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Integrals of Four Variables with Statistical Distribution associated with hyper geometric Function of Matrix Argument

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Abstract: In this chapter, we evaluated ten integrals associated with hypergeometric function of four variables of matrix argument with their statistical distribution. All the matrix involved were real positive definite symmetric of order m x m.
Keywords: Hypergeometric function, Matrix argument

I. INTRODUCTION

In this paper, we have evaluated forty integrals associated with hypergeometric function of four variables of matrix argument with their statistical distribution. All the matrices involved are real positive definite symmetric of order m x m. We will start with introducing matrix sequences, matrix series, and concepts analogous to convergence of series in scales variable. A matrix series is obtained by adding up the matrices in a matrix sequence. For example if A_0, A_1, A_2 ……… is matrix series given by

$$F(A) = \sum_{k=0}^{\infty} A_k \qquad (1.1)$$

If the matrix series is a power series than we will be considering powers of matrices and hence in this case the series will be define only for n x n matrices,for an n x n matrix A. consider the series.

$$g(A) = \sum_{k=0}^{\infty} a_k\, A_k \ldots\ldots\ldots\ldots\ldots \qquad (1.2)$$

where $a_0, a_1, \ldots\ldots a_k$ are scalars.

As in the case of scalars series, convergence of a matrix will be defined in terms of the sums.

A general hypergeometric series ${}_pF_q(.)$ in a real scalar variable X is defined as following :

$${}_pF_q(a_1, \ldots\ldots\ldots a_p ; b_1, \ldots\ldots\ldots b_q; X) = \sum_{r=0}^{\infty} \frac{(a_1)_r \ldots\ldots\ldots (a_p)_r\, X^r}{(b_1)_r \ldots\ldots\ldots (b_q)_r\, r!} \qquad (1.3)$$

For $(a)_m = a(a+1) \ldots\ldots (a+m-1)$

$(a)_0 = 1 \qquad a \neq 0$

For example: ${}_0F_0(\,;\,; X) = e^x$

${}_1F_0(\alpha;\,; X) = (I - x)^{-\alpha}$ for $|X| < 1$

In (1.3) there are p upper parameters $a_1, \ldots\ldots, a_p$ and q lower parameters $b_1, \ldots\ldots\ldots, b_q$. The series in (1.3) is convergent for all $|X| < \infty$ ifq $\geq$p, convergent for |X|<I if p = q+1, divergent for all X, $X \neq 0$,if p> q+1 and the convergence. Condition for X = 1 and X = -1 can also be worked out. A matrix series is in n x n matrix. A corresponding to the right side in (1.3) is obtained by replacing X by A, thus we may define a hypergeometric series in an n x n matrix. A as follows.

$${}_pF_q(a_1, \ldots\ldots\ldots a_p ; b_1, \ldots\ldots\ldots b_q; A) = q \geq p, \sum_{r=0}^{\infty} \frac{(a_1)_r \ldots\ldots\ldots (a_p)_r\, X^r}{(b_1)_r \ldots\ldots\ldots (b_q)_r\, r!} \qquad (1.4)$$

where $a_1, \ldots\ldots\ldots a_p, b_1 \ldots\ldots\ldots\ldots b_q$ are scalars.

The series in (1.4) is convergent for all A is q $\geq$ p convergent for p = q + 1 when the Eigen values of A are all less than 1 in absolute value and divergent when p > q + 1.

Similarly, it may be defined for two, three and four variables. Other definition involving in this chapter for four variables of m x m matrix. Exactly similar analogous in scalar variable due to Exton (1985). In what follows we shall take p, q, r, and s to be positive integers of the symbols X and Δ (n, a) stand the sequence of parameters of square positive definite matrices $X_1, X_2, \ldots\ldots X_r$ of order n x n and $\frac{\alpha}{n}, \frac{\alpha+1}{n}, \ldots\ldots \frac{\alpha+n-1}{n}$ respectively.

Also, in all be established here after, proper conditions of convergence of the involved are assumed. In which follows X, Y, Z, T, U etc. matrices are positive definite symmetric of same order m x m.

