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The Product - Normed Linear Space

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Abstract: In this paper, The combination of the structure of a vector space with the structure of a metric space naturally produces the structure of a normed space and a Banach space, i.e., of a complete linear normed space. The abstract definition of a linear normed space first appears around 1920 in the works of Stefan Banach (1892–1945), Hans Hahn (1879–1934) and Norbert Wiener (1894–1964). In fact, it is in these years that the Polish school around Banach discovered the principles and laid the foundation of what we now call linear functional analysis.
Keywords: Product-Normed Linear Space, Semi Inner Product Space, Completeness, Fixed Point Theorem

I. INTRODUCTION

In recent times, the solution space of a mathematical problem has become necessary condition for its existence. Without the solution space, the qualitative, as well as, quantitative property of the mathematical structure cannot be established. In [1], the author introduced the n−metrics. About 30 decades afterwards, the author in [2] narrowed then−metric space to two-dimensional space and also, obtained some topological propertiesassociated with the space.

A. *The Product Normed Linear Space*

1) *Definition:* Let X be a linear space over R (or C). A Norm on X is a nonnegative real-valued function on X, commonly denoted $\|\cdot\|$ such that:

a) $\|x\|=0$ if and only if $x=0$.

b) $\|\alpha x\|=|\alpha|\|x\|$ for all $\alpha\in R$ (or C) and for all $x\in X$.

c) $\|x+y\|\leq\|x\|+\|y\|$ for all $x,y\in X$.

If only properties (2) and (3) from above hold, then $\|\cdot\|$ is instead called a Seminorm.

B. *Semi Inner Product Space*

A Hilbert space can be thought of either as a complete inner product space, or as a Banach space whose norm satisfies the parallelogram law. In the theory of operators on a Hilbert space, it actually does not function as a particular Banach space, but rather as a particular inner product space.

C. *Completeness*

1) The completeness of the real numbers, which implies that there are no "holes" in the real numbers
2) Complete metric space, a metric space in which every Cauchy sequence converges
3) Complete uniform space, a uniform space where every Cauchy net in converges (or equivalently every Cauchy filter converges)
4) Complete measure, a measure space where every subset of every null setis measurable

D. *Fixed Point Theorem*

A fixed-point theorem is a result saying that a function F will have at least one fixed point (a point x for which F(x) = x), under some conditions on F that can be stated in general terms. Results of this kind are amongst the most generally useful in mathematics.
If g is a continuous function $g(x) \in [a,b]$ for all $x \in [a,b]$, then g has a fixed point in [a,b] . This can be proven by supposing that

$g(a) \geq a,\ g(b) \leq b$ (1)

$g(a) \geq - a,\ g(b) \leq - b$ (2)

Since g is continuous, the intermediate value theorem guarantees that there exists a $c \in [a,b]$ such that

$g(c) - c = 0$ (3)

so there must exist a c such that

$g(c) = c.$ (4)

so there must exist a fixed point $\in [a,b]$.

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II. CONCLUSION AND DISCUSSION

The norm is a continuous function on its vector space. All linear maps between finite dimensional vector spaces are also continuous. An isometry between two normed vector spaces is a linear map f which preserves the norm (meaning $\|f(v)\| = \|v\|$ for all vectors v). Isometries are always continuous and injective. A surjective isometry between the normed vector spaces V and W is called an isometric isomorphism, and V and are called isometrically isomorphic. Isometrically isomorphic normed vector spaces are identical for all practical purposes.

III. RESULT

Theorem : If X is a normed linear space over R (or C) with norm $\|\cdot\|$ and if $d:X\times X\to[0,\infty)$ is defined for all $x,y\in X$ by $d(x,y)=\|x-y\|$ then (X,d) is a metric space.

Proof : We show that d is a metric on X.

Let $x,y\in X$. Then,

$d(x,y)=\|x-y\|=\|-1(y-x)\|=|-1|\|y-x\|=1\|y-x\|=\|y-x\|=d(y,x)$ (1)

Let $x,y\in X$ and suppose that $d(x,y)=0$. Then,

$0=d(x,y)=\|x-y\|\Leftrightarrow x-y=0\Leftrightarrow x=y.$ (2)

Lastly, let $x,y,z\in X$. Then,

$d(x,z)=\|x-z\|=\|(x-y)+(y-z)\|\leq\|x-y\|+\|y-z\|=d(x,y)+d(y,z)$ (3)

Hence d is a metric on X and (X,d) is a metric space.

We give a special name to the metric space that can be obtained by a normed linear space.

A metric space need not have any kind of algebraic structure defined on it. In many applications, however, the metric space is a linear space with a metric derived from a norm that gives the "length" of a vector. Such spaces are called normed linear spaces.

In a nutshell, we have introduced the product-normed linear space and product-semi-normed linear space (product-semi-Banch space) which are endowed with some functional space. In addition, $P-NLS$ is endowed with fixed point.

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