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Estimation of Scale (θ) and Shape (α) Parameters of Power Function Distribution by Median Ranks Regression Method using Optimally Constructed Grouped Data

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Abstract: The objective of this paper is to estimate the parameters and also construct an Optimal Grouped sample in the absence of prior knowledge or guess values of parameters. In this heuristic algorithm, the Median Ranks Regression Method is used to find out Estimates the parameters of Power Function Distribution. We also computed Average Estimate (AE), Variance (VAR), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Efficiency (RE) for both the parameters under grouped sample based on 1000 simulations to assess the performance of the estimators.

Keywords: Power Function Parameters, Optimally Grouped sample, Median Ranks Regression

I. INTRODUCTION

Rider (1964) the name Power Function Distribution has been used. Johnson (1970) given that the moments of the power function distribution are simply the negative moments of the Pareto distribution. Ahsanullah and Kabir et al (1975) discussed the Estimation of the location and scale parameters of a Power function distribution. According to Dallas et al (1976), if Y is power function distribution then Y-1 is the Pareto distribution model. Cohen and Whitten et al (1980) used the estimation in the three parameter lognormal distribution.

Rosaiah et al (1991) studied the problem of asymptotically optimal grouping of sample into equiclass grouped sample for maximum likelihood estimation in two parameter gamma distribution. Vasudevarao et al (1994) considered the problem of asymptotically optimal grouping for maximum likelihood estimation in a two parameter Weibull distribution in the case of equispaced group. They also studied the same for maximum likelihood estimation of Weibull shape parameter when scale parameter is known in the case of unequispaced grouped samples.

Meniconi and Barry et al(1996) explore the performance of Power function distribution on electrical components and illustrated that power function distribution is most suitable distribution on electrical component data as compared to log-normal,Weibull and exponential models.

Theoretically, Kleiber et al (2003) studied power function distribution has an inverse relationship with the standard Pareto distribution, and it is also a special case of Pearson type I distribution.

Saran & Pandey et al (2004) estimate the parameters of Power Function Distribution and they also characterize this distribution. Balakrishna et al (2004) and Kantam et al (2005) constructed the Optimum group limits for un-eqi-spaced grouped sample using M. L. Estimation in Scaled Log-Logistic distribution.

Reliability Hotwire- The e-Magazine et al (2007) had mentioned the Correlation Coefficient tool in 'How our weibull distribution be good'.CH. Rama Mohan et al (2011) Studied Least Square Estimation of the Weibul parameters from an optimally constructed grouped sample.

Rahman, Roy & Baizid et al (2012) applied the Bayesian estimation method to estimate the parameters of Power Function Distribution. Zarrin et al (2013) applied power function distribution to assess component failure of semi-conductor device data by using both the maximum likelihood and Bayesian estimation methods.

A Comparison of Maximum Likelihood and Median Rank Regression for Weibull Estimation proposed by Ulrike Genschel William et al (2010).

Accelerated Life Test Modeling Using Median Rank Regression (2016) discussed by Austin J. Rhodes. Vijaya lakshmi et al (2018), studied the Estimation of Location (μ) and Scale (λ) for Two-Parameter Rayleigh Distribution by Median Rank Regression Method. Vijaya lakshmi, O. V. Raja Sekharam and G. V. S. R. Anjaneyulu (2018) proposed Estimation of Scale (θ) and Shape (α) Parameters of Power Function Distribution by Least Square Method Using Optimally Constructed Grouped Data. Vijaya lakshmi and Anjaneyulu studied Estimation of Location (μ) and Scale (λ) for Two Parameter Half Logistic Pareto Distribution (HLPD) by Median Rank Regression Method. The literature mentioned above, reveals that much attention seems to have been paid for inference based on grouped data from two parameter Power Function distribution. In this, when we have no prior Knowledge about the unknown parameters that we used to construct an asymptotically Optimal Groped Data, which can be used to estimation of parameters using Median Ranks Regression Method. The optimal group limits of a grouped sample from two parameter Power Function distribution constructed which are presented at the in the chapter as Table 4.7. Here we developed a practical procedure to construct an optimally grouped sample even when there is no prior knowledge or guess values of the parameters are given in section 4.2. In section 4.3 we made an attempt to study some problems of point estimation from grouped data based on Power Function distribution. The Median Ranks Regression method was used to estimate the parameters from such an optimally constructed grouped sample in two parameter Power Function distribution using the optimal group limits constructed and The Average Estimate (AE), Variance (VAR), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE) of the Scale parameter (θ) and Scale (α) are calculated for assessing the performance of the estimated parameters.