II. INTEGRALS OF FOUR VARIABLES WITH STATISTICAL DISTRIBUTION ASSOCIATED WITH HYPERGEOMETRIC FUNCTION OF MATRIX ARGUMENT

The formulae to be established are as following :

1) *Result 1:* $\int_0^I U^{a-\frac{m+1}{2}} (I-U)^{b-\frac{m+1}{2}} F_1(a, 1-a; c; \frac{U}{2}).$

$$F_1^4\left[a_1, a_1, a_1, a_2, b_1, b_1, b_1, b_1, c_1, c_2, c_3, c_4; X(I-U)^n, Y(I-U)^n, Z(I-U)^n, T(I-U)^n\right] dU$$

$$= LF_1^4\left[\begin{matrix} a_1, a_1, a_1, a_2, b_1, b_1, b_1, b_1, c_1, c_2, c_3, c_4; \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta(\frac{n}{2}, c+b+a), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X, Y, Z, T \end{matrix}\right] \quad (2.1)$$

where

$$F_1^4\left[a_1, a_1, a_1, a_2, b_1, b_1, b_1, b_1, c_1, c_2, c_3, c_4; (I-U)^n X, (I-U)^n Y, (I-U)^n Z, (I-U)^n T\right]$$

$$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q+r}(a_2)_s (b_1)_{p+q+r+s}}{p!q!r!s!(c_1)_p (c_2)_q (c_3)_r (c_4)_s} (I-U)^{np} X^p (I-U)^{nq} Y^q (I-U)^{nr} Z^r (I-U)^{ns} T^s$$

so that

$$F_1^4\left[a_1, a_1, a_1, a_2, b_1, b_1, b_1, b_1, c_1, c_2, c_3, c_4; (I-U)^n X, (I-U)^n Y, (I-U)^n Z, (I-U)^n T\right]$$

$$= F_1^4\left[(I-U)^n X, (I-U)^n Y, (I-U)^n Z, (I-U)^n T\right]$$

Then a probability density function (p.d.f.) of (3.2.1.) is given by: $F(U) = \dfrac{U^{a-\frac{m+1}{2}} (I-U)^{b-\frac{m+1}{2}} F_1(\chi_1) F_1^4(\chi_2)}{LF_1^4(\chi_3)}$

= 0 else where

where $\chi_1 = (a, a-1; c; \frac{U}{2})$

$$\chi_2 = \left[(I-U)^n X, (I-U)^n Y, (I-U)^n Z, (I-U)^n T\right]$$

$$\chi_1 = \left[\begin{matrix} a_1, a_1, a_2, a_1, b_1, b_1, b_1, b_1, c_1, c_2, c_3, c_4; \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta(\frac{n}{2}, c+b+a), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X, Y, Z, T \end{matrix}\right]$$

2) *Result 2:* $\int_0^I U^{a-\frac{m+1}{2}} (I-U)^{b-\frac{m+1}{2}} F_1(a, 1-a; c; \frac{U}{2}). F_2^4\left[(I-U)^n X, (I-U)^n Y, (I-U)^n Z, (I-U)^n T\right] dU$

$$= LF_2^4\left[\begin{matrix} a_1, a_1, a_2, a_2, b_1, b_1, b_1, b_1, c_1, c_2, c_3, c_4; \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta(\frac{n}{2}, c+b+a), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X, Y, Z, T \end{matrix}\right] \quad (2.2)$$

where

$$F_2^4\left(a_1, a_1, a_2, a_2, b_1, b_1, b_1, b_1, c_1, c_2, c_3, c_4; (I-U)^n X, (I-U)^n Y, (I-U)^n Z, (I-U)^n T\right)$$

$$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q}(a_2)_{r+s}(b_1)_{p+q+r+s}}{p!q!r!s!(c_1)_p(c_2)_q(c_3)_r(c_4)_s}(I-U)^{np}X^p(I-U)^{nq}Y^q(I-U)^{nr}Z^r(I-U)^{ns}T^s$$

Then a probability density function (p.d.f.) of (3.2.2) is given by: $F(U) = \dfrac{U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}}F_1(\chi_1)F_2^4(\chi_2)}{LF_2^4(\chi_4)}$

