



# IJRASET

International Journal For Research in  
Applied Science and Engineering Technology



---

# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

---

**Volume: 8      Issue: II      Month of publication: February 2020**

**DOI: <http://doi.org/10.22214/ijraset.2020.2007>**

**[www.ijraset.com](http://www.ijraset.com)**

**Call:  08813907089**

**E-mail ID: [ijraset@gmail.com](mailto:ijraset@gmail.com)**

# Analytical Design of Belleville Spring in Context with Load Carrying Capacity by using different Materials

Prof. Nitinchandra R Patel<sup>1</sup>, Vatsal K Dabhi<sup>2</sup>, Adnan H Fancy<sup>3</sup>

<sup>1</sup>Assistant Professor, <sup>2,3</sup>Undergraduate student, Department of Mechanical Engineering, G H Patel College of Engineering & Technology, Vallabh Vidhyanagar, Gujarat

**Abstract:** In this project we are going to use different materials which are in correspondence with the most usable material in the market which is used to make Belleville springs now using other materials which are alloys of the same material we will try to enhance the loading capacity and various other parameters which will help to improve the design of Belleville springs and in doing such we will use experimental data which we will gather through our experiments and research and using those data we will see whether the material that we have selected for manufacturing of the spring is really worth it or not.

**Keywords:** Belleville spring, Poisson's ratio, young modulus, spring rate

## I. INTRODUCTION

A disc spring is a conical shell which can be loaded along its axis either statically or dynamically [3]. The loads are normally applied to the upper inner edge and the lower outer edge. Either a single spring or a stack of springs can be used. A spring stack consists of either single springs or parallel spring sets. Disc springs are available either with or without contact flats. Although the disc spring has found a wider application during the last few decades, it is still an old established machine component. The original inventor is not known, but more than 130 years ago (on 26.12.1861 to be precise) Julien Francois Belleville of Dunkirk was granted French Patent Number 52399 for a spring design which already contained the principle of the disc spring. The importance of this invention achieved is unknown, but the fact that even today France and the Anglo-Saxon countries still speak of "Belleville Springs" infers its importance. Dissemination of this or similar springs. Today this tends to denote a disc spring of inferior quality, which still reflects the not always satisfactory design and function of springs at that time. This is no wonder considering that in the last century neither the theoretical conditions for calculations nor the necessary materials for manufacture were available. Not until 1917 did Fr. Dubois develop the theory on which the calculation of the disc spring is based in his dissertation "The Strength of the Conical Shell" at the ETH in Zurich. However, it still took several decades until this was adopted in practice. For a long time disc springs continued to be calculated – if at all – in accordance with the theory of the flat perforated plate. Then in 1936 two Americans, Almen and László, published a simplified method of calculation which allowed a quick and practically correct method for calculating disc springs. In the meantime the disc spring had been introduced into numerous areas of technology. Starting with applications in the construction of cutting and presswork tools, where the disc spring is especially advantageous because of the large number of variations possible with the same spring size, new applications were quickly found in machine, engine and motor vehicle manufacture[2]. These springs are widely used in applications where high spring rates and compact spring units are required [5]. Features of the Disc Spring: Compared with other types of springs, the disc spring has a number of advantageous properties, of which the following should be named:

Stock keeping is minimized, as the individual spring sizes can be combined universally. Because the springs are of an annular shape, force transmission is absolutely concentric.

- 1) Very large loads can be supported with a small installation space.
- 2) Depending on the dimensional relationships, its spring characteristic can be designed to be linear or regressive and with a suitable arrangement also progressive.
- 3) Due to the nearly unlimited number of possible combinations of individual disc springs, the characteristic curve and the column length can be further varied within additional limits.
- 4) High service life under dynamic load if the spring is properly dimensioned.
- 5) Provided the permissible stress is not exceeded, no impermissible relaxation occurs.
- 6) With suitable arrangement, a large damping effect may be achieved.



Fig.1 Disc springs

## II. ASSUMPTIONS IN DESIGN

Due to the relatively simple geometrical shape the complexity of disc springs in production and application is very often under rated. There are possibilities for mistakes in outlining a disc spring solution, which inevitably cause faulty design or even failures later on. Then it is very difficult to find better substitutes, because most of the times the installation space is fixed. With a correct design these problems are easy to avoid. The main difficulty is to realize these in the design stage to get an optimum disc spring solution. Since for most of the designers the disc spring is not daily bread and to many the rules for disc spring design are little known, the most important aspects are summarized here.

### A. Spring Force

The calculation of the force of a disc spring is based on a model by Almen and László. Its accuracy in the usable range of the character line of the spring is very good. Yet there is a slow rise at the beginning of the measured load/deflection curve, because disc springs never are perfectly symmetrical. They so to speak have to be pressed even. Also the spring force rises in the last part of the load/ deflection curve more than calculated, when the spring is loaded in between two parallel planes, since the leverage changes due to the never ideally even surfaces.

### B. Static Loading

In the design of a new disc spring a certain stress level should not be surpassed for static loading [6]. The maximum allowable limit is given by the reference stress  $\sigma_{om}$ . Its value should not exceed the value of the tensile strength  $R_m$  of the material to avoid plastic deformations of the spring, i.e. setting losses.

### C. Dynamic Loading

Most of the disc springs only can bear a limited dynamic load. The life time depends on the load span as well as on the load level. The number of cycles, which is to be expected under a certain load condition, can be estimated from fatigue diagrams. It is also necessary to preload disc springs in a dynamic application to at least 15% to 20% of their maximum deflection, to avoid compression-tension alternating stresses in the beginning of the deflection range of the spring.

