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FPGA Implementation of Efficient Window-Architecture Design Using Completely Scaling – Free CORDIC Algorithm

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Abstract- one of the most important step in spectral analysis is filtering, this paper presents an FPGA implementation of window function using CORDIC (co-ordinate rotation digital computer) to minimize area-delay product. Co-ordinate rotation digital computer (CORDIC) is an arithmetic technique, which performs two dimensional rotations using simple hardware components. This algorithm can be used for computing a wide range of functions including certain trigonometric, hyperbolic, linear and logarithmic functions, and can evaluate elementary functions such as cosine, sine, arctangent, sinh, cosh, tanh, ln, &exponential. Here FPGA implementation of various window functions using CORDIC algorithm by replacing linear CORDIC with multiple shift-add network and conventional circular CORDIC processor by a completely scaling free CORDIC processor to improve the area-time efficiency.
Keywords: CORDIC, field programmable gate array (FPGA), completely scaling-free , micro-rotation., pipelined.

I.   INTRODUCTION

CORDIC(Co-Ordinate Digital Computer) algorithm is a well known iterative technique to perform basic arithmetic operations which includes computation of trigonometric functions, estimation of vector magnitude etc . Commonly many hardware architecture are available for realizing FFT but the same is not true for window-architectures. The conventional hardware implementation of window functions uses lookup table which leads to area and time complexity with increase in word-length, in this they do not allow user defined variations in window – length. CORDIC is a very well-known algorithm because of its versatility and simple hardware implementation that only needs add & shifts operations. Though they allow user defined variations in window-length, latency is a major problem. By using two CORDIC processor in series, the overall latency can be hampered. In this paper, replacing conventional linear CORDIC processor with the multiple optimized shift-add network and conventional circular CORDIC with completely scaling-free CORDIC processor reduce the pipelining depth & area of the existing design which reduces the latency. The rest of this paper is organized as follows:- section II deals with the different window functions, conventional CORDIC algorithm and completely scaling-free CORDIC algorithm. Section-III deals with proposed architecture. Section-IV deals with FPGA implementation, section –V details the results, section-VI conclude the paper.

II.   BRIEF REVIEW OF THE CONVENTIONAL AND THE SCALING-FREE CORDIC

In signal processing, a window function is a mathematical function that is zero-valued outside of some chosen interval. For instance, a function that is constant inside the interval or zero elsewhere is called a rectangular window, which describes shape of its graphical representation. When another function or waveform/data-sequence is multiplied by a window function, the product is also zero-valued outside the interval; all that is left is the part where they overlap. There are different windows like hanning, hamming, blackman, kaizer etc. the windows can be selected based on spectrum or spectral characteristics.

The equation for the hanning & hamming windows are-

$w_{hann}=0.5+0.5\cos(\frac{2\pi n}{N-1})$

Where N is the length of the window.

$w_{hamm}=\alpha+\beta\cos s(\frac{2\pi n}{N-1})$

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Where β=1-α.

The constant is approximated to α=$\frac{25}{46}$, β=$\frac{21}{46}$ which achieves the side lobe cancellation.

A. Conventional CORDIC algorithm

CORDIC algorithm is based on rotation of vectors in two-dimensional coordinate space using simple shift & add operation. CORDIC algorithm can operate in two modes namely: rotation & vectoring. In rotation mode, the objective is to convert polar coordinate of a vector into Cartesian where as vice versa in vectoring mode through a series of iterations. The rotation trajectory can be linear, circular or hyperbolic depending upon the requirement. It consists of two operating modes, the rotation mode (RM), where a vector is $[x_A, y_A]$ rotated by an angle (θ) to obtain $[x_B, y_B]$ and the vectoring mode (vM) where CORDIC algorithm computes length r, angle θ towards the x-axis of a vector$[x_A, y_A]$. The coordinates of two vectors VA $[x_A, y_A]$ and VB $[x_B, y_B]$ separated by an angle θ which is related as:

$$\begin{bmatrix} xA \\ yB \end{bmatrix} = R \begin{bmatrix} xA \\ yA \end{bmatrix} \quad , \quad R=\begin{bmatrix} Cos\theta & -Sin\theta \\ Sin\theta & Cos\theta \end{bmatrix}$$