= 0 else where

where $\chi_4 = \begin{bmatrix} a_1, a_1, a_2, a_2, b_1, b_1, b_1, b_1, c_1, c_2, c_3, c_4; \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}) \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta(\frac{n}{2}, c+b+a), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X, Y, Z, T \end{bmatrix}$

3) *Result 3:* $\int_0^I U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}} F_1(a, 1-a; c; \frac{U}{2}).F_3^4\left[(I-U)^n X, (I-U)^n Y, (I-U)^n Z, (I-U)^n T\right]dU$

$$= LF_3^4\begin{bmatrix} a_1, a_1, a_2, a_3, b_1, b_1, b_1, b_1, c_1, c_2, c_3, c_4; \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta(\frac{n}{2}, c+b+a), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X, Y, Z, T \end{bmatrix} \quad (2.3)$$

where

$$F_3^4\left(a_1, a_1, a_2, a_3, b_1, b_1, b_1, b_1, c_1, c_2, c_3, c_4; (I-U)^n X, (I-U)^n Y, (I-U)^n Z, (I-U)^n T\right)$$

$$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q}(a_2)_{r+s}(b_1)_{p+q+r+s}}{p!q!r!s!(c_1)_p(c_2)_q(c_3)_r(c_4)_s}(I-U)^{np}X^p(I-U)^{nq}Y^q(I-U)^{nr}Z^r(I-U)^{ns}T^s$$

Then a (p.d.f.) of (2.3) is given by : $F(U) = \dfrac{U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}}F_1(\chi_1)F_3^4(\chi_2)}{LF_3^4(\chi_5)}$

= 0 else where

where $\chi_5 = \begin{bmatrix} a_1, a_1, a_2, a_3, b_1, b_1, b_1, b_1, c_1, c_2, c_3, c_4; \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}) \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta(\frac{n}{2}, c+b+a), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X, Y, Z, T \end{bmatrix}$

4) *Result 4:* $\int_0^I U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}} F_1(a, 1-a; c; \frac{U}{2}).F_4^4\left[(I-U)^n X, (I-U)^n Y, (I-U)^n Z, (I-U)^n T\right]dU$

$$= LF_4^4\begin{bmatrix} a_1, a_1, a_1, a_1, b_1, b_1, b_1, b_2, c_1, c_2, c_3, c_4; \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta(\frac{n}{2}, c+b+a), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X, Y, Z, T \end{bmatrix} \quad (2.4)$$

where $F_4^4\left(a_1, a_1, a_1, a_1, b_1, b_1, b_2, b_1, c_1, c_2, c_3, c_4; X, Y, Z, T\right)$

$$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q+r+s}(b_1)_{p+q+s}(b_2)_r}{p!q!r!s!(c_1)_p(c_2)_q(c_3)_r(c_4)_s} X^pY^qZ^rT^s$$

Then a (p.d.f.) of (2.4) is given by : $F(U) = \dfrac{U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}} F_1(\chi_1)F_4^4(\chi_2)}{LF_4^4(\chi_6)}$

= 0 else where

$$\text{where } \chi_6 = \left[\begin{matrix} a_1, a_1, a_1, a_1, b_1, b_1, b_2, b_1, c_1, c_2, c_3, c_4; \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}) \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta(\frac{n}{2}, c+b+a), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X, Y, Z, T \end{matrix}\right]$$

5) *Result5:* $\int_0^I U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}} F_1(a, 1-a; c; \frac{U}{2}).F_5^4\left[(I-U)^nX, (I-U)^nY, (I-U)^nZ, (I-U)^nT\right]dU$ =

$$LF_5^4\left[\begin{matrix} a_1, a_1, a_1, a_2, b_1, b_1, b_2, b_2, c_1, c_2, c_3, c_4; \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta(\frac{n}{2}, c+b+a), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X, Y, Z, T \end{matrix}\right] \quad (2.5)$$

where $F_5^4\left(a_1, a_1, a_1, a_2, b_1, b_1, b_2, b_2, c_1, c_2, c_3, c_4; X, Y, Z, T\right)$

$$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q+r}(a_2)_s(b_1)_{p+q}(b_2)_{r+s}}{p!q!r!s!(c_1)_p(c_2)_q(c_3)_r(c_4)_s} X^pY^qZ^rT^s$$

