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The Comparison of Degree of Complementary Dominating Set of Dominating Set using Euclidean Division Algorithm with Divisor 5 for Interval Graph G

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Abstract: Interval graphs, their importance over the years can be seen in the increasing number of researchers trying to explore the field. In this paper we presented the comparison of degree of complementary dominating set of dominating set using Euclidean division algorithm with divisor 5 for interval graph G corresponding to an interval family I. In particular we prove a necessary condition under which the complementary dominating set as well as the domination number.
Keywords: Interval family, Interval graph, Degree, Dominating set, Domination number, Complementary dominating set, Euclidean division algorithm.

I. INTRODUCTION

All graphs considered in this paper are finite, undirected, with no loop or multi edge. A graph is said to be a simple graph if it has no loops and no parallel edges[4, 7]. A graph is said to be connected if there is a path between every pair of vertices, otherwise it is said to be disconnected graph[3, 6, 8]. The number of edges incident with a vertex V is called degree of V and is denoted by d(v). If d(v) = 1 then the vertex v is said to be a pendent vertex. If d(v) = 0 then the vertex v is said to be an isolated vertex. Let G = (V, E) be a graph, a set $D \subseteq V$ is a dominating set of G if every vertex in V\D is adjacent to some vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set[9]. A minimum dominating set with minimum cardinality among all the dominating sets of a graph G is said to be Minimum dominating set.

II. PRELIMINARIES

Interval graphs are rich in combinatorial structures and have found applications in several disciplines such as traffic control, ecology, biology, computer science and particularly useful in cyclic scheduling and computers storage allocation problems etc. Have a representation of graph with intervals or arcs can be helpful in combinatorial problems of the graph, such as isomorphism testing and finding maximum independent set and cliques graphs.

Let I = $\{ I_1, I_2, I_3, I_4, \ldots\ldots\ldots\ldots, I_n \}$ be an interval family, where each I_i is an interval on the real line and $I_i = [a_i, b_i]$ for i = 1, 2, 3,..........,n. Here a_i is called the left end point and b_i is called the right end point. Without loss of generality, one can assume that, all end points of the intervals are distinct numbers between 1 and 2n. An interval of degree one is called a pendent interval. The intervals are named in the increasing order of their right end points. The graph G is an interval graph if there is one-to-one correspondence between the vertex set V and the interval family I.

Two vertices of G are joined by an edge if and only if their corresponding intervals in I intersect. That is if $I_i = [a_i, b_i]$ and $I_j = [a_j, b_j]$, then I_i and I_j will intersect if $a_i < b_j$ or $a_j < b_i$. According to the Euclidean division algorithm if we have two positive integers a and b, then there exists unique integers q and r which satisfies the condition a = bq + r where $0 \leq r < b$.

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III. PREREQUISITE RESULTS

A. *Result1*

Let I = $\{ I_1, I_2, I_3, I_4, \cdots\cdots\cdots, I_n \}$, $\forall n \geq 5$ be any finite interval family such that every interval I_i, i ≠ $\{n-1, n\}$ intersect the next two intervals only and let G(V,E) be an interval graph corresponding to an interval family I if there is one-to-one correspondence between the vertex set V and interval family I where $V = \{v_1, v_2, v_3, \ldots\ldots, v_n\}$ then

(i) $d(v_1) = d(v_n) = 2$ (ii) $d(v_2) = d(v_{n-1}) = 3$ and (iii) $d(v_3) = d(v_4) =$ ××××××××= $d(v_{n-2}) = 4$.

B. *Result2*

Let I= $\{ I_1, I_2, I_3, I_4, \ldots\ldots\ldots \ldots, I_n \}$, $\forall n \geq 5$ be any finite interval family such that every interval I_i, i ≠ $\{n-1, n\}$ intersects next two intervals only and let G be an interval graph corresponding to an interval family I. suppose n = 5q + r, where r = 0, 1, 2, 3, 4 and q is any positive integer. If r = 0, then the Minimum dominating set $D = \{ V_{5m-2} \;/\; m \in N, 1 \leq m \leq q \}$ and the domination number $\gamma(G) = q$ and then the following results are true.

$$(i) \sum_{v \in D} d(v) = 4q \quad (ii) \sum_{v \in D} d(v) = 4\gamma \quad (iii) \sum_{v \in D} d(v) = \frac{4n}{5}$$

C. *Result3*

Let I= $\{ I_1, I_2, I_3, I_4, \ldots\ldots\ldots \ldots, I_n \}$, $\forall n \geq 5$ be any finite interval family such that every interval I_i, i ≠ $\{n-1, n\}$ intersects next two intervals only and let G be an interval graph corresponding to an interval family I. suppose n = 5q + r, where r = 0, 1, 2, 3, 4 and q is any positive integer. If r = 1, then the Minimum dominating set $D = \{ V_{5m-2}, V_n \;/\; m \in N, 1 \leq m \leq q \}$ and the domination number $\gamma(G) = q+1$ and then the following results are true.

$$(i) \sum_{v \in D} d(v) = 4q+2 \quad (ii) \sum_{v \in D} d(v) = 4\gamma - 2 \quad (iii) \sum_{v \in D} d(v) = \frac{4n+6}{5}.$$

D. *Result4*

Let I= $\{ I_1, I_2, I_3, I_4, \ldots\ldots\ldots \ldots, I_n \}$, $\forall n \geq 5$ be any finite interval family such that every interval I_i, i ≠ $\{n-1, n\}$ intersects next two intervals only and let G be an interval graph corresponding to an interval family I. suppose n = 5q + r, where r = 0, 1, 2, 3, 4 and q is any positive integer. If r = 2, then two cases will arise

1) *Case (i):* The Minimum dominating set $D = \{ V_{5m-2}, V_{n-1} \;/\; m \in N, 1 \leq m \leq q \}$ and the domination number $\gamma(G) = q+1$ and then the following results are true.

$$(i) \sum_{v \in D} d(v) = 4q+3 \quad (ii) \sum_{v \in D} d(v) = 4\gamma - 1 \quad (iii) \sum_{v \in D} d(v) = \frac{4n+7}{5}.$$

Case(ii): The Minimum dominating set $D = \{ V_{5m-2}, V_n \;/\; m \in N, 1 \leq m \leq q \}$ and the domination number $\gamma(G) = q+1$ and then the following results are true.

$$(i) \sum_{v \in D} d(v) = 4q+2 \quad (ii) \sum_{v \in D} d(v) = 4\gamma - 2 \quad (iii) \sum_{v \in D} d(v) = \frac{4n+2}{5}.$$

E. Result5

Let I= $\{I_1, I_2, I_3, I_4, \ldots\ldots\ldots, I_n\}$, $\forall n \geq 5$ be any finite interval family such that every interval I_i, i $\neq \{n-1, n\}$ intersects next two intervals only and let G be an interval graph corresponding to an interval family I. suppose n = 5q + r, where r = 0, 1, 2, 3, 4 and q is any positive integer. If r = 3, then three cases will arise

1) *Case(i):* The Minimum dominating set $D = \{V_{5m-2}, V_{n-2} \ / \ m \in N, 1 \leq m \leq q\}$ and the domination number $\gamma(G) = q+1$ and then the following results are true.

$$(i)\sum_{v\in D} d(v) = 4q+4 \quad (ii)\sum_{v\in D} d(v) = 4\gamma \quad (iii)\sum_{v\in D} d(v) = \frac{4n+8}{5}$$

Case(ii): The Minimum dominating set $D = \{V_{5m-2}, V_{n-1} \ / \ m \in N, 1 \leq m \leq q\}$ and the domination number $\gamma(G) = q+1$ and then the following results are true.

$$(i)\sum_{v\in D} d(v) = 4q+3 \quad (ii)\sum_{v\in D} d(v) = 4\gamma - 1 \quad (iii)\sum_{v\in D} d(v) = \frac{4n+3}{5}$$

Case(iii): The Minimum dominating set $D = \{V_{5m-2}, V_{n} \ / \ m \in N, 1 \leq m \leq q\}$ and the domination number $\gamma(G) = q+1$ and then the following results are true.

$$(i)\sum_{v\in D} d(v) = 4q+2 \quad (ii)\sum_{v\in D} d(v) = 4\gamma - 2 \quad (iii)\sum_{v\in D} d(v) = \frac{4n-2}{5}$$

F. Result6

Let I= $\{I_1, I_2, I_3, I_4, \ldots\ldots\ldots, I_n\}$, $\forall n \geq 5$ be any finite interval family such that every interval I_i, i $\neq \{n-1, n\}$ intersects next two intervals only and let G be an interval graph corresponding to an interval family I. suppose n = 5q + r, where r = 0, 1, 2, 3, 4 and q is any positive integer. If r = 4, then two cases will arise

Case(i):The Minimum dominating set $D = \{V_{5m-2}, V_{n-2} \ / \ m \in N, 1 \leq m \leq q\}$ and the domination number $\gamma(G) = q+1$ and then the following results are true.

$$(i)\sum_{v\in D} d(v) = 4q+4 \quad (ii)\sum_{v\in D} d(v) = 4\gamma \quad (iii)\sum_{v\in D} d(v) = \frac{4n+4}{5}.$$

Case(ii):The Minimum dominating set $D = \{V_{5m-2}, V_{n-1} \ / \ m \in N, 1 \leq m \leq q\}$ and the domination number $\gamma(G) = q+1$ and then the following results are true.

$$(i)\sum_{v\in D} d(v) = 4q+3 \quad (ii)\sum_{v\in D} d(v) = 4\gamma - 1 \quad (iii)\sum_{v\in D} d(v) = \frac{4n-1}{5}.$$

G. Result7:

Let I= $\{I_1, I_2, I_3, I_4, \ldots\ldots\ldots, I_n\}$ be any finite interval family and let G be an interval graph corresponding to an interval family I. D be a minimum dominating set and D^c is a compliment dominating set of an interval graph G. And γ and γ^C be the domination numbers of an interval graph G with respect to D and D^C respectively then $\gamma + \gamma^C = n$.

