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Study to Linear Topological Space

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Abstract: In this theis several topies from Topology linear Algebra and Real Analysis are com-bined in the study of linear topological spaces. We begin with a brief look at linear spaces before moving on to study some basic properties of this structure of linear topological basis. Then we turn our attention to linear spaces with a metric topology. In particular we consider problems involving normed liner spaces bounded linear transformation and Hilbert spaces
Keywords: topology, linear algebra

I. INTRODUCTION

P. Thangavelu and Nithanantha Jothi introduced the concept of binary topology in (4). It is a single topological structure that carrier the subjects of a set x as well as the subsets of another set x for studying the information about the orderded pair (A.B.) of subset of x and y. A linear topological space endowed with a topology such that the vector addition and sclar multiplication are both continuos the theory of linear topological spaces provide a remarkable economy in discussion of many classical mathematical problems. We introduce the concept of binary topology to linear section 2. We define the binary linear jtopology. Section 3 Space (BLTS) We prove that the binary product of two linear topological spcae is a BLTS. Also we discuss to concept of locally convese BLTS and locally bounded BLTS and prove some of their properties. In section 4 we define binary metric and binary normal. The main result of this section is that the binary product preserve metrizablity and normbility. Section 5 deals with the construction of aBLTS using a family of binary seminorms.

II. PRELIMINARIES

1) *Definition:* Let x and y be any two non-empty and d (x) and g (y) be their power sets respectively. A binary topology from x to y is a binary structure M Í d (x) x d (y) that satisties the following arioms (f, f) and (x, y) ÎM

If (A_1, B_1) and (A_2, B_2) ÎM, then (A_1 Ç A_2, B_1 Ç B_2) ÎM.

If {(A_a, B_a) : a ÎD} is a family of members of M; then (U_d ÎDA_d, U_a ÎDB_a) ÎM.

If M is a binary topology from x to y then the triplet (x, y, m) is called a binary topology space and the members of M are called binary points of binary open sets. (C,D) is called binary colsed if (x | c, y | D) is binary open. The elements of x, x y are called the binary points of the binary topological space (x, y, m) yet (x, y, m) be a binary topological space and let (x, y) Î x x x The binary open set (A, B) is called a binary neighborhood of (x, y) if x Î A and ÎB. If x = y then M is called a binary topology on x and we write (x, M) as a binary space.

2) *Proposition:* Let (x, y, m) be a binary topological space. Then

(1) $_T$(M) = {A Í x : (A, B) ÎM for some B Í Y} is a topology on x.

T1(M) = {BÍ Y: (A, B) ÎM for some A Í x} is a topology on y.

III. BINARY LINEAR TOPOLOGY

1) *Definition:* A binary topology between two vector space is said to be binary linear if the two operation are continuous i. e, if V_1, and V_2 are vector space over the some field k and for every neighbourhoods U of (x1 + x2 , y1+ y2) ÎV_1 x V_2. ' two neighbourthoods U_1 and U_2 of (x_1, y_1) and (x_2, y_2) respectively that U_1 + U_2 Í U. Similarly for every neighbourthood W of (lx, ly) $v_1 \times v_2$ there exsits a neighbourthood w of (x, y) such that lw Í w. If M is a binary linary topology between two vector space V_1 and V_2 then triplet (V_1, V_2, M) is called a binary linear topological space (BLTS).
2) *Proposition:* If (V_1 T_1) and V_2, T_2) are two linear topologica space then (V_1, V_2, $T_1 \times T_2$) is a called the binary linear topological space.