Let y1, y2, …,yn be a raw sample of size ‘n’ dawn from two-parameter Power Function distribution with unknown scale(θ) and shape(α) parameters. The Probability density (p.d.f.) and cumulative distribution function (c.d.f.) of Power Function distribution are respectively given by

$$f(y;\theta,\alpha) = \frac{\alpha y^{\alpha-1}}{\theta^{\alpha}}, 0 < y < \theta,\ \alpha > 0,\ \theta > 0 \qquad (1.1.1)$$

$$F(y;\theta,\alpha) = \left(\frac{y}{\beta}\right)^{\alpha}, 0 < y < \theta,\ \alpha > 0, \theta > 0 \qquad (1.1.2)$$

$$F(Xi) = \frac{i-0.3}{N+0.4};\ i = 1,2,3,\dots,k$$

II. OPTIMALLY CONSTRUCTED GROUPED SAMPLE

In this section, we develop a practical procedure to construct an optimally grouped sample in the case when there is no a priori knowledge or guess values of the parameters. In this procedure, we will prefix the number of test units to be failed in each group of the optimal grouped sample and then we record some arbitrary time point after failure of the number of the test units that are to be failed in that group, but before starting the failure of a test unit in the next group. Suppose N is the number of test units put under a life-testing experiment which assumes the Power model (4.1.1) and suppose the experimenter wishes to obtain the grouped life-time data with k classes. Then Table 4.7 can be used to compute the expected number of test units to be failed in the time interval (ti-1, ti) and is given by

$$fi = Npi;\quad \text{For } i = 1, 2, \dots, k \qquad (1.2.1)$$

Where $pi = \frac{1}{\theta^{\alpha}}[(x_{i-1})^{\alpha} - (x_i)^{\alpha}]$

and

$$F(Xi) = \frac{i-0.3}{N+0.4};\ i = 1,2,3,\dots,k$$

1) fi is expected number of failures in the ith interval
2) xi‘ s are optimal group limits obtained from the above procedure
3) k is number of groups
4) N is total frequency

fi ‘s may be rounded to the nearest integers so that N=f1+ f2+…..+f k. Thus, the optimal group limits may be used to compute the expected optimum number of test units to be failed in ith interval (ti-1, ti), for i=1,2,…, k. Here, it may be noted that the experimenter has to observe the random time instants y1, y2, …, yk-1 so as the optimum pre-fixed number of units fi to be failed in the time interval (yi-1, ti) for i=1, 2,…, k taking to=0 and tk = ∞. In other words, record a random time instant after failure of first f1 test units, but before the failure of (f1+1)th test unit and to record a random time instant after failure of first f1+ f2 test units, but before the failure of (f1+ f2 +1)th test unit and so on. Further, it may be noted that it is difficult to record all exact failure times of the individual units but, it is not so difficult to note a random time instant between the failure times of two consecutive test units.

III. METHODOLOGY

A. Median Ranks Regression Estimation Of The Parameters From The Optimally Constructed Equispaced Grouped Sample

We know that t1, t2, …, tk-1the group limits of te optimally constructed grouped Sample using the procedure explained in the above section, are the observed values of the true asymptotic optimal group limits x1,x2, …, xk-1 where as their estimated values are given by

$$\hat{y}_i = \theta\,(x_i)^{\frac{1}{\hat{\alpha}}} \qquad (1.3.1)$$

where $\hat{\beta}$ and $\hat{\gamma}$ are obtained by using the principle of Median Ranks Regression method (MRR) is extensively used in reliability engineering and mathematics problems. According to the Median Ranks Regression method (MRR) linear relation between the two parameters taking the natural logarithm of above equation as follows

$$\log t_i = \log(\theta) + \left(\frac{1}{\alpha}\right) \log(x_i) \quad \text{for i= 1, 2, …, k-1} \qquad (1.3.2)$$

After simplification, we get

$Y_i = \log t_i$, $A = \log(\theta)$, $B = \frac{1}{\alpha}$,$X_i = \log(x_i)$

Thus, equation (1.3.2) is a linear equation and is expressed as

$$Y_i = A + BX_i$$

To compute a and d by simple linear regression we proceed as follows

$$\text{Let} \qquad S(A,B) = \sum_{i=1}^{k-1}\left(y_i - A - Bx_i\right)^2 \qquad (4.3.3)$$