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IV. MAIN THEOREMS

A. *Theorem 1*

Let I= $\{I_1, I_2, I_3, I_4, \ldots\ldots\ldots \ldots, I_n\}$, $\forall n \geq 5$ be any finite interval family such that every interval I_i, $i \neq \{n-1, n\}$ intersect next two intervals only and let G be an interval graph corresponding to an interval family I. suppose n = 5q + r, where r = 0, 1, 2, 3, 4 and q is any positive integer. Let γ be the domination number of D and γ^C be the complementary domination number of D^C. If r = 0 then the Minimum dominating set $D=\{V_{5m-2} \ / \ m \in N, 1 \leq m \leq q\}$ and $\gamma = q$ and then the following deductions are true.

$$(i) \sum_{v \in D^C} d(v) = 16q-6 \ (ii) \sum_{v \in D^C} d(v) = 4\gamma^C - 6 \ (iii) \sum_{v \in D^c} d(v) = \frac{16n-30}{5}$$

1) *Proof:* Let I = I= $\{I_1, I_2, I_3, I_4, \ldots\ldots\ldots \ldots, I_n\}$, $\forall n \geq 5$ be any finite interval family such that every interval I_i, $i \neq \{n, n-1\}$ intersect the next two intervals only and let G(V, E) be an interval graph if there is one to one correspondence between the vertex set V and the interval family I where V=$\{v_1, v_2, v_3, v_4, \ldots\ldots\ldots\ldots, v_n\}$.

By Euclidean division algorithm, n = 5q + r where r = 0, 1, 2, 3, 4 and q is any positive integer, n is a number of vertices in G.

Suppose r = 0 then we have n = 5q + 0 $\Rightarrow n = 5q \Rightarrow q = \frac{n}{5}$

Then we have $D=\{V_{5m-2} \ / \ m \in N, 1 \leq m \leq q\}$

$$D = \{v_3, v_8, v_{13}, \ldots\ldots\ldots\ldots, v_{5q-2}\}$$

$$D=\{v_3, v_8, v_{13}, \ldots\ldots\ldots\ldots, v_{n-2}\}$$ and γ = q

Then $D^C = \{v_1, v_2, v_4, v_5, v_6, v_7, \ldots\ldots\ldots\ldots, v_{n-1}, v_n\}$

and $\gamma^C = n - \gamma \Rightarrow \gamma^C = 5q - q \Rightarrow \gamma^C = 4q$

$\therefore \gamma^C = |D^C| = 4q$

$$(i) \sum_{v \in D^C} d(v) = d(v_1) + d(v_2) + d(v_4) + \ldots\ldots\ldots + d(v_{n-1}) + d(v_n)$$

$$= 2 + 3 + 4 + 4 + \ldots\ldots\ldots\ldots + 4 + 3 + 2$$

$$= (4 + 4 + 4 + 4 + \ldots\ldots\ldots + 4 + 4 + 4) - 6$$

$$= 4 (4q) - 6$$

$$\therefore \sum_{v \in D^C} d(v) = 16q - 6$$

$$(ii) \sum_{v \in D^C} d(v) = 16q - 6$$

$$= 16\left(\frac{\gamma^C}{4}\right) - 6$$

$$= 4\gamma^C - 6$$

$$\therefore \sum_{v \in D^C} d(v) = 4\gamma^c - 6$$

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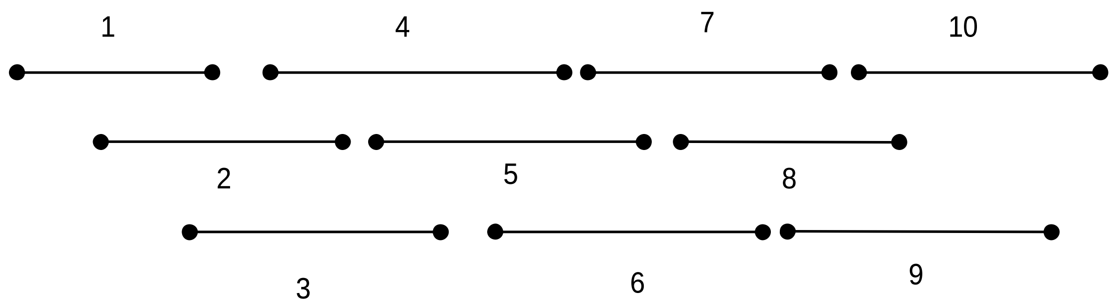
$$(iii)\sum_{v\in D^c} d(v) = 16q-6$$

$$= 16\left(\frac{n}{5}\right)-6$$

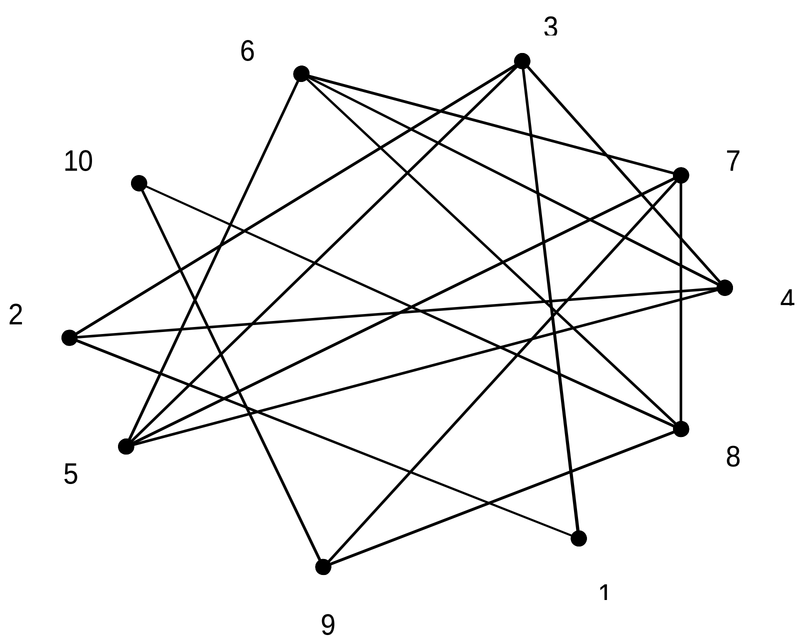
$$= \frac{16n}{5}-6 = \frac{16n-30}{5}$$

$$\therefore \sum_{v\in D^c} d(v) = \frac{16n-30}{5}$$

2) *Practical Problem:* Let I= $\{ I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9, I_{10} \}$ be an interval family and let G be an interval graph corresponding to an interval family I is as follows:



INTERVAL FAMILY I



INTERVAL GRAPH G

Here n = 10 = 5 × 2 + 0

This is of the form n = 5q + r and then q = 2 and r = 0.

If r = 0, then the Minimum dominating set D = { v_3, v_8 } and $\gamma = 2$ since theorem 1.

D^C = { v_1, v_2, v_4, v_5, v_6, v_7, v_9, v_{10}} and $\gamma^C = 8$

$$\sum_{v\in D^C} d(v) = d(v_1)+d(v_2)+d(v_4)+d(v_5)+d(v_6)+d(v_7)+d(v_9)+d(v_{10})$$

$$= 2 + 3 + 4 + 4 + 4 + 4 + 3 + 2$$

$$= 26$$

$$\therefore \sum_{v\in D^C} d(v) = 26$$

$$(i) \sum_{v\in D^C} d(v) = 16q-6 = 16\times 2-6 = 32-6 = 26$$

$$\sum_{v\in D^C} d(v) = 26$$

$$(ii) \sum_{v\in D^C} d(v) = 4\gamma^C - 6 = 4\times 8 - 6 = 32-6 = 26$$

$$\therefore \sum_{v\in D^C} d(v) = 26$$

$$(iii) \sum_{v\in D^C} d(v) = \frac{16n-30}{5} = \frac{16\times 10-30}{5} = \frac{160-30}{5} = \frac{130}{5} = 26$$

$$\therefore \sum_{v\in D^C} d(v) = 26$$

Hence theorem 1 is verified.