The key concept for realizing rotation using CORDIC algorithm is to express the desired rotation angle 'θ' as an aggregation of pre-defined elementary angle defined as:

$$\theta = \sum_{i=0}^{b} \mu i. \alpha i$$

Where $[x_B, y_B]$ is the final vector & 'θ' is the target angle of rotation and is expressed as the summation of $\alpha_i = tan^{-1}2^{-i}$, where 'b' is the word length and $\mu_i \epsilon \{1, -1\}$ is known as the direction of the vector rotation for its iteration.

The rotation matrix R requires, determining sine & cosine term, simplifies the rotation matrix by converting multiplication operations to shift, as the tangent of elementary angles are defined in negative power of two.

$$R_i = K_i \begin{bmatrix} 1 & -2^{-i} \\ 2^{-i} & 1 \end{bmatrix} \quad , \quad K_i = \frac{1}{\sqrt{1+2^{-2i}}}$$

The rotation matrix $R_i = K_i \begin{bmatrix} 1 & -\mu_i 2^{-i} \\ \mu_i 2^{-i} & 1 \end{bmatrix}$

Where μ_i = -1 for clockwise rotations & μ_i = 1 for anticlockwise rotations. In its original form , CORDIC algorithm suffers from major disadvantages like, scale-factor compensation , latency and optimal identification of micro-rotations.

B. Completely – scaling free CORDIC algorithm

In completely scaling-free CORDIC (coordinate rotation digital computer) algorithm, the coordinate rotation matrix is derived using 3rd order of approximation of Taylor series of sine and cosine function.

Taylor series is used for expansion of sine &cosine functions to avoid scaling operation. Suggests a generalized sequence of micro-rotation to have adequate range of the Taylor series.

To facilitate shift-add implementation of rotation matrix, the Taylor series coefficient 3! Is approximated as 2^3.

For $\alpha_i = 2^{-si}$
$$R_i = \begin{bmatrix} 1 - 2^{(2_{si}+1)} & -(2^{-si} - 2^{-(3si+3)}) \\ (2^{-si} - 2^{-(3si+3)} & 1 - 2^{-(2_{si}+1)} \end{bmatrix}$$

The elementary are defined as

$$\alpha_i = 2^{-si}$$

Where s_i is the number of shifts for the ith iteration

Depending on the order of Taylor series approximation, the largest elementary angle α_{max} & the S_{basic} (basic-shift) is used to realize the coordinate rotation matrix. $S_{basic} = 2$ and $\alpha_{max} = 0.25$ radians for 3rd order approximation. Any rotation angle, is realized by using n1 iterations of α_{max} and n2 iterations corresponding to other elementary angles , such that, the total number of iterations (n1+n2) is always a constants.

$$\theta = n1.\alpha_{max} + \sum^{n2 \ \ iterations} \alpha_{si}$$

Where $s_i > S_{basic}$ and n n1+ n2

To realize any rotation angle in the range [0,π/4], total of seven iterations are required which is extended to the entire coordinate space using the octant wave symmetry of sine and cosine function.

III.   ARCHITECTURE FOR IMPLEMENTING WINDOW FUNCTIONS

In this section, focusing on the pipelined architecture to generate window functions. The length of the window function is selected by the user at the run time. Currently, the architecture implements the hamming window, but with slight modifications it can be extended to other window functions as well. In this proposed architecture, the output bit width is set to 16-bits.

Fig-1 shows the block diagram for generating hamming window function.

