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Equilibrium Analysis of Famous Hammer and Scale Problem

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Abstract: The hammer and scale problem was prevalent on social media platform like Facebook and Instagram few years back. This question is still one of the most challenging physics problem often named as 'physics or strong glue'. In this document, detailed analysis of conditions of equilibrium is carried out using static approach by science of engineering mechanics. The entire thesis is conducted by analyzing free body diagrams of each individual member of setup and then stability curve is plotted in order to visualize stability ranges for the system. Afterwards, equations for getting optimum parameters are derived which are necessary to obtain for attainment of equilibrium of system. Finally the procedure for entire calculation is shown followed by the special case of attaining stability with help of glue is described.

Keywords: Center of gravity, center of mass, line of action (L.O.I) of force, normal reaction, seat of normal reaction

Figure 1 setup of famous hammer and scale problem

I. INTRODUCTION

As shown in figure, the system consists of wooden scale and hammer both resting on table. The hammer is in contact with scale at the furthermost end from table; and, at the another end, the hammer is restrained by a string connected to a scale. To check the conditions of equilibrium of whole system, we will first carry out analysis of all linear and rotational external agents such as gravitational force, normal reactions, tension on string and respective moments generated by those forces.

Along with that, the positions of center of gravity (center of mass) is illustrated in figures showing the general diagrams of system. One important observation can be made that center of gravity of scale is at geometric center or centroid as scale is assumed to be uniformly dense throughout the body. The center of gravity of hammer is located near to the head as it is presumably made up of material which is several times denser than the material of stock.

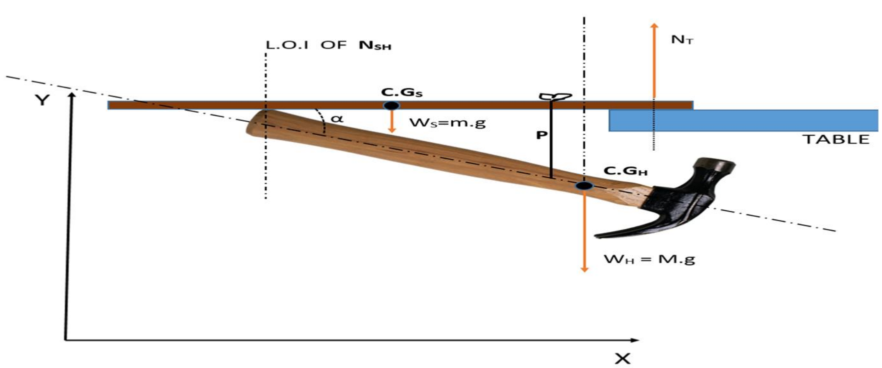

Figure 2 equilibrium position

II. ASSUMPTIONS

A. Analysis is carried out without considering the mechanical properties of material. All objects are treated as rigid bodies.
B. Frictional thrust force are neglected.
C. All loads and reactions are point forces.
D. Thickness and bending properties of scale is neglected

III. NOTATIONS

1) M = mass of hammer
2) m = mass of scale
3) P = tension on string
4) N_T = normal reaction on scale from table
5) N_{SH} = normal reaction between hammer and scale at the utmost end from table.
6) $C.G_S$ = center of mass of scale
7) $C.G_H$ = center of mass of hammer
8) α = angle between geometric axis of hammer and that of scale

IV. FREE BODY DIAGRAM OF SCALE

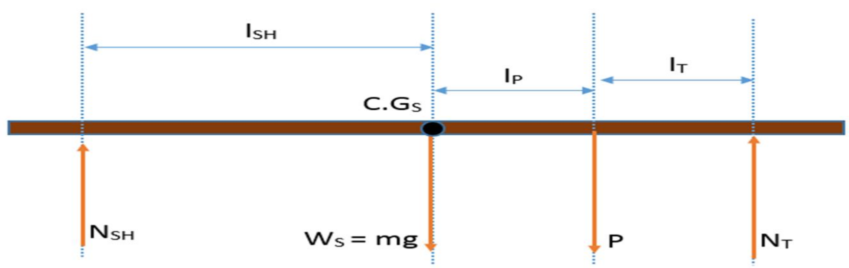

Figure 3 representation of forces exerting on scale

For force equilibrium,

$N_{SH} + N_T = W_S + P$ eqn (1)