B. *Theorem 2*

Let I={ $I_1, I_2, I_3, I_4,\ldots\ldots\ldots, I_n$ }, $\forall n \geq 5$ be any finite interval family such that every interval I_i, i ≠ $\{n-1, n\}$ intersect next two intervals only and let G be an interval graph corresponding to an interval family I. suppose n = 5q + r, where r = 0, 1, 2, 3, 4 and q is any positive integer. Let γ be the domination number of D and γ^c be the complimentary domination number of D^c. If r = 1 then the Minimum dominating set $D = \{V_{5m-2}, V_n \ / \ m \in N, 1 \leq m \leq q\}$ and $\gamma = q+1$ and then the following deductions are true.

$$(i) \sum_{v\in D^c} d(v) = 16q-4 \quad (ii) \sum_{v\in D^c} d(v) = 4\gamma^c - 4 \quad (iii) \sum_{V\in D^c} d(V) = \frac{16n-36}{5}.$$

1) *Proof:* Let I ={ $I_1, I_2, I_3, I_4,\ldots\ldots\ldots, I_n$ }, $\forall n \geq 5$ be any finite interval family such that every interval I_i, i ≠ $\{n-1, n\}$ intersect the next two intervals only and let G(V, E) be an interval graph if there is one to one correspondence between the vertex set V and the interval family I where V={$v_1, v_2, v_3, v_4, \ldots\ldots\ldots, v_n$}.

By Euclidean division algorithm, n = 5q + r where r = 0, 1, 2, 3, 4 and q is any positive integer, n is a number of vertices in G.

Suppose r = 1 then we have n = 5q + 1 $\Rightarrow q = \frac{n-1}{5}$

$$\text{Then we have } D = \{V_{5m-2}, V_n \ / \ m \in N, 1 \leq m \leq q\}$$

$$D = \{v_3, v_8, v_{13}, \ldots\ldots\ldots, v_{5q-2}, v_n\}$$

$$D=\{v_3,v_8,v_{13},..........,v_{n-3},v_n\} \text{ and } \gamma = q+1$$

Then $D^C=\{v_1,v_2,v_4,v_5,v_6,v_7,...........,v_{n-2},v_{n-1}\}$

and $\gamma^C = n-\gamma \Rightarrow \gamma^C = 5q+1-q-1 \Rightarrow \gamma^C = 4q$

$\therefore\ \gamma^C = |D^C| = 4q$

(i) $\sum_{v\in D^C} d(v) = \mathrm{d}(v_1) + \mathrm{d}(v_2) + \mathrm{d}(v_4) + \ldots\ldots\ldots + \mathrm{d}(v_{n-2}) + \mathrm{d}(v_{n-1})$

$= 2+3+4+4+ \ldots\ldots\ldots\ldots + 4+3 = (4+4+4+4+ \ldots\ldots\ldots + 4+4) - 4 = 4(4q) - 4$

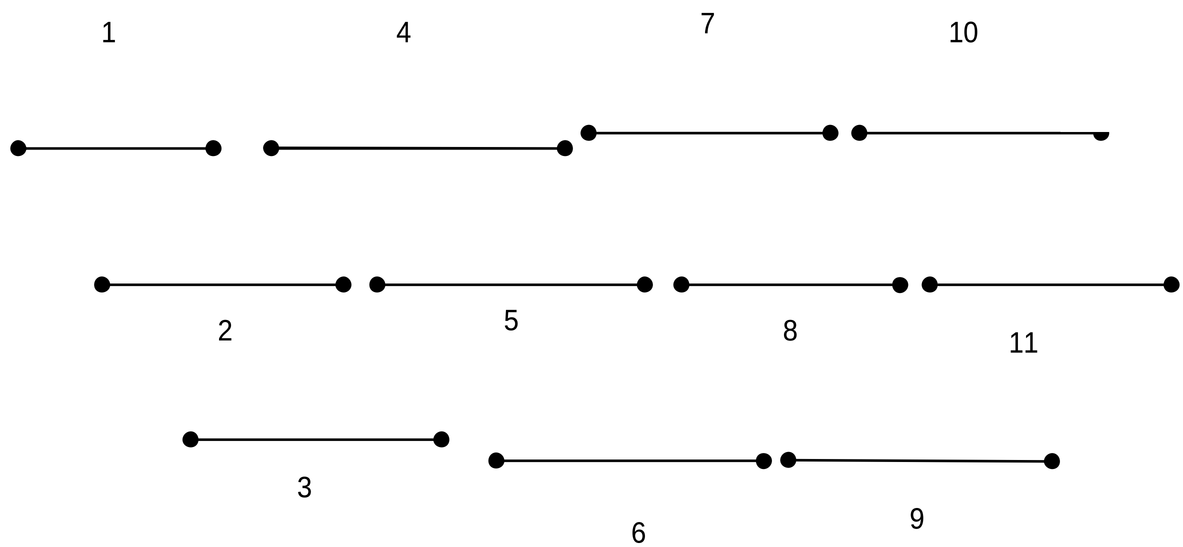
$\therefore \sum_{v\in D^C} d(v) = 16q-4$

$(ii) \sum_{v\in D^C} d(v) = 16q-4 = 16\left(\frac{\gamma^C}{4}\right)-4$

$\therefore \sum_{v\in D^C} d(v) = 4\gamma^C - 4$ $(iii) \sum_{v\in D^C} d(v) = 16q-4 = 16\left(\frac{n-1}{5}\right)-4 = \frac{16n-16}{5}-4 = \frac{16n-16-20}{5} = \frac{16n-36}{5}$

$\therefore \sum_{v\in D^C} d(v) = \frac{16n-36}{5}$

2) *Practical Problem:* Let I= $\{I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9, I_{10}, I_{11}\}$ be an interval family and let G be an interval graph corresponding to an interval family I is as follows:



INTERVAL FAMILY I

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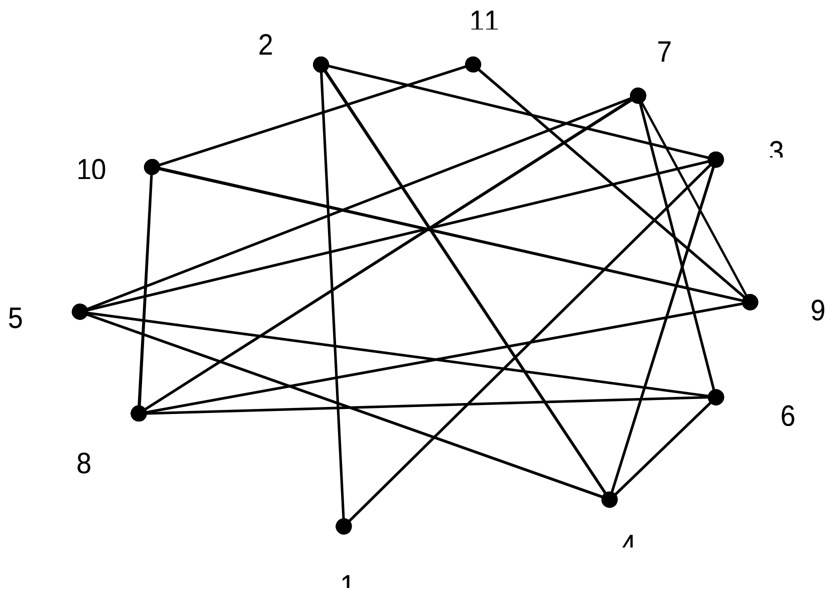

INTERVAL GRAPH G

Here n = 11= 5 × 2 + 1
This is of the form n = 5q + r and then q = 2 and r = 1.
If r = 1, then the Minimum dominating set D = { v_3, v_8, v_{11} }
$\gamma = 3$, since theorem 2.
D^C = { v_1, v_2, v_4, v_5, v_6, v_7, v_9, v_{10}} and $\gamma^C = 8$

$$\sum_{v\in D^C} d(v) = d(v_1)+d(v_2)+d(v_4)+d(v_5)+d(v_6)+d(v_7)+d(v_9)+d(v_{10})$$

$$= 2 + 3 + 4 + 4 + 4 + 4 + 4 + 3$$
$$= 28$$

$$\therefore \sum_{v\in D^C} d(v) = 28$$

$$(i) \sum_{v\in D^C} d(v) = 16q - 4$$

$$= 16(2) - 4$$
$$= 32 - 4 = 28$$

$$\therefore \sum_{v\in D^C} d(v) = 28$$

$$(ii) \sum_{v\in D^C} d(v) = 4\gamma^C - 4 = 4(8) - 4 = 32 - 4 = 28$$

$$\therefore \sum_{v\in D^C} d(v) = 28$$

$$(iii) \sum_{v\in D^C} d(v) = \frac{16n-36}{5} = \frac{16\times 11-36}{5} = \frac{176-36}{5} = \frac{140}{5} = 28$$

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$$\therefore \sum_{v\in D^C} d(v) = 28$$

Hence theorem 2 is verified.

