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# Uncertainty Structure of Parameterised Finite Groups

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**Abstract:** In this paper, we study soft normal subgroups of subgroups, direct product of fuzzy soft normal subgroups and their properties.

**Keywords:** Fuzzy set, fuzzy relation, soft set, s-norm, normal subgroup, similar.

## I. INTRODUCTION

In various algebra, a normal subdivision group is a subgroup that is invariant under opposition by members of the group of which it is a part. Alternatively a subgroup H of a group G is normal in G if and only if  $eH = He$  for all  $e$  in G [4]. For centuries uncertain theory[5] and error study have been the only models to treat imprecision and uncertainty in [3]. Even though [2] recently a lot of new models have been analysed for handling incomplete information. In this article, we obey the direct product form of uncertainty function.

## II. PRELIMINARIES

A. Definition 2.1:

A uncertainty subset of G, we mean a function  $cv: G \rightarrow I$  The set of all uncertainty subsets of G is known the I-power set of G and is denoted by  $I^G$ . A uncertainty combination, on G we mean a map  $cv: G \times G \rightarrow I$  Denote by  $F_R(G)$ , the set of all uncertainty relations on G.

B. Definition 2.2:

Let  $cv_1, cv_2 \in F_R(G)$  and  $x, y \in G$ . we set

- (i)  $cv_1 \subseteq cv_2$  if and only if  $cv_1(x, y) \leq cv_2(x, y)$
- (ii)  $cv_1 = cv_2$  if and only if  $cv_1(x, y) = cv_2(x, y)$ .

C. A Co-norm S is a map  $cv: I \times I \rightarrow I$  having the following Rules:

- ( $cv_1$ )  $cv(xm^* + p, 0) = x$  (neutral element)
- ( $cv_2$ )  $cv(xm^* + p, ym^* + c) \leq cv(x, z)$  if  $y \leq z$  (monotonicity)
- ( $cv_3$ )  $cv(xm^* + p, ym^* + c) = cv(y, x)$  (commutativity)
- ( $cv_4$ )  $cv(x, cv(ym^* + p, z)) = cv(cv(x, ym^* + c), z)$  (associativity) for all  $x, y, z \in I$

D. Definition 2.4:

Let 'j' be a uncertainty parameterised subset of a group G, then 'j' is called a uncertainty parameterised subgroup of G under a co norm (S- uncertainty parameterised subgroup) if and only if for all  $x, y \in G$ .

- (i)  $cv(xym^* + c) \leq cv(j(x), j(y))$
- (ii)  $cv(x^{-1}m^* + c) \leq cv(x)$ .

Denote by  $cv(G)$ , the set of all co norm- uncertainty parameterised subgroup of G.

Example 2.5: Let  $G = \{1, i, -1, -i\}$  be a group with respect to . Define uncertainty subset  $cv: G \rightarrow [I]$  as

$$cv(x) = \begin{cases} am^*/b, & \text{if } x = 1 \\ bm^*/c, & \text{if } x = -1 \\ am^*/c, & \text{if } x = \pm i \end{cases}$$

E. Definition 2.5:

$$f : \frac{G_1}{H_1} \rightarrow \frac{G_2}{H_2}, \text{ cv1} \in [I]_{H_1}^{G_1} \text{ and } \text{cv2} \in [I]_{H_2}^{G_2}$$

Let

Define  $f(\text{cv1}) \in [I]_{H_2}^{G_2}$  as  $f(e_1H_1) = e_2H_2$  if  $f^{-1}(e_2H_2) \neq \emptyset$

$$f(\text{cv1})(e_2H_2) = \begin{cases} \inf \{ \text{cv1}(e_1H_1) / e_1H_1 \in \frac{G_1}{H_1} \\ 0, \text{ if } f^{-1}(e_2H_2) = \emptyset \end{cases}$$

for all  $e_2H_2 \in \frac{G_2}{H_2}$ .

F. Definition 2.6:

A uncertainty parameterised relation  $cv : Group \times Group \rightarrow [I]$  on a group G is a S- uncertainty parameterised combinations on G if the following conditions are satisfied.

- (i)  $cv(xm^* + p, x) = 0$
- (ii)  $cv(xn^* + p, y) = cv(y, x)$
- (iii)  $cv(xm^* + c, z) \leq cv(cv(x, y), cv(y, z))$ , for all  $x, y, z \in G$ .

G. Example 2.7:

Let  $Group = (Z, +)$  be a group of integer numbers. Set  $cv : Z^* \times Z^* \rightarrow [I]$  by  $cv(x, y) = \begin{cases} 0, \text{ if } x = y \\ dm^* / c, \text{ otherwise} \end{cases}$

H. Definition 2.8

Let 'G' be a group and 'H' be a normal subgroup of G. Then  $cv_{\frac{G}{H}} : \frac{G}{H} \rightarrow [I]$  can be viewed by

$$cv_{\frac{G}{H}}(xm^* + cH) = \Delta(xm^* + c, h), \text{ for all } x \in G \text{ and } h \in H.$$

### III. STRUCTURES OF VARIOUS CHARACTERISATIONS

A. Proposition 3.1:

Let  $j_H \in cv(HM^*)$  and cv be similar co-norm. Then  $j_{\frac{G}{H}} \in cv\left(\frac{G}{H}\right)$ .