For moment equilibrium at $C.G_S$,

$N_T \times l_T = N_{SH} \times l_{SH} + P \times l_P$ eqn (2)

Here,

l_{SH} = perpendicular distance of normal reaction N_{SH} from C.G of scale
l_P = perpendicular distance of tension force in string from C.G of scale
l_T = perpendicular distance of normal reaction N_T from C.G of scale

V. FREE BODY DIAGRAM OF HAMMER

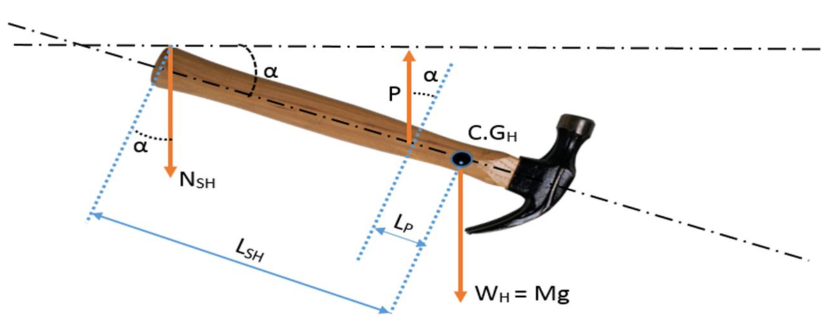

Figure 4 representation of forces exerting on hammer

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For force equilibrium,

$N_{SH} + W_H = P$ eqn(3)

For moment equilibrium at C.G$_H$,

$N_{SH} \times L_{SH} = P \times L_P$ eqn(4)

Here,

L_{SH} = perpendicular distance of normal reaction N_{SH} from C.G of hammer

L_P = perpendicular distance of tension force In string from C.G of hammer

In equation (4), W_H and L_{SH} are limited by a physical dimensions. Therefore, they are constants.

Putting value of N_{SH} from equation (3) in equation (4).

Therefore,

$(P - W_H) \times L_{SH} = P \times L_P$

$\therefore P(L_{SH} - L_P) = W_H \times L_{SH} = C$ (constant)

$\therefore \mathbf{P} = \frac{C}{(L_{SH} - L_P)}$ eqn(5)