Differentiating (1.3.3) w.r.t to A and B then equate to zero, we obtain the following two normal equations

$$\sum_{i=1}^{n} y_i = nA + B\sum_{i=1}^{n} x_i \quad (1.3.4)$$

$$\sum_{i=0}^{n} x_i y_i = A\sum_{i=1}^{n} x_i + B\sum_{i=0}^{n} x_i^2 \qquad (1.3.5)$$

Solving the above two equations for A and B, we obtain the Median Ranks Regression estimates (MRRE) of A and B as:

$$A = \bar{y} - B\,\bar{x}$$

$$B = \frac{\sum_{i=1}^{k-1} x_i y_i - \frac{\left(\sum_{i=1}^{k-1} x_i y_i\right)}{k-1}}{\sum_{i=1}^{k-1} x_i^2 - \frac{\left(\sum_{i=1}^{k-1} x_i\right)^2}{k-1}}$$

$$A = \frac{\sum_{i=1}^{k-1}\log t_i}{k-1} - B\,\frac{\sum_{i=1}^{k-1}\log x_i}{k-1} \qquad (4.3.6)$$

$$B = \frac{\sum_{i=1}^{k-1}\log(x_i)(\log t_i) - \frac{\left(\sum_{i=1}^{k-1}\log x_i\right)\left(\sum_{i=1}^{k-1}\log t_i\right)}{k-1}}{\sum_{i=1}^{k-1}(\log(x_i))^2 - \frac{\left(\sum_{i=1}^{k-1}\log x_i\right)^2}{k-1}} \qquad (1.3.7)$$

where $A = \log(\theta)$ and $B = \frac{1}{\alpha}$

$$\text{Therefore} \quad \hat{\theta} = \text{Antilog}\left\{\frac{\sum_{i=1}^{k-1}\log t_i}{k-1} - B\,\frac{\sum_{i=1}^{k-1}\log x_i}{k-1}\right\} \qquad (1.3.8)$$

$$\text{and} \quad \hat{\alpha} = \frac{\sum_{i=1}^{k-1}(\log(x_i))^2 - \frac{\left(\sum_{i=1}^{k-1}\log x_i\right)^2}{k-1}}{\sum_{i=1}^{k-1}\log(x_i)(\log t_i) - \frac{\left(\sum_{i=1}^{k-1}\log x_i\right)\left(\sum_{i=1}^{k-1}\log t_i\right)}{k-1}} \qquad (1.3.9)$$

The rationale for applying least square method is that for a given k, xi's, are fixed values and are can be borrowed from Table 1.5 where as ti's are random values and are obtained as observations from the experiment. . It may be noted that the least square estimates, $\hat{\theta}$ and $\hat{\alpha}$ obtained from the equations (1.3.8) and (1.3.9)

Performance Indices: Goodness of Fit Analysis:

B. Comparison of Median Ranks Regression estimators of Equispaced and Unequispaced Optimally Constructed Grouped Data

The Median Ranks Regression estimators of θ and $\boldsymbol{\alpha}$ namelly, $\hat{\theta}$ and $\hat{\boldsymbol{\alpha}}$ developed in the above section are in non-linear form and hence, it is very difficult to obtain the bias and variances of the estimators. Hence, we have resorted to Monte Carlo simulation to compute the, Average Estimate (AE), Variance (VAR), Standard Deviation (STD), Mean Absolute Deviation (MAD), Mean Square Error (MSE= Variance+Bias2), Simulated Error (SE) and Relative Absolute Bias (RAB), the performance of unequispaced Median Ranks Regression estimators $\hat{\theta}$ and $\hat{\boldsymbol{\alpha}}$

we compare with the corresponding equispaced least square estimators obtained from ungrouped sample as well as asymptotically optimal grouped sample based on variance.