C. Theorem 3

Let I= { $I_1, I_2, I_3, I_4,$......... ..., I_n }, $\forall n \geq 5$ be any finite interval family such that every interval I_i, i ≠ $\{n-1, n\}$ intersect next two intervals only and let G be an interval graph corresponding to an interval family I. suppose n = 5q + r, where r = 0, 1, 2, 3, 4 and q is any positive integer. Let γ be the domination number of D and γ^c be the compliment domination number of D^c. If r = 2 then two cases will arise

a) *Case (i):* $D = \{V_{5m-2}, V_{n-1} \;/\; m \in N, 1 \leq m \leq q\}$ and $\gamma = q+1$ and then the following deductions are true.

$$(i)\sum_{v\in D^c} d(v) = 16q-1 \quad (ii)\sum_{v\in D^c} d(v) = 4\gamma^c - 5 \quad (iii)\sum_{v\in D^c} d(v) = \frac{16n-37}{5}.$$

b) *Case (ii):* $D = \{V_{5m-2}, V_n \;/\; m \in N, 1 \leq m \leq q\}$ and $\gamma = q+1$ and then the following deductions are true.

$$(i)\sum_{v\in D^c} d(v) = 16q \quad (ii)\sum_{v\in D^c} d(v) = 4\gamma^c - 4 \quad (iii)\sum_{v\in D^c} d(v) = \frac{16n-32}{5}.$$

1) *Proof:* Let I = I= { $I_1, I_2, I_3, I_4,$......... ..., I_n }, $\forall n \geq 5$ be any finite interval family such that every interval I_i, i ≠ $\{n, n-1\}$ intersect the next two intervals only and let G(V, E) be an interval graph if there is one to one correspondence between the vertex set V and the interval family I where V={$v_1, v_2, v_3, v_4,$.....................,v_n}.

By Euclidean division algorithm, n = 5q + r where r = 0, 1, 2, 3, 4 and q is any positive integer, n is a number of vertices in G.

Suppose r = 2 then we have n = 5q + 2 $\Rightarrow q = \frac{n-2}{5}$ and then two cases will arise

a) *Case(i):* we have $D = \{V_{5m-2}, V_{n-1} \;/\; m \in N, 1 \leq m \leq q\}$

$$D = \{v_3, v_8, v_{13}, ..., v_{5q-2}, v_{n-1}\}$$

$$D = \{v_3, v_8, v_{13}, ..., v_{n-4}, v_{n-1}\} \text{ and } \gamma = q+1$$

Then $D^C = \{v_1, v_2, v_4, v_5, v_6, v_7, ..., v_{n-2}, v_n\}$ *and*

$$\gamma^C = n - \gamma \Rightarrow \gamma^C = 5q + 2 - q - 1 \Rightarrow \gamma^C = 4q + 1$$

Therefore $\gamma^C = |D^C| = 4q + 1$

$$(i) \sum_{v\in D^C} d(v) = d(v_1) + d(v_2) + d(v_4) + \ldots\ldots\ldots + d(v_{n-2}) + d(v_n)$$

= 2 + 3 + 4 + 4 +................ + 4 + 2
= (4 + 4 + 4 + 4 + ... + 4 + 4) – 5
= 4 (4q + 1) – 4
= 16q + 4 –5 = 16q – 1

$$\therefore \sum_{v\in D^C} d(v) = 16q - 1$$

$$(ii)\sum_{v\in D^C} d(v) = 16q - 1 = 16\left(\frac{\gamma^C - 1}{4}\right) - 1 = 4\gamma^C - 4 - 1 = 4\gamma^C - 5$$

$$\therefore \sum_{v\in D^C} d(v) = 4\gamma^C - 5$$

$$(iii) \sum_{v\in D^C} d(v) = 16q - 1 = 16\left(\frac{n-2}{5}\right) - 1 = \frac{16n-32-5}{5} = \frac{16n-37}{5}$$

$$\therefore \sum_{v\in D^C} d(v) = \frac{16n-37}{5}$$

b) **case** $(\mathbf{ii})$: We have $D=\{V_{5m-2}, V_n \ / \ m\in N, 1\le m\le q\}$

$$D=\{v_3, v_8, v_{13}, \ldots\ldots\ldots, v_{5q-2}, v_n\}$$

$D=\{v_3, v_8, v_{13}, \ldots\ldots\ldots, v_{n-4}, v_n\}$ and γ = q + 1

Then $D^C = \{v_1, v_2, v_4, v_5, v_6, v_7, \ldots\ldots\ldots\ldots, v_{n-2}, v_{n-1}\}$ *and*

$$\gamma^C = n - \gamma \Rightarrow \gamma^C = 5q + 2 - q - 1 \Rightarrow \gamma^C = 4q + 1$$

$$\therefore \gamma^C = |D^C| = 4q + 1$$

$$(\mathrm{i}) \sum_{v\in D^C} d(v) = \mathrm{d}(v_1) + \mathrm{d}(v_2) + \mathrm{d}(v_4) + \ldots\ldots\ldots + \mathrm{d}(v_{n-2}) + \mathrm{d}(v_{n-1})$$

= 2 + 3 + 4 + 4 +................ + 4 + 3 = (4 + 4 + 4 + 4 + ... + 4 + 4) – 4

= 4 (4q + 1) – 4 = 16q + 4 – 4 = 16q $\therefore \sum_{v\in D^C} d(v) = 16q$

$$(ii) \sum_{v\in D^C} d(v) = 16q$$

$$= 16\left(\frac{\gamma^C - 1}{4}\right)$$

$$= 4\gamma^C - 4$$

$$\therefore \sum_{v\in D^C} d(v) = 4\gamma^C - 4$$

$$(iii) \sum_{v\in D^C} d(v) = 16q$$

$$= 16\left(\frac{n-2}{5}\right)$$

$$= \frac{16n-32}{5}$$

$$\therefore \sum_{v\in D^C} d(v) = \frac{16n-32}{5}$$

2) *Practical Problem:* Let I=$\{ I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9, I_{10}, I_{11}, I_{12}\}$ be an interval family and let G be an interval graph corresponding to an interval family I is as follows:

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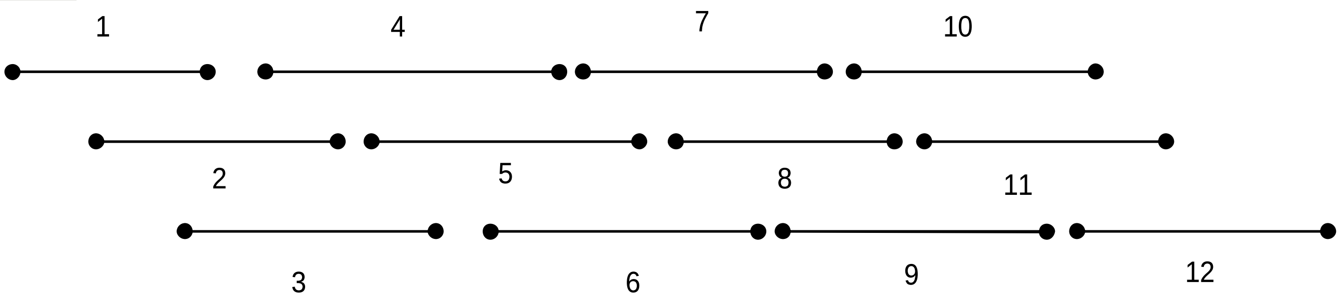
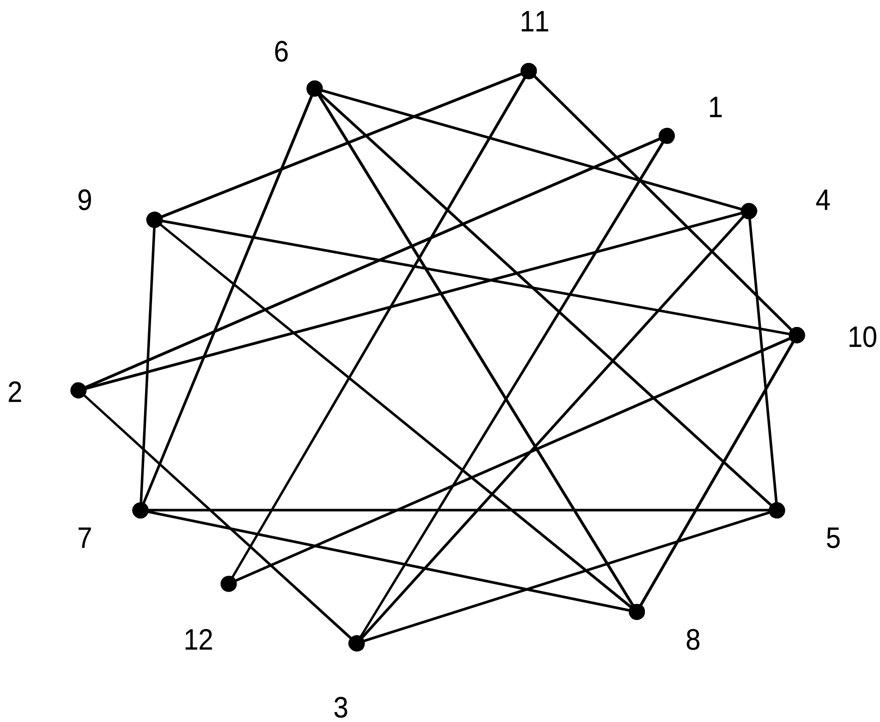


INTERVAL FAMILY I



INTERVAL GRAPH G

Here n=12 = 5 × 2 + 2

This is of the form n = 5q + r and then q = 2 and r = 2.