$\therefore P = f(L_P)$

VI. EQUILIBRIUM CONDITIONS

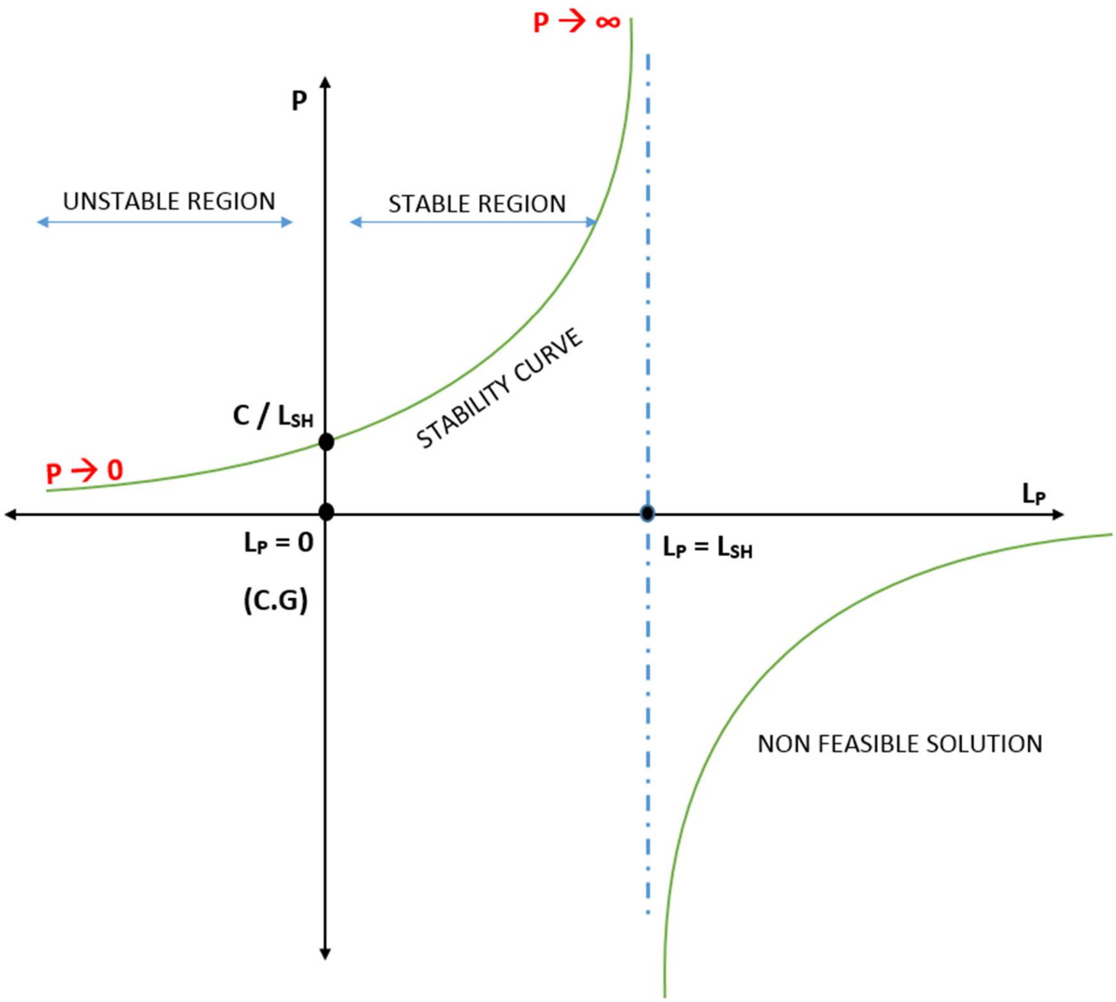

Figure 5 stability curve for static equilibrium of system

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We can make few observations from stability curve shown in figure 5 namely,

1) As shown in plot, the system will remain stable for $L_P \in [\,0\,, L_{SH}\,)$

2) At $L_P = 0\,, P = \frac{C}{L_{SH}} = W_H \times \frac{L_{SH}}{L_{SH}} = W_H$

(In this case, string is tied on C.G of hammer)

3) At $L_P \rightarrow L_{SH}$, tension force $P \rightarrow \infty$

(Hammer cannot be restrained by string and stability is lost)

Solution is not possible when L_P holds values beyond that of L_{SH} .