If $\hat{\omega}_{lm}$ is Median Ranks Method estimate of $\widehat{\omega}_m$, m=1, 2 where ω_m is a general notation that can be replaced by $\omega_1 = \theta, \omega_2 = \alpha$ based on sample l,(l=1,2,…,r) then The Average Estimate (AE), Variance (VAR), Standard Deviation (STD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) are given respectively by

$$\text{Average Estimate } (\widehat{\omega}_m) = \frac{\sum_{i=1}^{r} \widehat{\omega}_{lm}}{r}$$

$$\text{Variance } (\widehat{\omega}_m) = \frac{\sum_{i=1}^{r} (\widehat{\omega}_{lm} - \overline{\widehat{\omega}_{lm}})^2}{r}$$

$$\text{Standard Deviation } (\widehat{\omega}_m) = \sqrt{\frac{\sum_{i=1}^{r} (\widehat{\omega}_{lm} - \overline{\widehat{\omega}_{lm}})^2}{r}}$$

$$\text{Mean Absolute Deviation} = \frac{\sum_{i=1}^{r} Med(|\widehat{\omega}_{lm} - \overline{\widehat{\omega}_{lm}}|)}{r}$$

$$\text{Mean Square Error } (\widehat{\omega}_m) = \frac{\sum_{i=1}^{r} (\widehat{\omega}_{lm} - \omega_m)^2}{r}$$

$$\text{Relative Absolute Bias } (\widehat{\omega}_m) = \frac{\sum_{i=1}^{r} |(\widehat{\omega}_{lm} - \omega_m)|}{r\omega_m}$$

IV. CONCLUSION

A. Variances of the estimators are decreasing as number of groups increases.

B. The estimates obtained from optimal grouped sample with equispaced efficient than the unequispaced sample when number of sample increaases .

C. When compared with small sample, the estimators in large sample are more efficient. Random Generated values of Power Function Distribution

V. AN ILLUTRATION

A random sample of 200 observations is generated from a two-parameter Power function distribution with the **θ** = 4, **α** = 3 using R Software and the ordered sample is given below:

0.5643	0.68283	0.6911	0.97532	1.03033	1.27493	1.34438	1.35793	1.37605	1.39571
1.42772	1.50664	1.56168	1.58275	1.60049	1.61385	1.68193	1.73975	1.75911	1.7654
1.83026	1.85858	1.86084	1.92251	1.94428	2.03693	2.03803	2.06997	2.10523	2.13578
2.19005	2.20756	2.23161	2.24989	2.26332	2.33308	2.34107	2.36156	2.3817	2.40646
2.4133	2.45333	2.50304	2.53859	2.54071	2.54162	2.54213	2.54796	2.56023	2.58833
2.59008	2.61792	2.63247	2.63397	2.65159	2.67836	2.67836	2.6893	2.75054	2.76331
2.76442	2.77519	2.80024	2.82289	2.83388	2.83857	2.84116	2.86014	2.868	2.87637
2.88053	2.88664	2.89171	2.90234	2.91327	2.95076	2.95896	2.96527	2.96534	2.98169
3.01012	3.03096	3.04479	3.06924	3.07366	3.10118	3.10186	3.13844	3.14714	3.15311
3.15971	3.16642	3.17645	3.19837	3.20965	3.21275	3.22231	3.22369	3.23461	3.24516
3.24584	3.2595	3.26726	3.2819	3.28449	3.28751	3.29124	3.29244	3.32526	3.32797
3.33478	3.34447	3.36305	3.36713	3.37726	3.38011	3.40641	3.40971	3.42271	3.47709
3.49269	3.49684	3.50285	3.50878	3.51047	3.51507	3.51686	3.52939	3.545	3.55111
3.55214	3.55719	3.56012	3.56186	3.56407	3.56683	3.57097	3.58855	3.59229	3.60441
3.6154	3.61878	3.63457	3.64284	3.64416	3.65282	3.65861	3.67522	3.67527	3.67835
3.68766	3.69621	3.70747	3.70814	3.71659	3.71664	3.72201	3.72356	3.72731	3.72993
3.73866	3.74076	3.75794	3.78971	3.79003	3.80214	3.80245	3.80866	3.81381	3.82305
3.83982	3.85074	3.85438	3.85736	3.85819	3.86068	3.86748	3.86774	3.87422	3.87664
3.87924	3.89823	3.899	3.90011	3.90276	3.90447	3.93014	3.93452	3.93813	3.94383
3.95693	3.9571	3.95997	3.963	3.96734	3.96892	3.98111	3.98165	3.98829	3.99695

rouped the above data into an optimally grouped sample with 10 groups as explained below.