If r = 2, then Minimum dominating set D is either D = { v_3, v_8 , v_{11} } or D = { v_3, v_8 , v_{12} } and $\gamma = 3$ since theorem 3.

i) ***Case(i):*** Suppose D = { v_3, v_8 , v_{11} } and $\gamma = 3$ and then D^C = { v_1, v_2, v_4, v_5, v_6, v_7, v_9, v_{10}, v_{12} } and $\gamma^C = 9$

$$\sum_{v \in D^C} d(v) = d(v_1) + d(v_2) + d(v_4) + d(v_5) + d(v_6) + d(v_7) + d(v_9) + d(v_{10}) + d(v_{12})$$

$$= 2 + 3 + 4 + 4 + 4 + 4 + 4 + 4 + 2$$

$$= 31$$

$$\therefore \sum_{v \in D^C} d(v) = 31$$

$$(i) \sum_{v \in D^C} d(v) = 16q - 1 = 16(2) - 1 = 32 - 1 = 31$$

$\therefore \sum_{v\in D^C} d(v)=31$

$(ii) \sum_{v\in D^C} d(v)=4\gamma^C-5= 4(9) - 5 = 36 - 5 = 31$

$\therefore \sum_{v\in D^C} d(v)=31$

$(iii) \sum_{v\in D^C} d(v)=\frac{16n-37}{5}=\frac{16\times 12-37}{5}=\frac{192-37}{5}=\frac{155}{5}=31$

$\therefore \sum_{v\in D^C} d(v)=31$

ii) *Case(ii):* Suppose D = { v_3, v_8 , v_{12} } and γ = 3 and then D^C = { v_1, v_2, v_4, v_5, v_6, v_7, v_9, v_{10}, v_{11} } and γ^C = 9

$$\sum_{v\in D^C} d(v)=d(v_1)+d(v_2)+d(v_4)+d(v_5)+d(v_6)+d(v_7)+d(v_9)+d(v_{10})+d(v_{11})$$

= 2 + 3 + 4 + 4 + 4 + 4 + 4 + 4 + 3

= 32

$\therefore \sum_{v\in D^C} d(v)=32$

$(i) \sum_{v\in D^C} d(v)=16q= 16(2) = 32$

$\therefore \sum_{v\in D^C} d(v)=32$

$(ii) \sum_{v\in D^C} d(v)=4\gamma^C-4= 4(9) - 4 = 36 - 4 = 32$

$\therefore \sum_{v\in D^C} d(v)=32$

$(iii) \sum_{v\in D^C} d(v)=\frac{16n-32}{5}=\frac{16\times 12-32}{5}=\frac{192-32}{5}=\frac{160}{5}=32$

$\therefore \sum_{v\in D^C} d(v)=32$

Hence theorem 3 is verified.

D. Theorem 4

Let I={ $I_1, I_2, I_3, I_4,\ldots\ldots\ldots, I_n$ }, $\forall n\geq 5$ be any finite interval family such that every interval I_i, $i \neq \{n-1, n\}$ intersect next two intervals only and let G be an interval graph corresponding to an interval family I. suppose n = 5q + r, where r = 0, 1, 2, 3, 4 and q is any positive integer. If r = 3 then three cases will arise.

a) Case (i): $D=\{V_{5m-2}, V_{n-2} \ / \ m\in N, 1\leq m\leq q\}$ and $\gamma = q+1$ and then the following deductions are true.

$$(i) \sum_{v\in D^C} d(v)=16q+2 \quad (ii) \sum_{v\in D^C} d(v)=4\gamma^C-6 \quad (iii) \sum_{v\in D^C} d(v)=\frac{16n-38}{5}.$$

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b) Case (ii): $D=\{V_{5m-2},V_{n-1}\ /\ m\in N,1\le m\le q\}$ and $\gamma = q+1$ and then the following deductions are true.

$$(i)\sum_{v\in D^C} d(v)=16q+3\ \ (ii)\sum_{v\in D^C} d(v)=4\gamma^C-5\ \ (iii)\sum_{v\in D^C} d(v)=\frac{16n-33}{5}.$$

c) Case (iii): $D=\{V_{5m-2},V_n\ /\ m\in N,1\le m\le q\}$ and $\gamma = q+1$ and then the following deductions are true.

$$(i)\sum_{v\in D^C} d(v)=16q+4\ \ (ii)\sum_{v\in D^C} d(v)=4\gamma^C-4\ \ (iii)\sum_{v\in D^C} d(v)=\frac{16n-28}{5}.$$

1) *Proof:* Let I = I=$\{I_1,I_2,I_3,I_4,.........\ ...,\ I_n\}$, $\forall n\ge 5$ be any finite interval family such that every interval I_i, i $\neq \{n-1,n\}$ intersect the next two intervals only and let G(V, E) be an interval graph if there is one to one correspondence between the vertex set V and the interval family I where V=$\{v_1,v_2,v_3,v_4,.....................,v_n\}$.

By Euclidean division algorithm, n = 5q + r where r = 0, 1, 2, 3, 4 and q is any positive integer, n is a number of vertices in G.

Suppose r = 3 then we have n = 5q + 3$\Rightarrow q=\frac{n-3}{5}$

Case(i): we have $D=\{V_{5m-2},V_{n-2}\ /\ m\in N,1\le m\le q\}$

$$D=\{v_3,v_8,v_{13},...........,v_{5q-2},v_{n-2}\}$$

$$D=\{v_3,v_8,v_{13},...........,v_{n-5},v_{n-2}\}\text{ and }\gamma = q+1$$

Then $D^C=\{v_1,v_2,v_4,v_5,v_6,v_7,............,v_{n-1},v_n\}$ *and* $\gamma^C = n-\gamma \Rightarrow \gamma^C = 5q+3-q-1\Rightarrow \gamma^C = 4q+2$

$\therefore\ \gamma^C=|D^C|=4q+2$

$$(i)\ \sum_{v\in D^C} d(v)= d(v_1)+d(v_2)+d(v_4)+..........+d(v_{n-1})+d(v_n)$$

= 2 + 3 + 4 + 4 +................. + 3 + 2
= (4 + 4 + 4 + 4 + ... + 4 + 4) – 6
= 4 (4q + 2) – 6
= 16q + 8 - 6
= 16q + 2

$$\therefore \sum_{v\in D^C} d(v)=16q+2$$

$$(ii)\sum_{v\in D^C} d(v)=16q+2=16\left(\frac{\gamma^C-2}{4}\right)+2=4\gamma^C-8+2=4\gamma^C-6$$

$$\therefore \sum_{v\in D^C} d(v)=4\gamma^C-6$$

$$(iii)\sum_{v\in D^C} d(v)=16q+2=6\left(\frac{n-3}{5}\right)+2=\frac{16n-48+10}{5}=\frac{16n-38}{5}$$

$$\therefore \sum_{v\in D^C} d(v)=\frac{16n-38}{5}$$

case (ii): We have $D=\{V_{5m-2},V_{n-1}\ /\ m\in N,1\le m\le q\}$

$$D=\{v_3,v_8,v_{13},...........,v_{5q-2},v_{n-1}\}$$

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$$D=\{v_3,v_8,v_{13},..........,v_{n-5},v_{n-1}\} \text{ and } \gamma = q+1$$

Then $D^C=\{v_1,v_2,v_4,v_5,v_6,v_7,............,v_{n-2},v_n\}$ *and* $\gamma^C = n-\gamma \Rightarrow \gamma^C = 5q+3-q-1 \Rightarrow \gamma^C = 4q+2$

$\therefore\ \gamma^C=\left|D^C\right|=4q+2$

(i) $\sum_{v\in D^C} d(v)= \mathrm{d}(v_1)+\mathrm{d}(v_2)+\mathrm{d}(v_4)+..........+\mathrm{d}(v_{n-2})+\mathrm{d}(v_n)$