The circuit consists of angle generator,(WCM) window coefficient multiplier, CSF CORDIC processor(CCP), FIFO. The angle generator generates θ & 2θ two angle value required for hamming window function. window coefficient multiplier(WCM) multiplies the input signal samples with the window constants using an shift-add network.CCP is used for generating the cosine term in window function. FIFO is used for proper synchronization between window coefficient having cosine terms and constants.
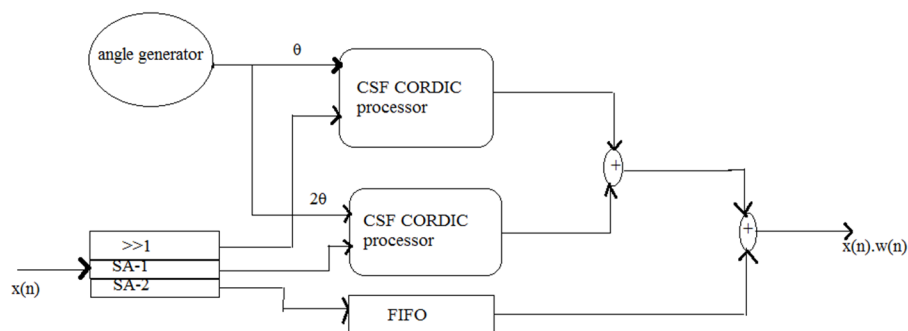

Fig 1: block diagram for generating hamming function

A.   Completely scaling free CORDIC processor(CCP)
The CCP is pipelined implementation of the proposed completely scaling-free CORDIC algorithm. A total of seven iterations are required. Since each pipeline stage performs iteration, the proposed CCP pipeline is seven stages long. Each stage is a combination

of three blocks (I)the coordinate calculation unit, (II) micro-rotation sequence generator(MRSG) ,(III) the shift-index calculation. The coordinate calculation unit implements using shift-add implementation. The shift- index calculation computes the necessary shifts, required by the calculation unit. The micro-rotation is as shown in the figure-3
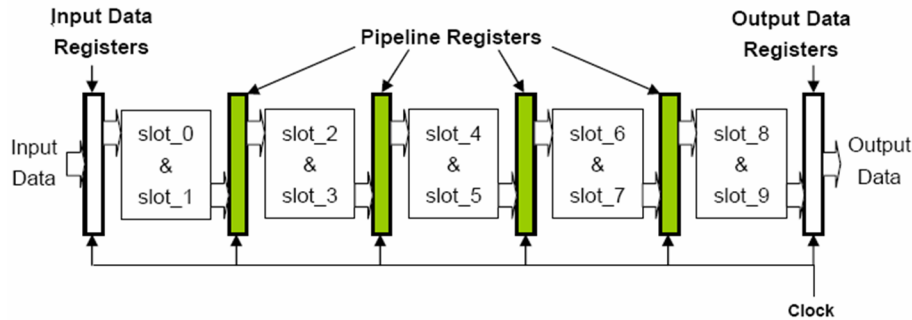
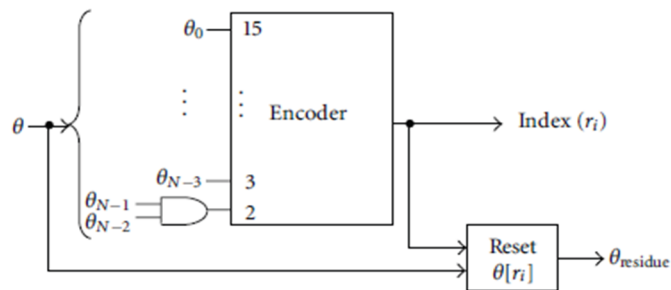

Fig 2: pipeline architecture

Fig 3: micro-rotation sequence generator(MRSG)

.The micro rotation sequence generator selects appropriate elementary angle for the current CORDIC iteration. Using refined elementary angles, the micro-rotations can be identified using the circuit shown in fig-3. It consists of a priority encoder and a reset circuitry. The input to the micro rotation sequence generator is the rotation angle θ   [N-1:0] , where N is the word length. The priorities of the encoder are hooked in the reverse order with θ_{N-1}

Having the highest priority and θ_0 the least. The reset circuitry resets a bit of the input rotation angle to generate the residue angle for next CORDIC iteration. Since, the micro rotation sequence generates the shift-index r_i for one CORDIC iteration, it is required in every stage of the CORDIC pipeline. Even though, the coordinate calculation unit of the proposed completely scaling free CORDIC is more complex than the conventional CORDIC ,the overall gate count of the proposed window architecture is reduced.