Then a (p.d.f.) of (2.5) is given by : $F(U) = \dfrac{U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}} F_1(\chi_1)F_5^4(\chi_2)}{LF_5^4(\chi_7)}$

= 0 else where

$$\text{where } \chi_7 = \left[\begin{matrix} a_1, a_1, a_1, a_2, b_1, b_1, b_2, b_2, c_1, c_2, c_3, c_4; \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}) \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta(\frac{n}{2}, c+b+a), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X, Y, Z, T \end{matrix}\right]$$

6) *Result 6:* $\int_0^I U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}} F_1(a, 1-a; c; \frac{U}{2}).F_6^4\left[(I-U)^nX, (I-U)^nY, (I-U)^nZ, (I-U)^nT\right]dU$

$$= LF_6^4\left[\begin{matrix} a_1, a_1, a_1, a_2, b_1, b_2, b_3, b_1, c_1, c_2, c_3, c_4; \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta(\frac{n}{2}, c+b+a), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X, Y, Z, T \end{matrix}\right] \quad (2.6)$$

where $F_6^4\left(a_1,a_1,a_1,a_2,b_1,b_2,b_3,b_1,c_1,c_2,c_3,c_4;X,Y,Z,T\right)$

$$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q+r}(a_2)_s(b_1)_{p+s}(b_2)_q(b_3)_s}{p!q!r!s!(c_1)_p(c_2)_q(c_3)_r(c_4)_s} X^p Y^q Z^r T^s$$

Then a (p.d.f.) of (3.2.6) is given by : $F(U) = \dfrac{U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}} F_1(\chi_1) F_6^4(\chi_2)}{LF_6^4(\chi_8)}$

= 0 else where

$$\text{where } \chi_8 = \begin{bmatrix} a_1, a_1, a_1, a_2, b_1, b_2, b_3, b_1, c_1, c_2, c_3, c_4; \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}) \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta(\frac{n}{2}, c+b+a), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X, Y, Z, T \end{bmatrix}$$

7) *Result 7:* $\int_0^I U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}} F_1(a, 1-a; c; \frac{U}{2}).F_7^4\left[(I-U)^n X, (I-U)^n Y, (I-U)^n Z, (I-U)^n T\right] dU$

$$= LF_7^4 \begin{bmatrix} a_1, a_1, a_2, a_2, b_1, b_2, b_1, b_2, c_1, c_2, c_3, c_4; \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta(\frac{n}{2}, c+b+a), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X, Y, Z, T \end{bmatrix} \quad (2.7)$$

where $F_7^4(a_1,a_1,a_2,a_2,b_1,b_2,b_1,b_2,c_1,c_2,c_3,c_4;X,Y,Z,T)$

$$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q}(a_2)_{r+s}(b_1)_{p+r}(b_2)_q(b_3)_s}{p!q!r!s!(c_1)_p(c_2)_q(c_3)_{r+s}(c_4)_s} X^p Y^q Z^r T^s$$

Then a (p.d.f.) of (3.2.7) is given by : $F(U) = \dfrac{U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}} F_1(\chi_1) F_7^4(\chi_2)}{LF_7^4(\chi_9)}$

= 0 else where

$$\text{where } \chi_9 = \begin{bmatrix} a_1, a_1, a_2, a_2, b_1, b_2, b_1, b_2, c_1, c_2, c_3, c_4; \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}) \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta(\frac{n}{2}, c+b+a), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X, Y, Z, T \end{bmatrix}$$

8) *Result 8:* $\int_0^I U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}} F_1(a, 1-a; c; \frac{U}{2}).F_8^4\left[(I-U)^n X, (I-U)^n Y, (I-U)^n Z, (I-U)^n T\right] dU$

$$= LF_8^4 \begin{bmatrix} a_1, a_1, a_2, a_2, b_1, b_2, b_1, b_3, c_1, c_2, c_3, c_4; \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta(\frac{n}{2}, c+b+a), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X, Y, Z, T \end{bmatrix} \quad (2.8)$$

where $F_8^4(a_1,a_1,a_2,a_2,b_1,b_2,b_1,b_3,c_1,c_2,c_3,c_4;X,Y,Z,T)$

$$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q}(a_2)_{r+s}(b_1)_{p+r}(b_2)_q(b_3)_s}{p!q!r!s!(c_1)_p(c_2)_q(c_3)_r(c_4)_s} X^p Y^q Z^r T^s$$