Here, we have N= 200 and k= 10. From Table -5.1 for k=10, asymptotic optimal group limits are x_1= 0.5987 , $x_{2=}$ 0.9875, x_3=1.09899, x_4 =1.2576, x_5 = 1.7954, x_6=1.9879 , x_7=2.0583, x_8=2.321, x_9=2.5214.

Table 5.2. Optimally Constructed Groups of a sample for Power Function Distribution

optimal class interval	Frequency(fi)	Cumulative frequency
0-0.5987	2.96≈3	3
0.5987-0.9875	11.85≈12	15
0.9875-1.09899	39.02≈39	54
1.09899-1.2576	19.24≈19	73
1.2576-1.7954	21.36≈21	94
1.7954-1.9879	38.33≈38	132
1.9879-2.0583	22.41≈22	154
2.0583-2.321	11.56≈12	166
2.321-2.5214	11.22≈11	177
>2.5214	23.45≈23	200

Now we compute the expected frequencies f_1, f_2, $f_{3,}$ $f_{4,}$ f_5, f_6, f_7 $f_{8,}$ f_9, f_{10} which can be obtained by using above values in (5.2.1) and are given by

f_1= 3, f_2=12, f_3=39, f_4=19, f_5 = 21, f_6 =38, f_7 =22, f_8 =12, f_9 =11, f_{10}=23
Now we have to choose t_0=0, t_1 t_2, $t_{3,}$…,$t_{10=}$ ∞ such that f_i (for i= 1,…,10) observations have fallen in the interval (t_{i-1},t_i). Here, t_1 is chosen as a random value in between 3^{rd} and 4^{th} order observation. Similarly t_2 is chosen as a random value in between 15^{th} and 16th order observation t_3 and soon t_9 observation choosen same as their respective random order values. Thus we get t_1=3.3713, t_2=3.9832, $t_{3=}$4.1278, t_4 =4.3176, t_5=4.8616, t_6 =5.0295,t_7=5.0882, t_8 =5.2960, t_9=5.4442.

Tale 5.2
The least square estimation of Power function distribution from the above optimally constructed grouped data can computed as follows

	ti	xi	log(xi)	log(ti)	log(xi)*log(ti)	log(xi)^2
	3.371292316	0.5987	-0.222790742	0.527796	-0.117588154	0.049635715
	3.983263403	0.9875	-0.005462896	0.600239	-0.003279043	0.0000298
	4.127856314	1.09899	0.040993741	0.615725	0.025240853	0.001680487
	4.317584383	1.2576	0.099542529	0.635241	0.063233479	0.009908715
	4.861613153	1.7954	0.254161221	0.68678	0.174552944	0.064597926
	5.029500271	1.9879	0.298394534	0.701525	0.209331176	0.089039298
	5.088184875	2.0583	0.313508674	0.706563	0.221513592	0.098287689
	5.296045466	2.321	0.36567514	0.723952	0.264731141	0.133718308
	5.44428152	2.5214	0.401641748	0.735941	0.295584459	0.161316094
Total	41.5196217	14.62679	1.545663949	5.933761	1.133320449	0.608214075

The median ranks estimate of Power function parameters $\boldsymbol{\theta}$, $\boldsymbol{\alpha}$ from the above optimally constructed grouped sample can be computed as follows from the equation (1.3.8) and (1.3.9) the least square estimates are $\tilde{\boldsymbol{\theta}}$ = 2.065176317, $\tilde{\boldsymbol{\alpha}}$ = -0.383769847.

Performance of median ranks estimation of Scale (θ) and Shape (α) Parameters of Power Function distribution obtained from optimally constructed grouped sample k= 10, 16,18,20 when Scale (θ) = 4 and Shape (α) = 3 with the equations (1.3.8) and (1.3.9) are Presented in the table 1.4. We have resorted to Monte Carlo simulation to compute The Average Estimate (AE), Variance (VAR), Mean Square Error (MSE), Relalated Error (RE), Relative Absolute Bias (RAB), of the Scale (θ) and Shape (α) Parameters of Power Function distribution are unknown under complte 1000 simulations based on N=50(50)300 generated from Power Function distribution with different Scale and Shape parameters.

A. Conclusion

1) Variances of the estimators are decreasing as number of groups increases.
2) The estimates obtained from optimal grouped sample variance increases the efficient is increased.
3) When compared with small sample, the estimators in large sample are more efficient.