= 2 + 3 + 4 + 4 +................ + 4 + 2
= (4 + 4 + 4 + 4 + ... + 4 + 4) – 5
= 4 (4q + 2) – 5 = 16q + 8 – 5 = 16q + 3

$\therefore \sum_{v\in D^C} d(v)=16q+3$

$(ii) \sum_{v\in D^C} d(v)=16q+3=16\left(\frac{\gamma^C-2}{4}\right)+3=4\gamma^C-8+3=4\gamma^C-5$

$\therefore\ \sum_{v\in D^C} d(v)=4\gamma^C-5$

$(ii) \sum_{v\in D^C} d(v)=16q+3=16\left(\frac{n-3}{5}\right)+3=\frac{16n-48+15}{5}=\frac{16n-33}{5}$

$\therefore\ \sum_{v\in D^C} d(v)=\frac{16n-33}{5}$

Case(iii): we have $D=\{V_{5m-2},V_n \ / \ m\in N, 1\le m\le q\}$

$$D=\{v_3,v_8,v_{13},..........,v_{5q-2},v_n\}$$

$$D=\{v_3,v_8,v_{13},..........,v_{n-5},v_n\} \text{ and } \gamma = q+1$$

Then $D^C=\{v_1,v_2,v_4,v_5,v_6,v_7,............,v_{n-2},v_{n-1}\}$ *and*

$\gamma^C = n-\gamma \Rightarrow \gamma^C = 5q+3-q-1 \Rightarrow \gamma^C = 4q+2$

$\therefore\ \gamma^C=\left|D^C\right|=4q+2$

(i) $\sum_{v\in D^C} d(v)= \mathrm{d}(v_1)+\mathrm{d}(v_2)+\mathrm{d}(v_4)+..........+\mathrm{d}(v_{n-2})+\mathrm{d}(v_{n-1})$

= 2 + 3 + 4 + 4 +................ + 4 + 3
= (4 + 4 + 4 + 4 + ... + 4 + 4) – 4
= 4 (4q + 2) – 4 = 16q + 8 - 4 = 16q + 4

$\therefore \sum_{v\in D^C} d(v)=16q+4$

$(ii) \sum_{v\in D^C} d(v)=16q+4=16\left(\frac{\gamma^C-2}{4}\right)+4=4\gamma^C-8+4=4\gamma^C-4$

$\therefore\ \sum_{v\in D^C} d(v)=4\gamma^C-4$

$(iii) \sum_{v\in D^C} d(v)=16q+4=16\left(\frac{n-3}{5}\right)+4=\frac{16n-48+20}{5}=\frac{16n-28}{5}$

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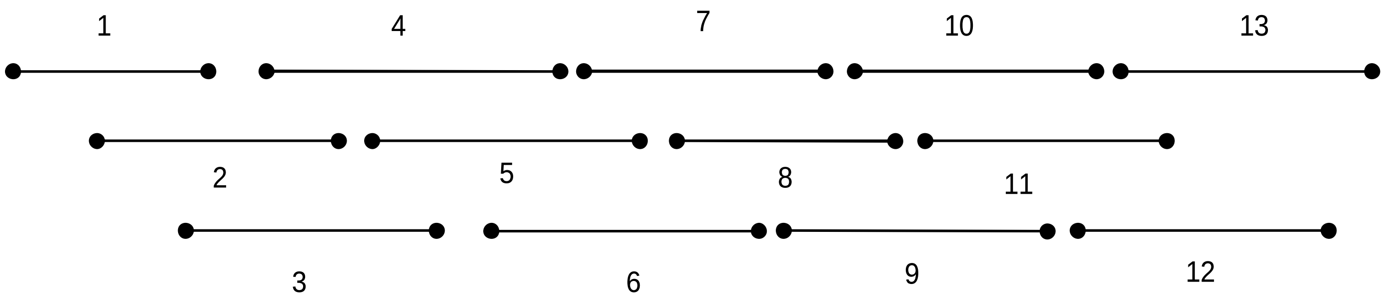
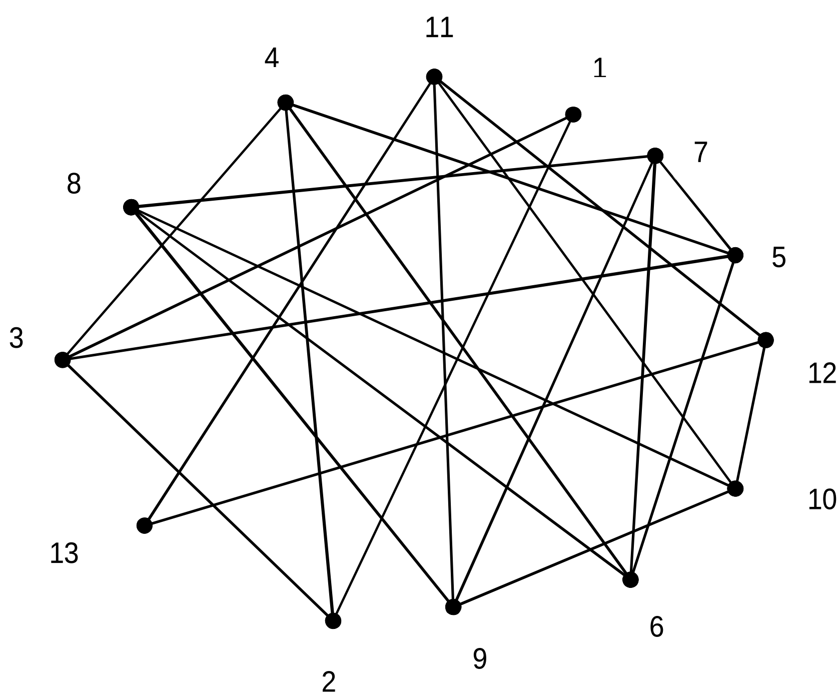
$$\therefore \sum_{v \in D^C} d(v) = \frac{16n-28}{5}$$

2) *Practical Problem:* Let I= $\{ I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9, I_{10}, I_{11}, I_{12}, I_{13} \}$ be an interval family and let G be an interval graph corresponding to an interval family I is as follows:



INTERVAL FAMILY I



INTERVAL GRAPH G

Here n =13= 5 × 2 + 3
This is of the form n = 5q + r and then q = 2 and r = 3.
If r = 3, then the Minimum dominating set D is D = { v_3, v_8 , v_{11} } or D = { v_3, v_8 , v_{12} } or D = { v_3, v_8 , v_{13} }, $\gamma = 3$ since theorem 4.

a) *Case(i):* Suppose D = { v_3, v_8 , v_{11} } and $\gamma = 3$
and then D^C = { v_1, v_2, v_4, v_5, v_6, v_7, v_9, v_{10}, v_{12} ,v_{13}} and $\gamma^C = 10$

$$\sum_{v\in D^C} d(v) = d(v_1)+d(v_2)+d(v_4)+d(v_5)+d(v_6)+d(v_7)+d(v_9)+d(v_{10})+d(v_{12})+d(v_{13})$$

= 2 + 3 + 4 + 4 + 4 + 4 + 4 + 4 + 3 + 2

= 34

$$\therefore \sum_{v\in D^C} d(v) = 34$$

$$(i) \sum_{v\in D^C} d(v) = 16q+2 = 16(2) + 2 = 32 + 2 = 34$$

$$\therefore \sum_{v\in D^C} d(v) = 34$$

$$(ii) \sum_{v\in D^C} d(v) = 4\gamma^C - 6 = 4(10) - 6 = 40 - 6 = 34$$

$$\therefore \sum_{v\in D^C} d(v) = 34$$

$$(iii) \sum_{v\in D^C} d(v) = \frac{16n-38}{5} = \frac{16\times 13-38}{5} = \frac{208-38}{5} = \frac{170}{5} = 34$$

$$\therefore \sum_{v\in D^C} d(v) = 34$$

b) *Case(ii):* Suppose D = { v_3, v_8 , v_{12} } and $\gamma = 3$

and D^C = { v_1, v_2, v_4, v_5, v_6, v_7, v_9, v_{10}, v_{11} ,v_{13}} and $\gamma^C = 10$

$$\sum_{v\in D^C} d(v) = d(v_1)+d(v_2)+d(v_4)+d(v_5)+d(v_6)+d(v_7)+d(v_9)+d(v_{10})+d(v_{11})+d(v_{13})$$