Then a (p.d.f.) of (2.8) is given by : $F(U) = \dfrac{U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}}F_1(\chi_1)F_8^4(\chi_2)}{LF_8^4(\chi_{10})}$

= 0 else where

$$\text{where } \chi_{10} = \left[\begin{array}{l} a_1, a_1, a_2, a_2, b_1, b_2, b_3, b_4, c_1, c_2, c_3, c_4; \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta(\frac{n}{2}, c+b+a), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X, Y, Z, T \end{array}\right]$$

9) *Result 9:* $\int_0^I U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}} F_1(a, 1-a; c; \frac{U}{2}).F_9^4\left[(I-U)^n X, (I-U)^n Y, (I-U)^n Z, (I-U)^n T\right]dU$

$$= LF_9^4\left[\begin{array}{l} a_1, a_1, a_1, a_2, b_1, b_1, b_1, b_1, c_1, c_2, c_3, c_1; \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta(\frac{n}{2}, c+b+a), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X, Y, Z, T \end{array}\right] \quad (2.9)$$

where $F_9^4(a_1,a_1,a_1,a_2,b_1,b_1,b_1,b_1,c_1,c_2,c_3,c_1;X,Y,Z,T)$

$$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q+r}(a_2)_s(b_1)_{p+q+r+s}}{p!q!r!s!(c_1)_{p+s}(c_2)_q(c_3)_r} X^p Y^q Z^r T^s$$

Then a (p.d.f.) of (2.9) is given by : $F(U) = \dfrac{U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}}F_1(\chi_1)F_9^4(\chi_2)}{LF_9^4(\chi_{11})}$

= 0 else where

$$\text{where } \chi_{11} = \left[\begin{array}{l} a_1, a_1, a_1, a_2, b_1, b_1, b_1, b_1, c_1, c_2, c_3, c_1; \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta(\frac{n}{2}, c+b+a), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X, Y, Z, T \end{array}\right]$$

10) *Result 10:* $\int_0^I U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}} F_1(a, 1-a; c; \frac{U}{2}).F_{10}^4\left[(I-U)^n X, (I-U)^n Y, (I-U)^n Z, (I-U)^n T\right]dU$

$$= LF_{10}^4\left[\begin{array}{l} a_1, a_1, a_2, a_2, b_1, b_1, b_1, b_1, c_1, c_2, c_1, c_3; \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta(\frac{n}{2}, c+b+a), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X, Y, Z, T \end{array}\right] \quad (2.10)$$

where $F_{10}^4(a_1,a_1,a_2,a_2,b_1,b_1,b_1,b_1,c_1,c_2,c_1,c_3;X,Y,Z,T)$

$$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q}(a_2)_{r+s}(b_1)_{p+q+r+s}}{p!q!r!s!(c_1)_{p+r}(c_2)_q(c_3)_s} X^p Y^q Z^r T^s$$

Then a p.d.f. $F(U) = \dfrac{U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}}F_1[\chi_1]F_{10}^4[\chi_2]}{LF_{10}^4[\chi_{12}]}$

= 0 else where

$$\text{where } \chi_{12} = \left[\begin{array}{l} a_1, a_1, a_2, a_2, b_1, b_1, b_1, b_1, c_1, c_2, c_1, c_3; \Delta(n,b), \Delta(\frac{n}{2}, \frac{c+b}{2}), \\ \Delta(\frac{n}{2}, \frac{1+c+b}{2}), \Delta(n, c+b), \Delta(\frac{n}{2}, c+b+a), \Delta(\frac{n}{2}, \frac{1+c+b-a}{2}); X, Y, Z, T \end{array}\right]$$