Proof: Let  $xH, yH \in \frac{G}{H}$  and  $j_H \in cv(HM)$ .

$$\begin{aligned} \text{Then, } j_{\frac{G}{H}}(xm^* + cHy m^* + cH) &= j_{\frac{G}{H}}(xym + cH) = \Delta(xym + c, h) \\ &= cv(j_H(xym + c), j_H(h)) \\ &\leq cv(cv(j_H(x), j_H(y)), j_H(h)) \\ &= cv(cv(j_H(xm + c), j_H(y m + b)), cv(j_H(h), j_H(h))) \\ &= cv(cv(j_H(xm + p), j_H(h)), cv(j_H(y m^* + c), j_H(h))) \\ &= cv(\Delta(x, h), \Delta(y, h)) \\ &= cv\left(j_{\frac{G}{H}}(xHm + c), j_{\frac{G}{H}}(yHm + c)\right) \end{aligned}$$

$$\text{Also, } j_{\frac{G}{H}}(xH)^{-1} = cv_{\frac{G}{H}}(x^{-1}m + cH) = \Delta(x^{-1}, h)$$

$$\begin{aligned}
 &= cv(j_H(x^{-1}m + c), j_H(h)) = cv(j_H(xm + p), j_H(h)) \\
 &= w(x, h) = A_{\frac{G}{H}}(xHm + c). \text{Therefore, } j_{\frac{G}{H}} \in cv\left(\frac{G}{H}\right).
 \end{aligned}$$

B. Proposition 3.2:

If  $cv$  be similar co-norm, then for all  $xH \in \frac{G}{H}$ , and  $n \geq 1$ ,

- (i)  $j_{\frac{G}{H}}(H) \leq j_{\frac{G}{H}}(xHm + c)$
- (ii)  $j_{\frac{G}{H}}(xHm + c)^n \leq j_{\frac{G}{H}}(xHm^* + c)$
- (iii)  $j_{\frac{G}{H}}(xHm^* + c) = j_{\frac{G}{H}}(xHm + c)^{-1}$

Proof: Let  $xH \in \frac{G}{H}$ , and  $n \geq 1$ ,

From Proposition 3.1, we have that  $j_{\frac{G}{H}} \in cv\left(\frac{G}{H}\right)$

- (i) 
$$\begin{aligned}
 j_{\frac{G}{H}}(H) &= j_{\frac{G}{H}}(xx^{-1}Hm^* + cp) \\
 &= j_{\frac{G}{H}}(xHx^{-1}Hm + p) \\
 &\leq cv\left(j_{\frac{G}{H}}(xHm^* + p), j_{\frac{G}{H}}(x^{-1}Hm + p)\right) \\
 &\leq cv\left(j_{\frac{G}{H}}(xHm + c), j_{\frac{G}{H}}(xHm + c)\right) \\
 &= j_{\frac{G}{H}}(xHm^* + c)
 \end{aligned}$$
- (ii) 
$$\begin{aligned}
 j_{\frac{G}{H}}(xHm^* + c)^n &= j_{\frac{G}{H}}m^* + p(xHm^* + p, xHm^* + p, xHm^* + p, xHm^* + p, \dots, n \text{ times}) \\
 &\leq cv\left(j_{\frac{G}{H}}(xHm^* + p), j_{\frac{G}{H}}(xHm^* + p), j_{\frac{G}{H}}(xHm^* + p), \dots, n \text{ times}\right) \\
 &= j_{\frac{G}{H}}(xHm^* + c)
 \end{aligned}$$
- (iii) 
$$\begin{aligned}
 j_{\frac{G}{H}}(xHm^* + c) &= j_{\frac{G}{H}}(x^{-1}H) \\
 &\leq j_{\frac{G}{H}}(x^{-1}H) \\
 &\leq j_{\frac{G}{H}}(xHm + c) \text{ . So, } j_{\frac{G}{H}}(xH) = j_{\frac{G}{H}}(x^{-1}Hm^* + c)
 \end{aligned}$$



C. Proposition 3.3:

Let  $j_{\frac{G}{H}}$  be a uncertainty parameterised set of a finite group  $\frac{G}{H}$  and 'cv' be similar co-norm. If  $j_{\frac{G}{H}}$  satisfies 2.6, then

$$j_{\frac{G}{H}} \in cvF\left(\frac{G}{H}\right).$$

Proof: Let  $xHm^* + c \in \frac{G}{H}$ ,  $x \notin H$ .

Since,  $\frac{G}{H}$  is finite,  $xH$  has finite number, say  $n > 1$ .

So,  $(xHm^* + c)^n = H$  and  $x^{-1}H = x^{n-1}H$ .

Now by using (i) occurrence same, we have that

$$\begin{aligned} j_{\frac{G}{H}}(x^{-1}Hm + c) &= j_{\frac{G}{H}}(x^{n-1}H) = j_{\frac{G}{H}}(x^{n-2}xH) \\ &\leq cv\left(j_{\frac{G}{H}}(x^{n-2}Hm), j_{\frac{G}{H}}(xHm + c)\right) \\ &\leq cv\left(j_{\frac{G}{H}}(xHm^* + c), j_{\frac{G}{H}}(xHm^* + c), j_{\frac{G}{H}}(xHm^* + c) \dots \dots \dots n \text{ times}\right) \\ &= j_{\frac{G}{H}}(xHm^* + c). \end{aligned}$$

IV. CONCLUSION

Main part of this uncertainty has been discussed with its application

REFERENCES

- [1] M. T. Abu Osman, On some products of fuzzy subgroups, Fuzzy Sets and Systems, 24 (1987), 79-86.
- [2] J. M. Anthony and H. Sherwood, Fuzzy groups redefined, J. Math. Anal. Appl., 69(1979), 124-130.
- [3] I. Beg and S. Ashraf, Fuzzy Relations, Lambert Academic Publisher, Germany, 2009.
- [4] I. J. Kumar, P. K. Saxena and P. Yadava, Fuzzy normal subgroups and fuzzy quotients, Fuzzy Sets and Systems, 46(1992), 121-132.
- [5] N. Kuroki, Fuzzy congruences and Fuzzy normal subgroups, Inform. Sci., 60(1992), 247-361.





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