a) *Proof:* By proposition 2. 3, (V_1, V_2, T_1 x T_2) is a binary topological space. If remains to show that T_2 x T_2 is a binary linear topology let (x_1, x_2), (y_1, y_2) ÎV1 × V2 and (A_1, A_2) be a neighbourhood of [(x_1, x_2) + (y_1, y_2)]. Then x_1 + y_1 Î A_1 and x_2 + y_2 Î A_2. Since A_1 Î T_1 ÎT_1 and T_2, and T_1 and T_2 are linear topological there exist neighhbourthood B_1 and C_1 of x_1 and y_1 respectively in T_1 such that B_1 + C_1 Î A_1 and neighbourhood B_2 and C_2 of x_2 and y_2 respectively in T_2 such that B_2 + C_2 Í A_2. Then in T_1 × T_2 (B_1, B_2) is a neighbourhood (B_1, that (B_1, B_2) + (C_1, C_2) = T_1 × T_2 (B_1, B_2) Now Let (A_1, A_2) be a neighborhood of l (x_1,x_2) T_1 × T_2 Then A1 is a neighborhood of lx_1 in T_1 and T_2 A_2 is a x_2 in T_2. So there exists two B_1 and B_2 off x_1 and x_2 respectively such that lB_1 Í A_1 and B_2 Í A_2. This implies that (B_1, B_2) is a neighbourhood of (x_1, x_2) such that l(B_1, B_2) Í (A_1, A_2). Thus T_1 × T_2 is a binary linear topology.

3) *Proposition:* If (V_1. V_2, M) is a BLTS, then a(M) = {A Í V_1 : (A, B) Î M for some B
ÎV_2} is a linear topology on V_1 and a (M) = {B Í V_2) (A, B) Î M for come A Í V_1} is a linear topology on V_2.

a) *Proof:* By proposition a (M) are both topologies in V_1 a V_2 respectively. Let x_1, y_1 Î V_1 and A ÎT(M) contains x, + y_1. Then for some x_2, y_2 Î v_2 there exsists B Í V_2 such that (x_1 ++ y_1, x_2 + y_2) Î (A, B) Where (A, B) Î M, since M is a binary linear topology, there exist (E_1, E_2) and F_1, F_2) in M such that (x_1, x_2) (e_1, e_2), (y_1, y_2) Î (f_1, f_2) and (E_1, E_2) + (F_1 + F_2) Í (A, B). x, Î E_1, Y_1 Î F1and (E_1, E_2) by the definition of binary sets. Also E_1 and F_1 $Î_1$ (M) by the construction of (T). Similary for lxÎA. Where for lxÎA. Where A ÎT(M) we can find also a linear of x say U such that ÎUÍA. Thus $_T$(M) is linear topology in the same say we can prove that (M) topology.

4) *Definition:* A local base of a binary linear topology (V_1, V_2, M) is the base Consists of the neighborhood of a binary points (x, y)

5) *Definition:* A set (A, B) Îd (V_1) × d (V_2) is convese if for all pairs (x_1, x_2), (y_1, y_2) Î (A, B) l (x_1, x_2) + (1 - l) (y_1, y_2) Î (A, B) lÎ (0, 1).

6) *Definition:* A binary topology is called locally conver if there exsist a local base at (0, 0) whose members are conver.

7) *Definition:* A BLTS is locally bounded of (0, 0) as a bounded neighbourhood, i, e, a neighbourhood (E, F) such that (N, M) ÎNo. the set of neighbourhood of (0, 0) there exists S Î R such that t> S, (E, F) Í t (N, M). Let (V_1, V_2, M) be a BLTS. Then for every (w_1, w_2) Î No. ' balanced and symmetic sets (x_1, y_1), (x_2, y_2) Î No such that (x_1, x_2) t (x_2, y_2) c (w_1, w_2).

a) *Proof :* If (w_1, w_2) ÎNo then w_1 and w_2 are neighbourhood of O in (v_1, T (M) and (V_2, T respectively by the property of linear topologices there exists symmetrie balanced neighbourhood of 0, x_1, x_2 Î T(M) and y1 + y_2 CW_2 Now, x_1, y_1 are Þ aÎR with |a| £ 1, a x_1 c x_1 and y_1 cy_1.

So a(x_1, y_1) = (a x_1, y_1) C (x_1, y_1) thus (x, y_1) and (x_2, y_2) are balanced by the symmetry of x1 and y1 we get x1 = -x_1, y_1 = -y_1 Þ (x_1, y_1) = (-x_1, y_1) thus (x_1, y_1) is symmetric and similarly (x_2, y_2) is also symmetric. (x_1 y_1) + (x_2, y_2) = (x_1 + x_2, y1 +y_2) C (w_1, w_2).

