exist at $x = 0$, so $x = 0$ is not ordinary point.

Also $x P(x) = \frac{(1+x)}{5}$ and $x^2 Q(x) = \frac{-1}{5}$

Both of these exist at $x = 0$

2) $x = 0$ is a regular singular point of (1).

Let the series solution of (1), about x=0, be give by

$Y = \sum_{r=0}^{\infty} a_r x^{m+r}$, $a_r \# 0$ …………………….. (2)

Diff. $Y' = \sum_{r=0}^{\infty} a_r^{(m+r)} x^{m+r-1}$

and $Y'' = \sum_{r=0}^{\infty} a_r^{(m+r)(m+r-1)} x^{m+r-2}$

Substituting in (1), we get

$\sum_{r=0}^{\infty} a_r [5(m+r)(m+r-1) x^{m+r} + (m+r)(1+x) x^{m+r} - x^{m+r}] = 0$

$\sum_{r=0}^{\infty} a_r [5(m+r)(m+r-1) x^{m+r} + (m+r) x^{m+r} + (m+r) x^{m+r+1} - x^{m+r}] = 0$

$\sum_{r=0}^{\infty} a_r [\{5(m+r)(m+r-1) + (m+r)-1\} x^{m+r} + (m+r) x^{m+r+1}] = 0$

$\sum_{r=0}^{\infty} a_r [\{5(m+r)+1\}(m+r-1) x^{m+r} + (m+r) x^{m+r+1}] =$ ------- (3)

Which is an identity and so we can equate to zero the coefficients of various power of x.

Equating to zero the coefficient of lowest power of x i. e. of x^m , the indicial equation is

$$a_0 (5m+1)(m-1) = 0 = m = \frac{-1}{5}, 1$$

3) $a_0 \# 0$ as it is the coefficient of first term with which we write the series.

Thus the roots of the indicial equation are unequal and their difference is not an integer.

A goin equating to zero the coefficient of x^{m+r} (lowest power term) of x is m and r) we get.

$a_r \{5(m+r)+1\}(m+r-1) + a_{r-1}(m+r-1) = 0$

$= a_r \{5(m+r)+1\} + a_r -1 = 0$

$= a_r = \frac{-ar-1}{5(m+r)+1}, r \geq 1$ ------- (4)

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Putting r=1,2,3, ------------ in (4), we get

$a_1 = \frac{-a\,0}{5m+6}$, $a_2 = \frac{-a1}{5m+11} = \frac{a\,0}{(5m+6)(5m+11)}$

$a_3 = \frac{-a0}{(5m+6)(5m+11)(5m+16)}$

and so on

4) from (2), we get

$$y= a_0 x^m + a_1 x^{m+1} + a_2 x^{m+2} a_3 x^{m+3} + \ldots\ldots\ldots\ldots$$

or $y= a_0 x^m[\,1 - \frac{x}{(5m+6)} + \frac{x^2}{(5m+6)(5m+11)} - \frac{x^3}{(5m+6)(5m+11)(5m+16)} \ldots \ldots\,]$

When m = -1/5, taking a_0 = P, we have

$$Y = px^{-1/5}\,[1 - \frac{x}{5} + \frac{x^2}{50} - \frac{x^3}{50} + \cdots]$$

= PU (say)

Which is one solution of the given equation.

Again, when m=1, taking a_0 =q, we have

$$Y = qx\,[1 - \frac{x}{11} + \frac{x^2}{176} - \frac{x^3}{3696} + \cdots]$$

= QV (say)

Which is the other solution of the given equation.

Hence the complete solution of the given equation is

Y = PU+QV

When P and Q are arbitrary constants.

V. CONCLUSION

From the discussion, it can be concluded that:

A. Forms solution Expanded Power Series (Frobenius)

Method) At singular point any placement in the form of differential equation:

$$Y'' + \frac{P(x)}{x} y' + \frac{q(x)}{x2} y = 0$$

Where $P(x)$ and $q\,(x)$ analytic in $x = 0$, has at least one solution that can be written in the form

$Y(x) = x^r \sum_{n=0}^{\infty} kn\, x^n$

Where is an indicator of the roots of equations differential equations.

B. Both Form Solution Variable Differential Equation Conferential Equation

The second solution form of variable coefficient differential equation are mutually lineally independent with $r_1 \geq r_2$ where r_1 and r^2 are the equation indicator $r^2 + (P_0 - 1)r + q_0 = 0$, will have the first solution that can be written in form

$Y_1\,(x) = x^{r,} \sum_{n=0}^{\infty} kn\, x^n$

And has their cases form a second solution

1) *Case 1:* The roots are different and the difference is not an inker.

$$Y_2\,(x) = x^{r2} \sum_{n=0}^{\infty} kn\, x^n$$

2) *Case 2:* The second difference is an integer root.

$$Y_2\,(x) = Vy_1\,(x) \text{ in } x + x^{r2} \sum_{n=0}^{\infty} Kn\, x^r,\ (x > 0)$$

3) *Case 3:* Twin roots

$$Y_2\,(x) = Y_1\,(x) \text{ In } x + x^{r+1} \sum_{n=0}^{n} kn\, x^r,\ (x > 0)$$

With the K1 K2, K3 ------------------------ Kn are constants.

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