III. SOLUTION OF INTEGRALS

One of the proofs is expressing the quadruple hypergeometric function in terms of equivalent series, in the integrand of the (2.1). We find that the integral becomes

$$\int_0^I U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}}\,F_1(a,1-a;c;\frac{U}{2})$$

$$F_1^4\left[a_1,a_1,a_1,a_2,b_1,b_1,b_1,b_1,c_1,c_2,c_3,c_4;(I-U)^nX,(I-U)^nY,(I-U)^nZ,(I-U)^nT\right]dU$$

or

$$\int_0^I U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}}\,F_1(a,1-a;c;\frac{U}{2})\sum_{p,q,r,s=0}^{\infty}A_{r,s}^{p,q}\left[(I-U)^nX,(I-U)^nY,(I-U)^nZ,(I-U)^nT\right]dU \tag{3.1}$$

where $A_{r,s}^{p,q}$ stands for the expression

$$\sum_{p,q,r,s=0}^{\infty}\frac{(a_1)_{p+q+r}(a_2)_s(b_1)_{p+q+r+s}}{p!q!r!s!(c_1)_p(c_2)_q(c_3)_r(c_4)_s}\left[(I-U)X\right]^p\left[(I-U)Y\right]^q\left[(I-U)Z\right]^r\left[(I-U)T\right]^s$$

We assume that the series is uniformly convergent in the region of integration, the inversion of integration and summation is infinite, then integral

$$\int_0^I U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}}\,F_1(a,1-a;c;\frac{U}{2})\,A_{r,s}^{p,q}\left[X^p(I-U)^{p\ n}Y^q(I-U)^{q\ n}Z^r(I-U)^{m}T^s(I-U)^{sn}\right]dU \tag{3.2}$$

$$=\sum_{p,q,r,s=0}^{\infty}A_{r,s}^{p,q}X^pY^qZ^rT^s\int_0^I U^{a-\frac{m+1}{2}}(I-U)^{b-\frac{m+1}{2}}\,F_1(a,1-a;c;\frac{U}{2}).\left[(I-U)^{n(p+q+r+s)}\right]dU$$

$$=\sum_{p,q,r,s=0}^{\infty}A_{r,s}^{p,q}X^pY^qZ^rT^s\int_0^I U^{a-\frac{m+1}{2}}(I-U)^{b+n(p+q+r+s)\,\frac{m+1}{2}}\,F_1(a,1-a;c;\frac{U}{2})dU$$

$$=\sum_{p,q,r,s=0}^{\infty}A_{r,s}^{p,q}X^pY^qZ^rT^s\,\frac{\Gamma_m(c)\Gamma_m(b+n(p+q+r+s))\,\Gamma_m\left(\frac{c+b+n+(p+q+r+s)}{2}\right)}{\Gamma_m(c+b+n(p+q+r+s))\,\Gamma_m\left[\frac{(c+b+n(p+q+r+s)-a}{2}\right]}$$

$$.\frac{\Gamma_m\left(\frac{1+c+b+n(p+q+r+s)}{2}\right)}{\Gamma_m\left[\frac{(1+c+b+n(p+q+r+s)-a}{2}\right]} \tag{3.3}$$

$$=L\sum_{p,q,r,s=0}^{\infty}\frac{(a_1)_{p+q+r}(a_2)_s(b_1)_{p+q+r+s}}{p!q!r!s!(c_1)_p(c_2)_q(c_3)_r(c_4)_s}$$

$$\frac{(b)_{n(p+q+r+s)}\left(\frac{c+b}{2}\right)_{n(p+q+r+s)}\left(\frac{1+c+b}{2}\right)_{n(p+q+r+s)}}{(c+b)_{n(p+q+r+s)}\left[\frac{(c+b+a)}{2}\right]_{\frac{n}{2}(p+q+r+s)}\left[\frac{(1+c+b-a)}{2}\right]_{\frac{n}{2}(p+q+r+s)}}X^pY^qZ^rT^s$$

By applying the below mentioned result, we get

$$(a)k_1=k^{k1}\prod_{j=1}^{k}\left\{\frac{(\alpha+j-1)}{k}\right\}$$

where k is a positive integral and non-negative, and finally after calculations, we arrive at the result (1).Similarly, other integrals from (2.2) to (2.10)are all proved. Therefore, ten direct results have been quoted and proved in a similar manner.