8) Proposition: Let V_1 and V_2 be real vector space and U_1 be a convex set in V_1 and U_1 be a convex set in V_2 then (U_1, U_2) is convex d (V_1) × d (V_2).

a) *Proof :* Let (x_1, y_1) Î (U_1, U_2) for i = 1, 2 then x1Î U_1 Î U_2 for i = 1, 2 Þ lx_1 + (1 - l) x_2 Î
U1 for 0 £ > Í 1. So (lx_1 + (x_1, y_1) + (1 - l) y_2) (U_1, U_2). Consider l(x_1 , y_1) + (1 - l) (x_2, y_2)
= (lx_1, ly_1) + (1 - l) y_2) ¬ U_1, U_2) for) £ > Í 1. Thus (U_1, U_2) is convex.

9) *Corollary:* If (V_1, T_1) and (V_2, T_2) are both locally convex topological vector spaces then their binary product (V_1, V_2, T_1 × T_2) is locally convex BLTS.

10) *Proposition:* Let U_1 and U_2 be bounded sets in two uleal vecotor spaces V_1 and V_2 respectively then bounded.

a) *Proof:* Since U_1 is bounded for every neighbourhood e1Î No. (V_1). 's1 ÎR such that t> E_2 ÎNo. (V_2), ' s_2 ÎR such that t > S1, V1 C tE1. Similarly for every neighbourhood E_2 ÎNo. (V_2), ' s2 Î R such that t > S_2, U_2 C tE_2. Let T_1 Î R correspound to E and T_2 Î to f then t > t_1, U_1 C tE and t > t_2. U_2 c tf. So t> S, where S = max (t_1, t_2), U1 c tE and U_2 ctf i.e (U_1 U_2) Ì t (E, F), t >S. Thus (U_1, U_2) is bounded.

11) Corollary : If (V_1 T_1) and (V_2 T_2) are both locally bounded topological vector spaces, then their binary product (V_1, V_2 T1 × T_2) is a locally bounded BLTS.

12) Proposition: Let (V_1, T_1) be a topological vector space and V_2 be another vector space such that map T : V_1®V_2 is an isomo rphism. Then T_2 = {T(A) : A ÎT_1} is a linear topology in V_2 and hence T_1 × T_2 is a binary linear topology from V_1 to V2.

a) *Proof:* Since T is an isomorphism, T (f) = f and T(V_1) = V_2 and So f V_2 and So f V_2 ÎT_2. Let A. B ÎT_2. Then A = T(A) and B = T(B) for some A' and B' Î T_1. So A' Ç B' Î T_1 (A' Ç B') Î T_2 T (A' Ç B') = T (A') Ç (B') = A Ç B Thus. A Ç B Î T_2. Now Let {Aa} . . $_T$ T_2 for some index set. T then exists (Ba). . $_T$ eT_1 Such that Aa = T (Ba) for each aeT Then U . . $_T$ ÎT_2 for each aeT. So x_1 + y_1 ¬ A and U. . $_T$ Aa = U. . $_T$ T (Ba). Then B_1, B_2 Î T_2 and x_1 ÎA_1 Þ x_2 = T (x_1) ÎT (A_1) = B_1 y_1.

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IV. BINARY MERITABLE AND BINARY NORMABLE BLTS

1) *Definition:* A binary metric on two sets V_1 and V_2 is a map d : $(V_1 \times V_2) \times (V_1 \times V_2)$ ® R satisfying the following axioms : If (x_1, x_2) (y_1, y_2) Î $V_1 \times V_2$ then.

df $[(x_1, x_2), (y_1, y_2)]$ ³ 0 d $[(x_1, x_2), (y_1, y_2)]$ = d $[(y_1, y_2), (x_1, x_2)]$ and.

d $[(x_1, x_2), (y_1, y_2)]$ £ d $[(x_1, x_2), (z_1, z_2)]$ + d $[(z_1, z_2), (y_1, y_2)]$ for every (z_1, z_2) Î $v_1 \times v_2$

d $[(x_1, x_2), (y_1, y_2)]$ = 0 Û $x_1 = x_2$ and $y_1 = y_2$.