= 2 + 3 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 2

= 35

$$\therefore \sum_{v\in D^C} d(v) = 35$$

$$(i) \sum_{v\in D^C} d(v) = 16q+3 = 16(2) + 3 = 32 + 3 = 35$$

$$\therefore \sum_{v\in D^C} d(v) = 35$$

$$(ii) \sum_{v\in D^C} d(v) = 4\gamma^C - 5 = 4(10) - 5 = 40 - 5 = 35$$

$$\therefore \sum_{v\in D^C} d(v) = 35$$

$$(iii) \sum_{v\in D^C} d(v) = \frac{16n-33}{5} = \frac{16\times 13-33}{5} = \frac{208-33}{5} = \frac{175}{5} = 35$$

$$\therefore \sum_{v\in D^C} d(v) = 35$$

c) *Case(iii):* Suppose D = { v_3, v_8 , v_{13} }and $\gamma = 3$

and then D^C = { v_1, v_2, v_4, v_5, v_6, v_7, v_9, v_{10}, v_{11},v_{12}} and $\gamma^C = 10$

$$\sum_{v\in D^C} d(v) = d(v_1)+d(v_2)+d(v_4)+d(v_5)+d(v_6)+d(v_7)+d(v_9)+d(v_{10})+d(v_{11})+d(v_{12})$$

= 2 + 3 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 3

= 36

$\therefore \sum_{v\in D^C} d(v) = 36$

$(i) \sum_{v\in D^C} d(v) = 16q + 4 = 16(2) + 4 = 32 + 4 = 36$

$\therefore \sum_{v\in D^C} d(v) = 36$

$(ii) \sum_{v\in D^C} d(v) = 4\gamma^C - 4 = 4(10) - 4 = 40 - 4 = 36$

$\therefore \sum_{v\in D^C} d(v) = 36$

$(iii) \sum_{v\in D^C} d(v) = \frac{16n-28}{5} = \frac{16\times 13-28}{5} = \frac{208-28}{5} = \frac{180}{5} = 36$

$\therefore \sum_{v\in D^C} d(v) = 36$

Hence theorem 4 is verified.

E. Theorem5

Let I={ $I_1, I_2, I_3, I_4, ...\ ...,\ I_n$ }, $\forall n \geq 5$ be any finite interval family such that every interval I_i, i $\neq \{n-1, n\}$ intersect next two intervals only and let G be an interval graph corresponding to an interval family I. suppose n = 5q + r, where r = 0, 1, 2, 3, 4 and q is any positive integer. If r = 4, then two cases will arise

a) *Case (i):* $D = \{V_{5m-2}, V_{n-2}\ /\ m \in N, 1 \leq m \leq q\}$ and $\gamma = q + 1$ and then the following deductions are true.

$(i) \sum_{v\in D^C} d(v) = 16q + 6 \quad (ii) \sum_{v\in D^C} d(v) = 4\gamma^C - 6 \quad (iii) \sum_{v\in D^C} d(v) = \frac{16n-34}{5}$.

b) *Case (ii):* $D = \{V_{5m-2}, V_{n-1}\ /\ m \in N, 1 \leq m \leq q\}$ and $\gamma = q + 1$ and then the following deductions are true.

$(i) \sum_{v\in D^C} d(v) = 16q + 7 \quad (ii) \sum_{v\in D^C} d(v) = 4\gamma^C - 5 \quad (iii) \sum_{v\in D^C} d(v) = \frac{16n-29}{5}$.

1) *Proof:* Let I={ $I_1, I_2, I_3, I_4, ...\ ...,\ I_n$ }, $\forall n \geq 5$ be any finite interval family such that every interval I_i, i $\neq \{n-1, n\}$ intersect the next two intervals only and let G(V, E) be an interval graph if there is one to one correspondence between the vertex set V and the interval family I where V={$v_1, v_2, v_3, v_4, ..., v_n$}.

By Euclidean division algorithm, n = 5q + r where r = 0, 1, 2, 3, 4 and q is any positive integer, n is a number of vertices in G.

Suppose r = 4 then we have n = 5q + 4 $\Rightarrow q = \frac{n-4}{5}$

Case (i): we have $D = \{V_{5m-2}, V_{n-2}\ /\ m \in N, 1 \leq m \leq q\}$

$$D = \{v_3, v_8, v_{13}, ..., v_{5q-2}, v_{n-2}\}$$

$$D = \{v_3, v_8, v_{13}, ..., v_{n-5}, v_{n-2}\} \text{ and } \gamma = q + 1$$

Then $D^C = \{v_1, v_2, v_4, v_5, v_6, v_7, ..., v_{n-1}, v_n\}$ *and*

$\gamma^c = n - \gamma \Rightarrow \gamma^c = 5q + 4 - q - 1 \Rightarrow \gamma^c = 4q + 3$

$\therefore\ \gamma^c = |D^C| = 4q+3$

(i) $\sum_{v\in D^C} d(v) = d(v_1) + d(v_2) + d(v_4) + \ldots\ldots\ldots + d(v_{n-1}) + d(v_n)$

$= 2 + 3 + 4 + 4 + \ldots\ldots\ldots + 3 + 2$
$= (4 + 4 + 4 + 4 + \ldots\ldots\ldots + 4 + 4) - 6$
$= 4(4q + 3) - 6$
$= 16q + 12 - 6$
$= 16q + 6$

$\therefore \sum_{v\in D^C} d(v) = 16q+6$

(ii) $\sum_{v\in D^C} d(v) = 16q+6 = 16\left(\frac{\gamma^C-3}{4}\right)+6 = 4\gamma^C - 12 + 6 = 4\gamma^C - 6$

$\therefore \sum_{v\in D^C} d(v) = 4\gamma^C - 6$

(iii) $\sum_{v\in D^C} d(v) = 16q+6 = 16\left(\frac{n-4}{5}\right)+6 = \frac{16n-64}{5}+6 = \frac{16n-64+30}{5} = \frac{16n-34}{5}$

$\therefore \sum_{v\in D^C} d(v) = \frac{16n-34}{5}$

case (ii): We have $D = \{V_{5m-2}, V_{n-1}\ /\ m\in N, 1\le m\le q\}$

$D = \{v_3, v_8, v_{13}, \ldots\ldots\ldots, v_{5q-2}, v_{n-1}\}$

$D = \{v_3, v_8, v_{13}, \ldots\ldots\ldots, v_{n-5}, v_{n-1}\}$ and $\gamma = q + 1$

Then $D^C = \{v_1, v_2, v_4, v_5, v_6, v_7, \ldots\ldots\ldots, v_{n-2}, v_n\}$ *and*

$\gamma^C = n - \gamma \Rightarrow \gamma^C = 5q + 4 - q - 1 \Rightarrow \gamma^C = 4q + 3$

$\therefore\ \gamma^C = |D^C| = 4q+3$

(i) $\sum_{v\in D^C} d(v) = d(v_1) + d(v_2) + d(v_4) + \ldots\ldots\ldots + d(v_{n-2}) + d(v_n)$

$= 2 + 3 + 4 + 4 + \ldots\ldots\ldots + 4 + 2$
$= (4 + 4 + 4 + 4 + \ldots\ldots\ldots + 4 + 4) - 5$
$= 4(4q + 3) - 5$
$= 16q + 12 - 5$
$= 16q + 7$

$\therefore \sum_{v\in D^C} d(v) = 16q+7$

(ii) $\sum_{v\in D^C} d(v) = 16q+7 = 16\left(\frac{\gamma^C-3}{4}\right)+7 = 4\gamma^C - 12 + 7 = 4\gamma^C - 5$

$\therefore \sum_{v\in D^C} d(v) = 4\gamma^C - 5$

(iii) $\sum_{v\in D^C} d(v) = 16q+7 = 16\left(\frac{n-4}{5}\right)+7 = \frac{16n-64}{5}+7 = \frac{16n-64+35}{5} = \frac{16n-29}{5}$