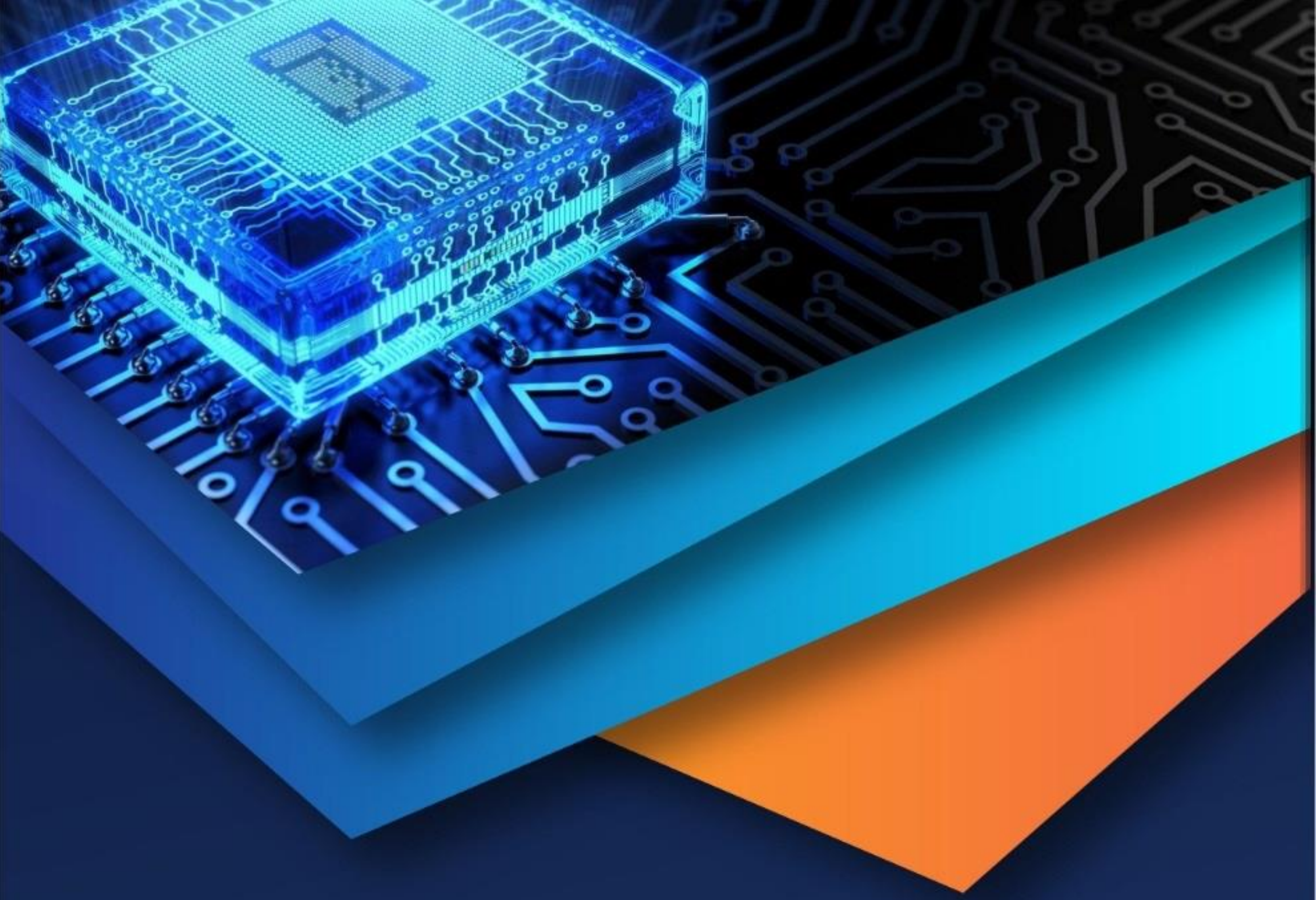
Table 1.4
Power Function distribution
The Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE) of the Scale parameter (θ) when Shape parameter (α) is known under complete data based on 1000 simulations. Population parameter values are θ= 4 & α = 3.