B.   Window coefficient multiplier (WCM)

WCM unit multiplies the input samples with the hamming window coefficients(α, β) . In radix-2 representation systems, multiplication with 0.5 is equivalent to single shift. Therefore, multiplication with α=0.5 is realized using a hardwired shifter for hamming window, the coefficient α is represented in 16-bit fixed point format is 0010_0010_1000_0000 and the coefficient β is represented 0001_1101_1000_0000. The complexity of the WCM unit is equivalent to six 16-bit adders, as hardwired shifters do not incur any hardware costs.

C.   Angle generator

Angle generator generates the two angles given by $\theta = (\frac{2\pi n}{N-1})$, $2\theta = \frac{4\pi n}{N-1}$ where N is   a multiple of 2 such that $N=2^M$. The difference between the consecutive values of θ is given by $\Delta\theta = \theta n+1 - \theta n$

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$\Delta\theta = (\frac{2\pi n}{N-1})$

For N=2^M, $\Delta\theta = \frac{2\pi}{2^M-1}$

$\Rightarrow \Delta\theta = \frac{2\pi}{2^M}).(1-2^{-M})^{-1}$

Using binomial theorem (B.T.), we simplify (15c) to the following:

using B.T.$(1-x)^{-1} = 1+x+x^2+x^3+x^4+\cdots$

B.T. of $= \Rightarrow$

$(1-2^{-M})^{-1} = 11+2^{-M}+2^{-2M}+\ldots\ldots\ldots.$

$\Delta\theta = \frac{2\pi}{2^M}+\frac{2\pi}{2^{2M}}+\frac{2\pi}{2^{3M}}$

$\Rightarrow \Delta\theta = (2\pi >> M)+(2\pi >> 2M)+(2\pi >> 3M).$

Generally in most signal processing applications, not less than 16-point DFT is used which implies $N \geq 16$ and $M \geq 4$. Therefore, only three terms of binomial expansion are sufficient for 16 bit accuracy as follow

fourth term of B.T. $\Delta\theta 4 = \frac{2\pi}{2^{4M}}$

$= \Rightarrow \Delta\theta 4 = 2\pi >> 4M.$

The angles in the windowing function are uniformly distributed over the entire coordinate space. The CCP unit handles angles in the range of [0, $\pi/4$]. Therefore, the TGU divides the entire coordinate space into octants, so that the input angle to CCP always lies in the range [0, $\pi/4$]. The octants are distinguished as shown in Figure3; the angle generator also generates signals for proper octant mapping of values generated by CCP. The TGU requires three 16 bit adders, two barrel-shifters and one encoder.
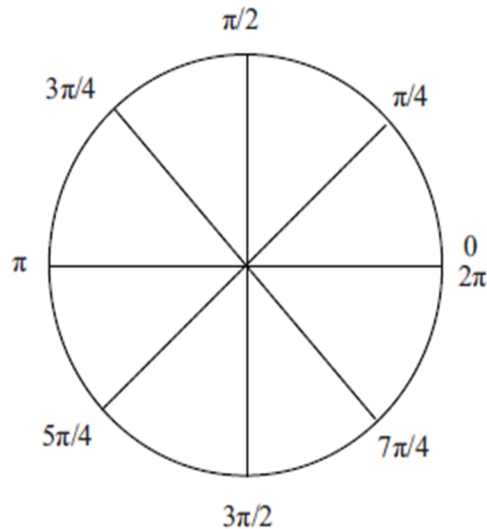