VII. SOME GEOMETRIC RELATIONS

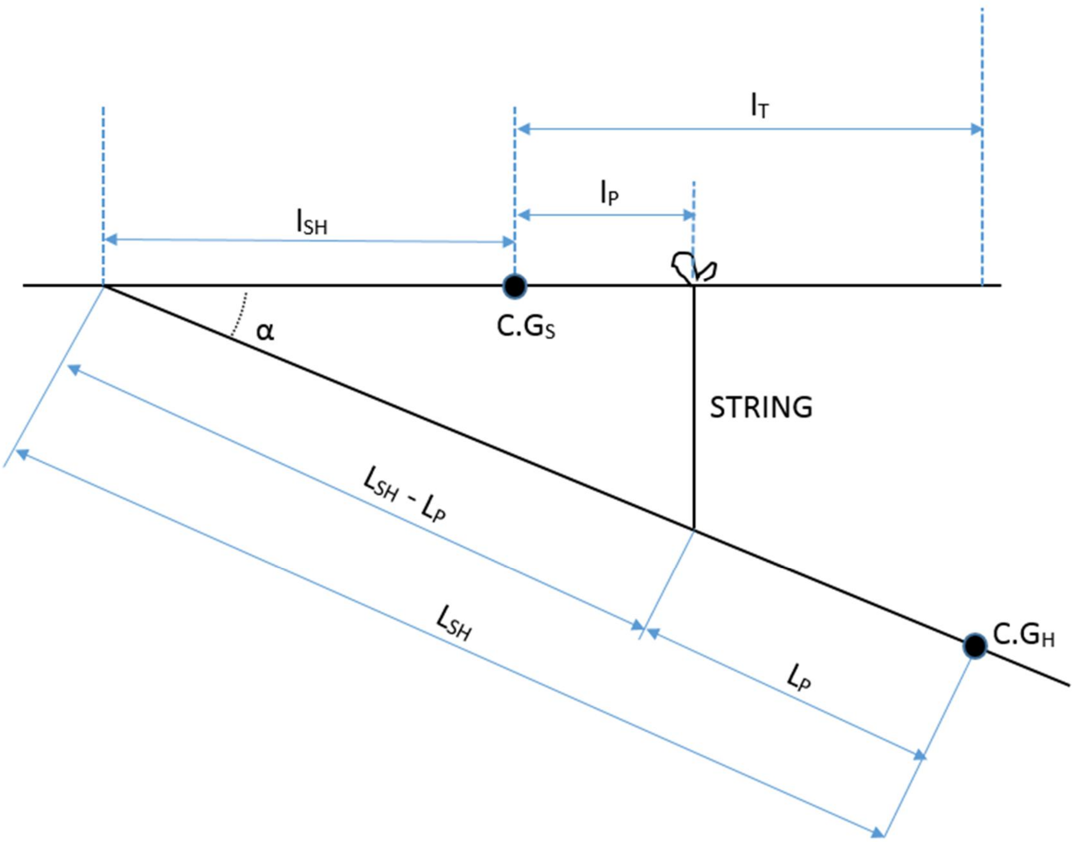

Figure 6 relation between perpendicular distances of L.O.I

From figure 6,

$(\,L_{SH} - L_P) \times \cos\alpha = l_{SH} + l_P$ eqn(6)

And,

$\alpha = \tan^{-1}(\frac{S}{l_{SH} + l_P})$ eqn(7)

Where S = half string length.

From equation (1) and equation (3),

$N_{SH} + N_T = W_S + P$

$\therefore (\,P - W_H\,) + N_T = W_S + P$

$\therefore \mathbf{N_T = W_S + W_H}$ eqn(8)

Total reaction offered by table is constant and is equal to the total weight of system.

Now, from equation (2),

$N_T \times l_T = N_{SH} \times l_{SH} + P \times l_P$

Substituting the value of,

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1) $\mathbf{P} = \frac{\mathbf{C}}{(\mathbf{L_{SH}} - \mathbf{L_P})}$ from equation (5)
2) $\mathbf{N_T} = \mathbf{W_S} + \mathbf{W_H}$ from equation (8) and,
3) $N_{SH} = P - W_H$ from equation (3)

Therefore, D

$$(W_S + W_H) \times l_T = (P - W_H) \times l_{SH} + P \times l_P$$

$$\therefore (W_S + W_H) \times l_T = P(l_{SH} + l_P) - (W_H \times l_{SH})$$

$$\therefore (W_S + W_H) \times l_T = \frac{C}{L_{SH} - L_P}(l_{SH} + l_P) - (W_H \times l_{SH})$$