IV. CONCLUSION

This paper deals with classical special function and generalized hypergeometric function of matrix argument (for positive definite symmetric matrix and hermitian positive definite matrix in complex case). We derived various important formulas which have a wide range of applications in the field of Mathematical Sciences especially multivariable distribution theory.

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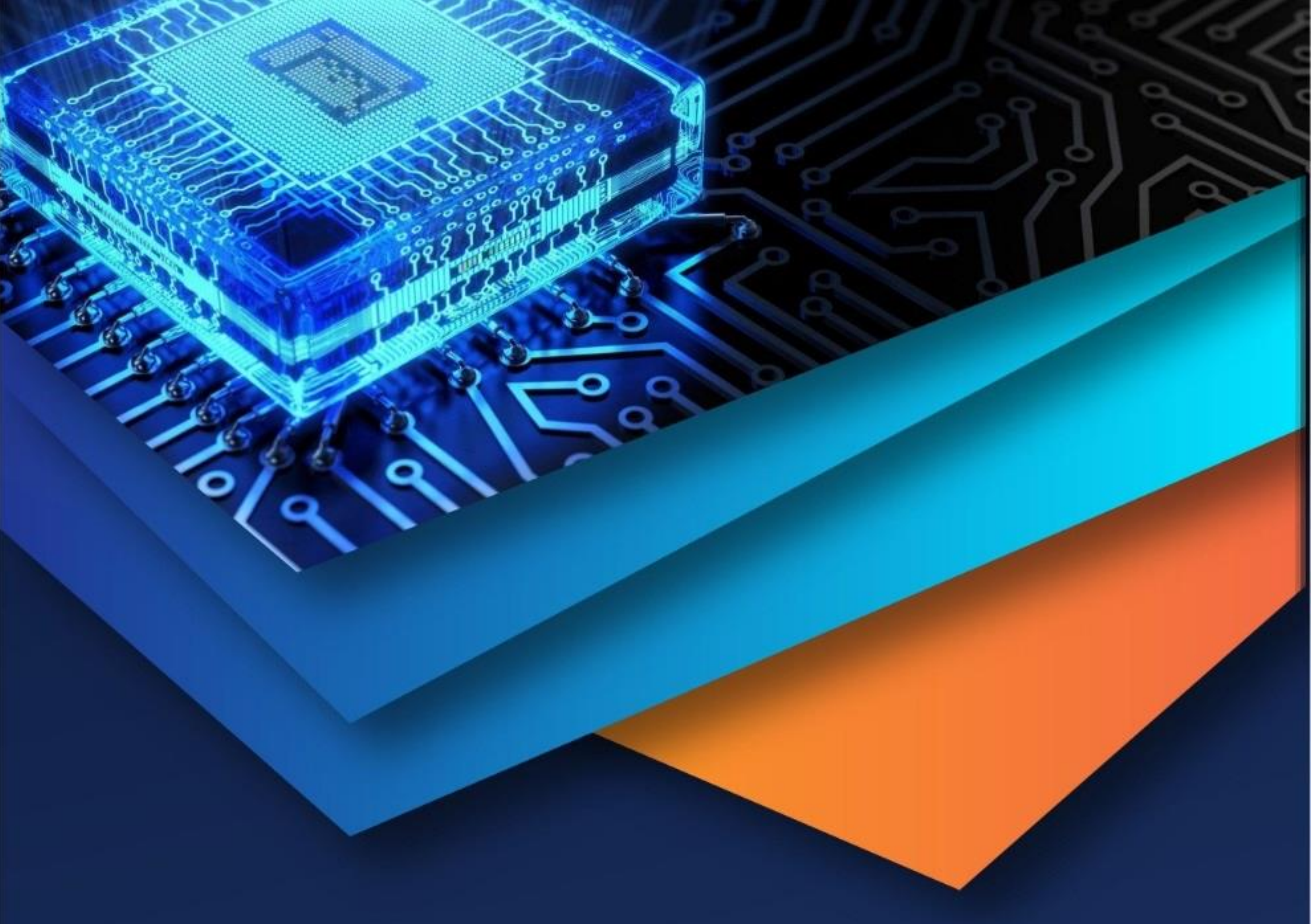

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