Fig 4 : Mapping Of Coordinate Space Into Octants

D.   Window Function

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Fig :3 compares the hamming window generated using the proposed processor with that of MATLAB inbuilt function hamming () for $N = 16$
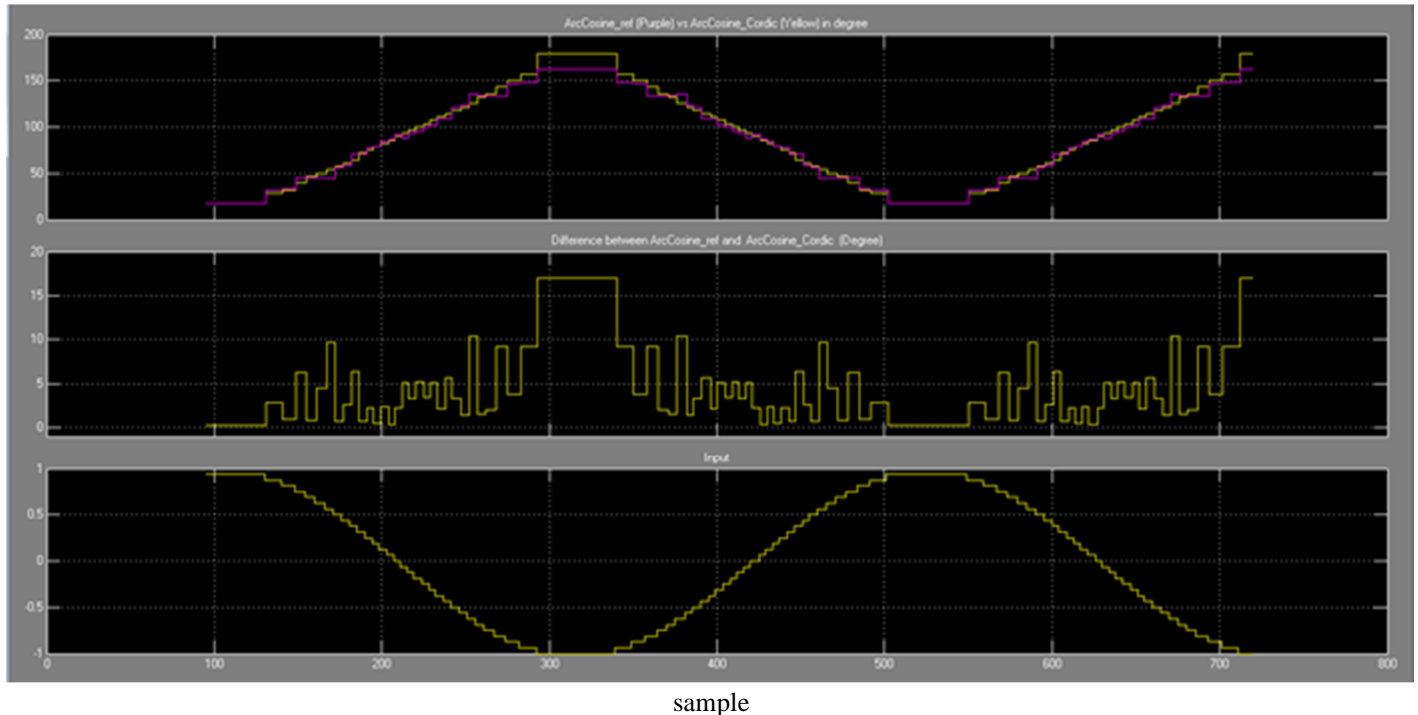
amplitude

sample

fig 5: Matlab output to generate sine wave

purple line- Matlab hamming
yellow line-proposed.

IV.   FPGA IMPLEMENTATION

The proposed architecture is coded in verilog, simulated in Xilinx ISE 12.2i design suite to be mapped on Xilinx virtex 2pro(x C2VP50-6FF1148) device. For 16 bit implementation, the proposed design consumes 960 slices and 1920 4-Input LUTs, with a maximum operating frequency of 101.23 MHz. The total delay of 8.48 nsec is distributed as 76.9% logic delay and 23.1% route delay. The total gate count of the proposed design is 16800.