But, from equation (6),

$$\frac{l_{SH} + l_P}{L_{SH} - L_P} = \cos\alpha \text{ , and } C = W_H \times L_{SH}$$

$$\therefore (W_S + W_H) \times l_T = (W_H \times L_{SH}) \times \cos[\tan^{-1}(\frac{S}{l_{SH} + l_P})] - (W_H \times l_{SH})$$

$$\therefore \mathbf{\cos\alpha} = [(\mathbf{W_S} + \mathbf{W_H}) \times \mathbf{l_T} + (\mathbf{W_H} \times \mathbf{l_{SH}})] \div (\mathbf{W_H} \times \mathbf{L_{SH}}) \qquad \text{eqn (9)}$$

Where, $\alpha = \tan^{-1}(\frac{S}{l_{SH} + l_P})$

VIII. SOLUTON PROCEDURE

A. M and m are known
B. Find $C.G_S$ and $G.G_H$ and locate it on respective objects
C. Measure L_{SH}
D. Find $W_S = mg$ and $W_H = Mg$
E. Take string of length 2S and fold it half ways.
F. Take any feasible values of l_{SH} and l_T
G. Find value of l_P accordingly
H. Find value of α from equation (7)
I. Tie string at distance l_P from $C.G_S$ as shown in figure of the setup.

IX. SPECIAL CASE

As shown in equation (5), we can observe at $L_P = 0$, $P = W_H$. This case will eliminate reaction N_{SH} and it will bring unbalanced state of system.

In such a case, glue should be applied at a distance l_g from $C.G_S$ in order to hold the system and prevent the unbalanced situation.

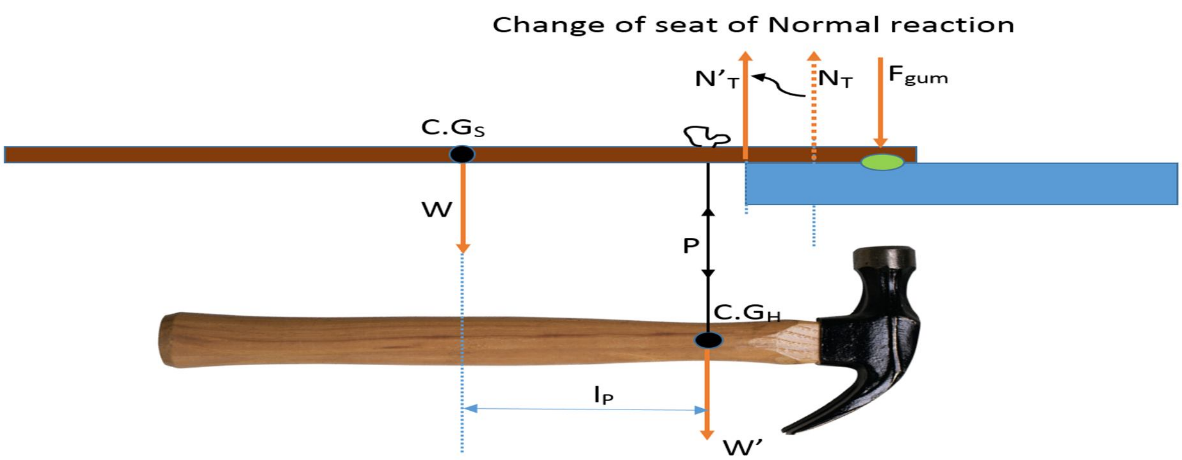

Figure 7 equilibrium position of system using glue

In this case, seat of normal reaction N_T is shifted to position l'_T and the value becomes N'_T with introduction of glue force F_{gum}. Another thing worth mentioning is that, in order to hold hammer in equilibrium position, the string is needed to be tied at $C.G_H$.