N	K	AE	VAR	MSE	RAB	RE
50	14	4.19297	0.39855	0.037236260	0.0024121	0.0000096
	16	4.37777	0.37924	0.142707600	0.0047221	0.0000189
	18	4.14397	0.3863	0.020727810	0.0017996	0.0000072
	20	4.18335	0.44635	0.033618500	0.0022919	0.0000092
100	14	4.35052	0.30858	0.122866300	0.0087631	0.0000175
	16	4.12707	0.30501	0.016146470	0.0031767	0.0000064
	18	4.28403	0.33617	0.080672590	0.0071007	0.0000142
	20	4.15833	0.35124	0.025067730	0.0039582	0.0000079
150	14	4.5556	0.22732	0.308687500	0.0208349	0.0000278
	16	4.65125	0.22754	0.424126500	0.0244219	0.0000326
	18	4.63046	0.21271	0.397477800	0.0236422	0.0000315
	20	4.56747	0.20962	0.322022600	0.0212801	0.0000284
200	14	3.9937	0.0834	0.000039689	0.0003150	0.0000003
	16	3.98399	0.08524	0.000256479	0.0008007	0.0000008
	18	3.99881	0.08245	0.000001423	0.0000596	0.0000001
	20	3.98541	0.08756	0.000212790	0.0007294	0.0000007
250	14	3.99036	0.08577	0.000092940	0.0006025	0.0000005
	16	3.98208	0.08472	0.000321297	0.0011203	0.0000009
	18	3.93527	0.08498	0.004189878	0.0040456	0.0000032
	20	3.9783	0.08562	0.000470808	0.0013561	0.0000011
300	14	3.93916	0.0805	0.003701601	0.0045631	0.0000030
	16	3.95668	0.08193	0.001877052	0.0032494	0.0000022
	18	3.99073	0.08344	0.000086017	0.0006956	0.0000005
	20	3.95701	0.08588	0.001848183	0.0032243	0.0000021
350	14	4.00686	0.07921	0.000047000	0.0005999	0.0000003
	16	3.99498	0.07257	0.000025164	0.0004389	0.0000003
	18	3.98318	0.07199	0.000283081	0.0014722	0.0000008
	20	3.99961	0.07381	0.000000151	0.0000340	0.0000000
400	14	4.15348	0.01395	0.023555090	0.0153477	0.0000077
	16	4.14158	0.01444	0.020045290	0.0141581	0.0000071
	18	4.15447	0.01448	0.023859940	0.0154467	0.0000077
	20	4.15967	0.01436	0.025493660	0.0159667	0.0000080
450	14	4.15224	0.00352	0.023176080	0.0171267	0.0000076
	16	4.15218	0.00349	0.023158500	0.0171202	0.0000076
	18	4.1503	0.00373	0.022589320	0.0169085	0.0000075
	20	4.15238	0.00355	0.023219890	0.0171428	0.0000076
500	14	4.08102	0.00107	0.006564962	0.0101281	0.0000041
	16	4.07819	0.00109	0.006113074	0.0097733	0.0000039
	18	4.07784	0.00101	0.006058249	0.0097293	0.0000039
	20	4.08124	0.00104	0.006600007	0.0101551	0.0000041

Asymptotic optimum group limits Y_i (i=1, 2, …, k-1) in the form $Y_i = \alpha \left(\frac{y}{\theta}\right)^{\alpha}$ ($t_0 = 0, t_\infty$) to estimate Power Function Scale (θ) = 4 and Shape (α) = 3 from a grouped sample are given by

Table-5.5

k	x1	x2	x3	x4	x5	x6	x7	x8	x9	x11	x12	x13	x14	
3	0.913	1.75												
4	0.893	2.013	3.256											
5	0.784	2.568	3.021	2.145										
6	0.658	1.356	2.453	2.0147	2.568									
7	0.587	2.013	2.018	2.897	3.2145	3.254								
8	0.237	0.856	1.256	1.078	2.147	2.014	3.586							
9	0.365	0.475	1.352	1.982	2.589	2.658	3.214	3.689						
10	0.214	0.586	1.025	1.874	1.874	2.247	2.985	3.487	3.986					
11	0.147	0.201	0.021	1.863	0.269	0.478	1.024	2.548	2.457	3.88				
12	0.25	0.5790.	0.745	1.269	3.248	1.385	1.458	2.004	2.879	3.254	3.568			
13	0.536	0.247	0.658	1.124	2.781	1.852	1.985	2.147	2.014	2.1478	3.247	3.894		
14	0.214	0.452	0.546	0.8552	1.258	1.358	1.478	1.547	2.982	2.698	2.0145	3.0214	3.925	



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