A.   Area and latency comparison:

Table III compares the area and latency of the proposed design with the existing architecture variants. The area of conventional circular CORDIC processor is calculated using Xilinx CORDIC IP. It is optimized for circular CORDIC computation with maximum pipelining for 16-bit word length. The complexity of 16-bit scaling free CORDIC is computed to be equivalent to 1000 1-bit full adders and gate count is 20122 and 597 1-bit registers, which requires 16776 gates for implementation. The complexity of the 16-bit linear conventional CORDIC is equivalent to 512 1-bit full adder and 768 1-bit register, which amounts to 12288 gates. The other units like angle generator unit, FIFO, adders required for realizing the window processor are common for the proposed as well as the existing design.

B.   Latency

The throughput of all the design is same that is, one data sample per clock cycle, while the latency is different and is closely related to number of iterations in circular CORDIC and linear CORDIC processor when designs are operating at the same frequency. The 16-bit linear CORDIC processor uses 16-stages long pipeline. The conventional circular CORDIC processor again uses 16-stages pipeline for 16 bit word length. For the same 16-bit word length, the scaling-free CORDIC processor uses 12-stages long pipeline. Therefore, the latency of existing design in conventional circular CORDIC is 32 stages, while with scaling free is 28 stages . The

new redesigned- CORDIC pipeline is 7 stages long.

C. Delay

The delay is the time required to generate one set of window coefficients for window length of N when the design is operating at the max clock frequency. The critical path for the proposed design is the angle generator. By using angle generator the existing design using conventional circular CORDIC and scaling free circular CORDIC is designed. The angle generator for the proposed design and the existing design using the conventional circular CORDIC and the scaling free circular CORDIC generates the angle in the range$[-\pi/4, \pi/4]$, the clock frequency for the existing design is 101.983MHz and for other designs including the proposed design is 107.008MHz.

Table I: FPGA features used in proposed architecture

Top level source type	HDL
Preferred language	Verilog
Family	Spartan-3
Device	Xa3s100
Package	vQg100
Speed	-4
Simulator	Isim

Table II: summary report of the CSF CORDIC

Area	960 slices, 1920 LUTs
Latency	7 pipelining
Maximum clock frequency	107.008MHz
Delay	8.48ns
Total gate-count	16800
Type of architecture	Pipelined
Window size	Variable

Table III: area-latency comparison of proposed window-architecture with existing architecture

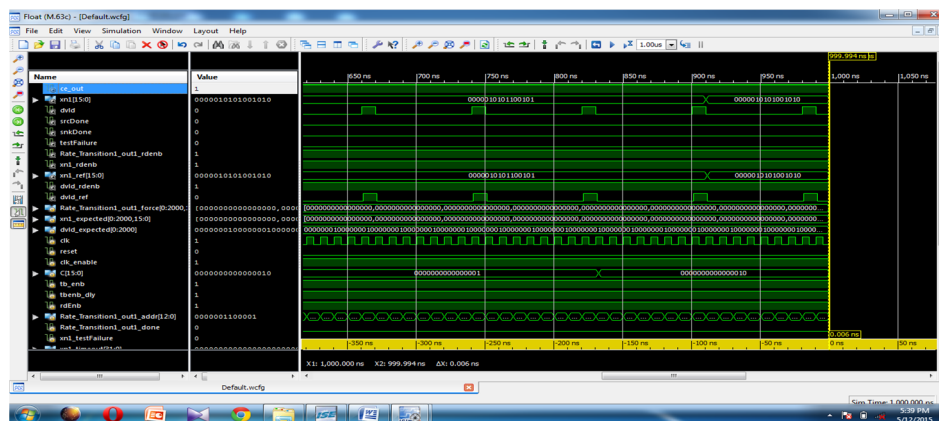| parameter	Proposed design	ED-SFC
Gate-count	16800	76961
latency	58.28%	28

OUTPUT RESULT:

Fig 6: simulation result

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V.   CONCLUSION

In this paper, an area-time efficient CORDIC based processor for computing window function. Currently the architecture implementing the hamming window function, with slight modification, can be extended to other window functions as well. Proposed design is of word length 16 bit. Computing the window function by saving the area, reduces the latency and optimizing the delay for maximum extent. In the existing design, replacing the linear CORDIC processor with multiple shift-add units and the circular conventional CORDIC processor with the completely scaling-free CORDIC processor.

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