For moment equilibrium at $C.G_S$,

$$-(P \times l_P) - (F_{gum} \times l_g) + (N'_T + l'_T) = 0$$

$$\therefore F_{gum} = [(N'_T \times l'_T) - (P \times l_P)] \div l_g \qquad \text{eqn (10)}$$

Where,

$N'_T = F_{gum} + W_S + P$ = changed normal reaction from table (located at the edge of table)

l_P = distance between $C.G_S$ and $C.G_H$

l_g = perpendicular distance of line of action of glue force F_{gum} from $C.G_S$

l'_T = perpendicular distance of line of action of force N'_T from $C.G_S$

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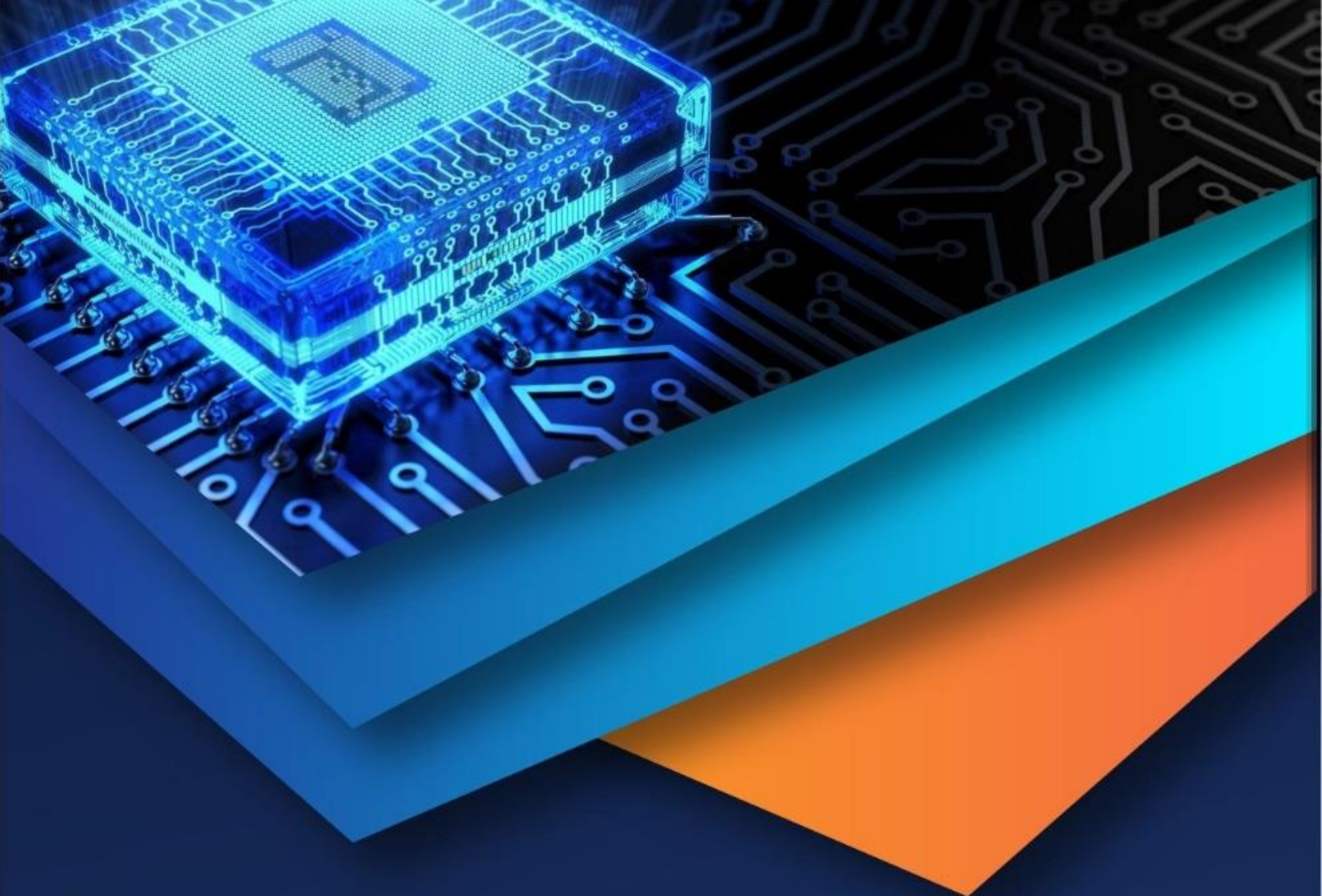

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