2) *Definition:* Let (v_1, v_2, M) be a BLTS. A binary topology M is metrizable with a binary metric d if for any (x, y) in some binary open set (A, B) Î M, ' g > 0 Such that B, (x, y)

CB where pi is the projection map to V_1 for i = 1, 2.

3) *Proposition:* If (V_1, T_1) and (V_2, T_2) are two linear topological space such that T_1 and T_2 are both metrizable with metrics d_1 and d_2 respectively then $T_1 \times T_2$ are both metrizable with metrics d_1 and d_2 respectively then $T_1 \times T_2$ is binary metrizable.

a) *Prof:* Consider the map d : $(V_1 \times V_2) \times (V_1 \times V_2)$ R defined by

$$d((x_1, x_2), (y_1, y_2)) = \frac{d_1 (x_1, y_1) + d_2 (x_2, y_2)}{2}, (x_1, x_2), (y_1, y_2) ¬ (v_1 \times v_2)$$

If $(x_1, x_2), (y_1, y_2)$ Î $v_1 \times v_2$ then

(1) $$d [(x_1, x_2) (y_1, y_2)] = \frac{d_1 (x_1, y_1) + d_2 (x_2, y_2)}{2} ³ 0, \text{since } d_1.$$

(x_1, y_1) and $d_2 (x_2, y_2)$ are both non-negative.

(2) $$d [(x_1, x_2), (y_1, y_2)] = \frac{d_1 (x_1, y_1) + d_2 (x_2, y_2)}{2} \quad 0 Û d_1 (x_1 y_1) = 0 \text{ and } d_2 (x_2, y_2) = 0.$$

This happens if and only x1 = x_2 and $y_1 = y_2$ i. e. when $(x_1, y_1) = (x_2, y_2)$

(3) $$d[(x_1, x_2) = d (y_1, y_2)] = \frac{d_1 (x_1, y_1) + d_2 (x_2, y_2)}{2} = \frac{d_1 (y_1 + y_1) + d_2}{2}$$

$(y_2, x_2) = d (y_1, y_2), x_1 x_2))$ and if (z_1, z_2) Î $v_1 \times v_2$

(4) $$d[(x_1, x_2), (y_1, y_2)] = \frac{(x_1, y_1) + d_2 (x_2, y_2)}{2} = \frac{£ (d_1 (x_1 , z_1) + d_1}{2}$$

$$(z_1, y_1)] + [d_2 (x_2, z_2) + d_2 (z_2, y_2)]^2 = \frac{d_1 (x_1, z_1) + d_2 (x_2, z_2)}{2} + \frac{d_1 (z_1 , y_1) +}{2}$$

$d_2 (x_2, y_2) = d[(x_1, x_2), (z_1, z_2)] + d [(z_1, z_2), (y_1, y_2)]$.

Thus dis a binary metric let (A, B) Î $T_1 \times T_2$ and (x, y) Î (A, B) Then x Î A Î T_1 and y Î B Î T_2 since T_1 and T_2 are metrizable. 'G_1, G_2 >0 with respect to d_1 and $d_2$respectively such that Br_1 (x) CA and Br_2 (y) c B. i. e if $d_1(x_1\ x_1) < n$ then x_1 Î Br_1 (x) and if $d_2 (y_1\ y_1) < r_2$ then y_1 ÎBr_2 (y) Þ (x, y,) ¬ (A, B) let r = min (r_1, r_2) and (u, v) Î B6/2 (x, y) then (x, y), (u, v) < r/2, i, e, d1 $(x_1\ y) + d_2 (y, v) < r/2$. So $d_1 (x, u) + d_2 (y, v) < r/2$, i, e $d_1\ d_1 (x_1, u_1) + d_2 (y, v)$

2 <r/2. So d1 (x, u) + d_2 (y, v) <r<r_1 and d (y, v) <r<r_2 Hence u Î Br_1

(x) c and ut Br_2 (y) CB. Thus (u, v) Î (A, B)showing that Br/2 (xy) C (A, B).