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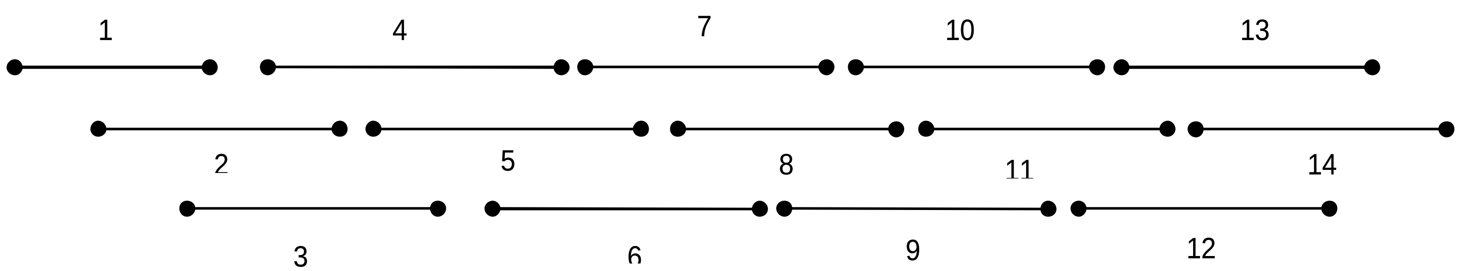
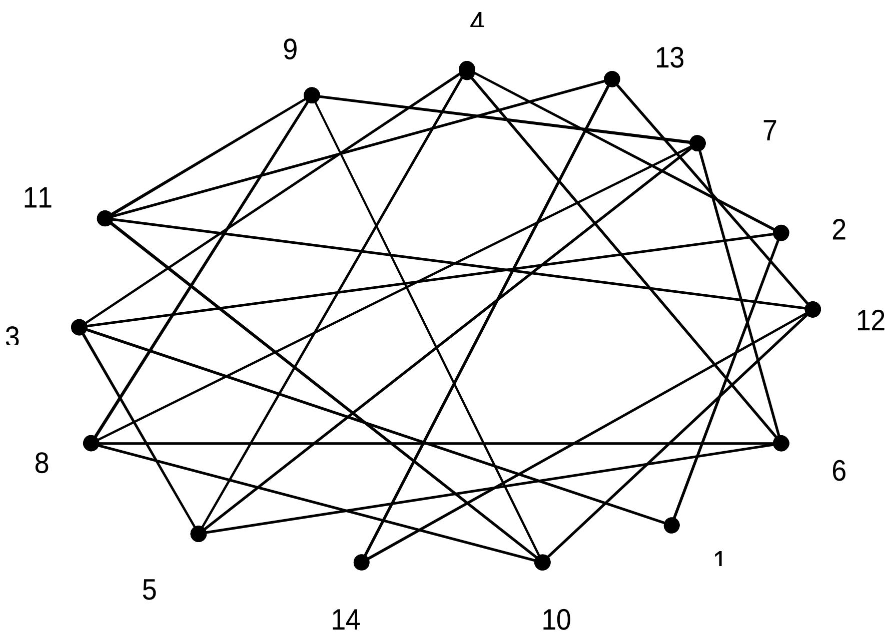
$$\therefore \sum_{v \in D^C} d(v) = \frac{16n-29}{5}$$

2) *Practical Problem:* Let I= $\{ I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9, I_{10}, I_{11}, I_{12}, I_{13}, I_{14} \}$ be an interval family and let G be an interval graph corresponding to an interval family I is as follows



INTERVAL FAMILY I



INTERVAL GRAPH G

Here n = 14 = 5 × 2 + 4
This is of the form n = 5q + r and then q = 2 and r = 4.
If r = 4, then Minimum dominating set either D = { v_3, v_8 , v_{12} } or D = { v_3, v_8 , v_{13} }and $\gamma = 3$ since theorem 5.

a) *Case(i):* Suppose D = { v_3, v_8 , v_{12} } and $\gamma = 3$

D^C = { v_1, v_2, v_4, v_5, v_6, v_7, v_9, v_{10}, v_{11},v_{13},v_{14}} and $\gamma^C = 11$

$$\sum_{v \in D^C} d(v) = d(v_1) + d(v_2) + d(v_4) + d(v_5) + d(v_6) + d(v_7) + d(v_9) + d(v_{10}) + d(v_{11}) + d(v_{13}) + d(v_{14})$$

= 2 + 3 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 3 + 2

= 38

$$\therefore \sum_{v\in D^C} d(v) = 38$$

$$(i) \sum_{v\in D^C} d(v) = 16q+6 = 16(2) + 6 = 32 + 6 = 38$$

$$\therefore \sum_{v\in D^C} d(v) = 38$$

$$(ii) \sum_{v\in D^C} d(v) = 4\gamma^C - 6 = 4(11) - 6 = 44 - 6 = 38$$

$$\therefore \sum_{v\in D^C} d(v) = 38$$

$$(iii) \sum_{v\in D^C} d(v) = \frac{16n-34}{5} = \frac{16\times 14-38}{5} = \frac{224-34}{5} = \frac{190}{5} = 38$$

$$\therefore \sum_{v\in D^C} d(v) = 38$$

b) *Case(ii):* Suppose D = { v_3, v_8 , v_{13} } and $\gamma = 3$

D^C = { v_1, v_2, v_4, v_5, v_6, v_7, v_9, v_{10}, v_{11},v_{12},v_{14}} and $\gamma^C = 11$

$$\sum_{v\in D^C} d(v) = d(v_1)+d(v_2)+d(v_4)+d(v_5)+d(v_6)+d(v_7)+d(v_9)+d(v_{10})+d(v_{11})+d(v_{12})+d(v_{14})$$

= 2 + 3 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 2

= 39

$$\therefore \sum_{v\in D^C} d(v) = 39$$

$$(i) \sum_{v\in D^C} d(v) = 16q+7 = 16(2) + 7 = 32 + 7 = 39$$

$$\therefore \sum_{v\in D^C} d(v) = 39$$

$$(ii) \sum_{v\in D^C} d(v) = 4\gamma^C - 5 = 4(11) - 5 = 44 - 5 = 39$$

$$\therefore \sum_{v\in D^C} d(v) = 39$$

$$(iii) \sum_{v\in D^C} d(v) = \frac{16n-29}{5} = \frac{16\times 14-29}{5} = \frac{224-29}{5} = \frac{195}{5} = 39$$

$$\therefore \sum_{v\in D^C} d(v) = 39$$

Hence theorem 5 is verified.

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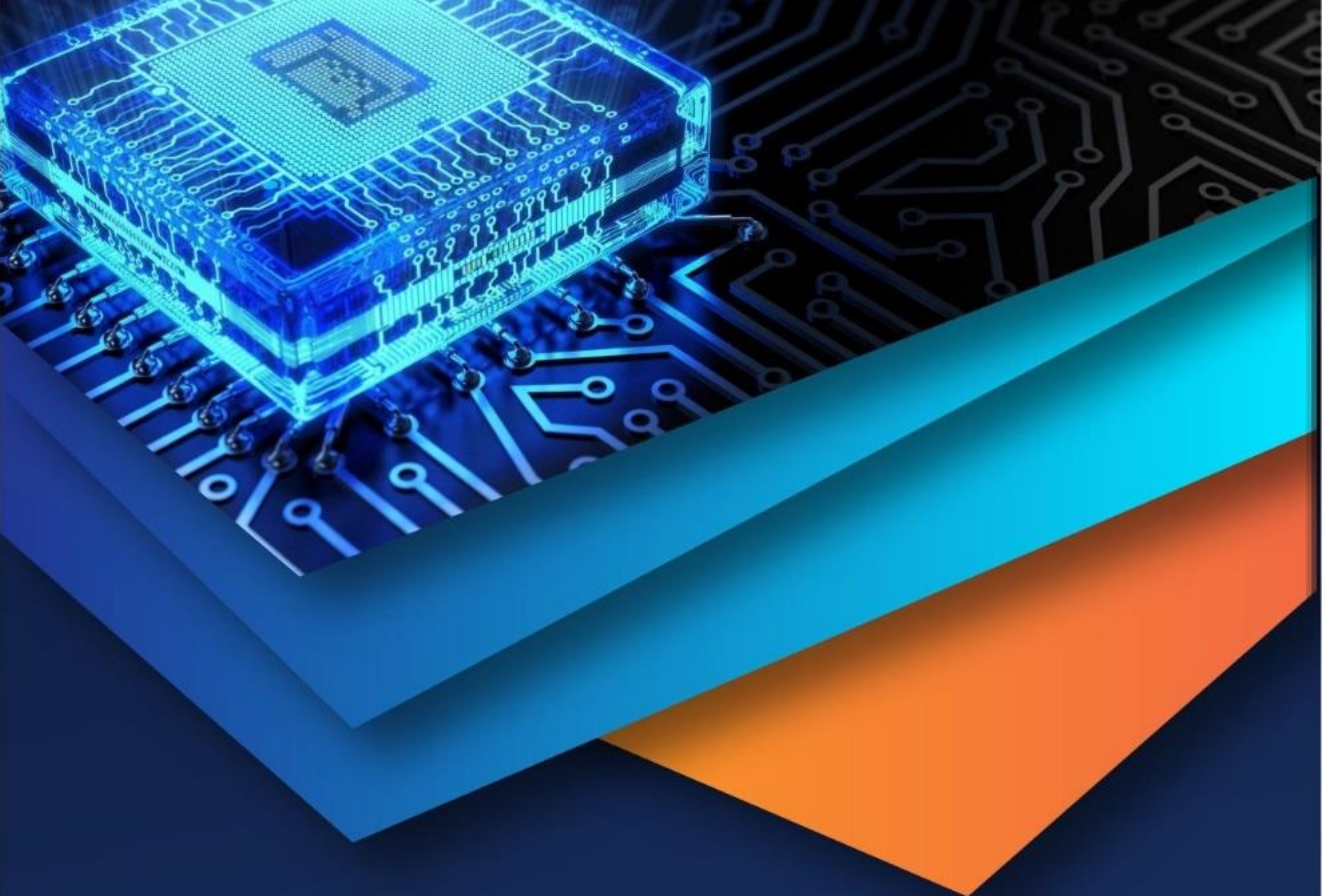

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