4) *Definition:* A binary seminorm on two vector space V_1 and V_2 is a map ||.|| : $V_1 \times V_2$ ®Rsuch that for each (x_1 x_2) (y_1 y_2) $V_1 \times V_2$.

|| (x_1, x_2) || ³ 0

|| a (x_1, x_2) || = |a | || (x_1, x_2) ||

|| (x_1, x_2) + (y_1, y_2) || £ || (x_1, x_2) || + || (y_1, y_2) ||.

A binary seminarm becomes a binary norm if the following condition holds.

|| (x_1, x_2) || = 0 Û (x_1, x_2) = (0, 0)

5) *Proposition:* If (V_1, T_1) and (V_2, T_2) are both normable topological vector space, then their ubinary product is binary normable.

a) *Proof:* Let $||.||_1$ and $||.||_2$ be the norms corres ponding to t_1 and t_2 respectively. Then we get two metrics d_1 and d_2 defind by d_1 ((x_1, x_2) (y_1, y_2(= ||(x_1, x_2) - (y_1, y_2) || i, i = 1, 2 and (x_1,x_2) (y_1, y_2) $v_1 \times v_2$ with which t_1 and t_2 are metrizable respectively. So by proposition $T_1 \times T_2$ is metrizable with which T_1 and d_1 (x_1, y_1) + d_2 (x_2, y_2) V (x_1, x_2), (y_1, y_2) ($v_1 \times v_2$) Hence the binary norm ||.|| defind by || (x_1, x_2) Î $v_1 \times v_2$ corresponds to the topology $T_1 \times T_2$ but

this norm is same as $\frac{||.||_1 + ||.||_2}{2}$ since $||(x_1. x_2)|| = d\,(x_1, x_2)\,(0, 0) = \frac{d_1 (x_1, 0) + d_2 (x_2, 0)}{2}$

$$= \frac{|| x_1 - 0||_1 + ||x_2 - 0||_2}{2} = \frac{|| x_1 ||_1 + ||x_2||_2}{2}$$

6) *Lemma:* Let V_1 and V_2 be two vector space and P be a binary seminorm on $V_1 \times V_2$

Then there exists two seminorm P_1 and P_2 on V_1 and V_2 respectively.

a) *Proof:* Let P_1 : V_1 ® R be defined by P_1 (x) = infy {P(x, y) : y Îv_2} since P (x, y) ³ 0 , (x, y) ¬ $V_1 \times V_2$, P1 (x) ³ 0 x ÎV_1 for x ÎV_1 and aÎK, P_1 (ax) = inf (P(ax,y) : y Îv_2)

inf |a| P (x ¹ y) : y Îv_2) 2y

|a| inf (x ¹ y) : y Îv_2) 2y

= | d | P1 | x |

for x, y Îv1 P1 (x + y) = inf (P (x+y, z) : z Î v2)

= $\inf\limits_{z = z_1 + z_2}$ (P (x+y. z_1 + z_2) : z = z_1 + z_2 Î v_2)

= $\inf\limits_{z_1\, z_2}$ (P (x, z_1) + (y. z_2) : z_1, z_2 Î v_2)

= £ $\inf\limits_{z_1\, z_2}$ {(P (x, z_1) + P (y_1 z_2) : z_1, z_2 Î v_2}

Thus P1 (x + y) P1 (x) + P1 (y)

Hence P1 is a seminors on V1 and family P2 : V2 R defined by P2 (y) = infx {P(x, y): x V1} is a seminorm on V2.

V. CONCLUSION

In This paper we have introduced the concept of linear topological space to situation in which we have to deal with two vector space and a topology between the spaces. This helps to study both the space simultaneourly. The concept of topological vector space is well used in mathematics engineering and science and particularly is auantum mechains. **Heance** our theory of Binary linear Topological space helps in the future development of such ares.

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