### D. Material

The best material for disc springs is standard spring steel [4]. It is always used as long as there are no particular circumstances, which may necessitate a special material. In general special materials have a lower tensile strength and most of the times a different Young's modulus compared to the standard spring steels. Therefore springs out of these materials generally cannot be designed with the same free height, which means that the spring forces are lower.

### E. Temperature

The different materials have different temperature ranges. Too high temperatures may have a tempering offset, which again results in a loss of force and, in extreme cases, in plastic deformation.

### F. Corrosion

Disc springs can be protected against corrosion either by suitable coatings or by using corrosion resistant materials. Such materials are only available in a limited variety.

## III. DESIGN METHODOLOGY

### A. Analytical Design Of The Belleville Spring

#### 1) Diameter Ratio: ( $\delta$ )

The diameter ratio is given by,

$$\therefore \delta = \frac{D_o}{D_i}$$

Where,  $D_o = OD$ , mm

$D_i = ID$ , mm

$H =$  Height of spring, mm

$T =$  Thickness of spring, mm

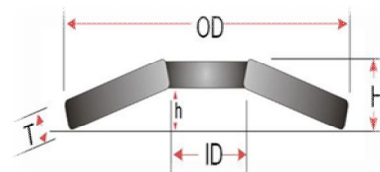


Fig. 2 Cross section of Belleville spring

It is the most important dimensionless parameter. It is used to find the spring constants for calculations.

2) *Spring Constants: (K1, K2, K3)*

The spring constants are depended on diameter ratio ( $\delta$ ) up to which the spring can show and for that the calculations are given by;

$$\therefore K_1 = \frac{1}{\pi} \cdot \frac{\left(\frac{\delta-1}{\delta}\right)^2}{\frac{\delta+1}{\delta-1} - \ln \delta}$$

$$\therefore K_2 = \frac{6}{\pi} \cdot \frac{\frac{\delta}{\ln \delta} - 1}{\ln \delta}$$

$$\therefore K_3 = \frac{3}{\pi} \cdot \frac{\delta-1}{\ln \delta}$$

Also, It is useful to find the spring force (F), the stresses ( $\sigma$ ) at different cross sections and other parameters of the spring.

3) *Spring Force of the Single Spring: (F)*

The spring force is depended on modulus of elasticity of material (E), Poisson's ratio

$$\therefore F = \frac{4E}{1-\mu^2} \cdot \frac{t^4}{K_1 \cdot D_e^2} \cdot \frac{S}{t} \left[ \left(\frac{h}{t} - \frac{S}{t}\right) \left(\frac{h}{t} - \frac{S}{2t}\right) + 1 \right]$$

4) *Stresses Induced in the Spring: ( $\sigma$ )*

The stresses at different cross section in spring [1] as shown in the figure

3 are given as:

$$\therefore \sigma_{OM} = \frac{-4F}{1-\mu^2} \cdot \frac{t^2}{K_1 \cdot D_e^2} \cdot \frac{S}{t} \cdot \frac{3}{\pi}$$

$$\therefore \sigma_1 = \frac{-4E}{1-\mu^2} \therefore \cdot \frac{t^2}{K_1 \cdot D_e^2} \cdot \frac{S}{t} \left[ K_2 \left(\frac{h}{t} - \frac{S}{2t}\right) + K_3 \right]$$

$$\therefore \sigma_2 = \frac{-4E}{1-\mu^2} \cdot \frac{t^2}{K_1 \cdot D_e^2} \cdot \frac{S}{t} \left[ K_2 \left(\frac{h}{t} - \frac{S}{2t}\right) - K_3 \right]$$

$$\therefore \sigma_3 = \frac{-4E}{1-\mu^2} \cdot \frac{t^2}{K_1 \cdot D_e^2} \cdot \frac{S}{t} \cdot \frac{1}{\delta} \left[ (K_2 - 2K_3) \left(\frac{h}{t} - \frac{S}{2t}\right) - K_3 \right]$$

$$\therefore \sigma_4 = \frac{-4E}{1-\mu^2} \cdot \frac{t^2}{K_1 \cdot D_e^2} \cdot \frac{S}{t} \cdot \frac{1}{\delta} \left[ (K_2 - 2K_3) \left(\frac{h}{t} - \frac{S}{2t}\right) + K_3 \right]$$

5) *Stiffness:* Spring rate is calculated by,

$$\therefore R = \frac{dF}{dS} = \frac{4E}{1-\mu^2} \cdot \frac{t^3}{K_1 \cdot D_e^2} \cdot \left[ \left(\frac{h}{t}\right)^2 - 3 \cdot \frac{h}{t} \cdot \frac{S}{t} + \frac{3}{2} \cdot \left(\frac{S}{t}\right)^2 + 1 \right]$$

$$\therefore R = \frac{F_2 - F_1}{S_2 - S_1} \text{ N/mm}$$

6) *Work Done:* Work done is in (J) Joule is calculated by,

$$\therefore W = \int_0^S F \cdot ds = \frac{2E}{1-\mu^2} \cdot \frac{t^5}{K_1 \cdot D_e^2} \cdot \left(\frac{S}{t}\right)^2 \left[ \left(\frac{h}{t} - \frac{S}{2t}\right)^2 + 1 \right]$$

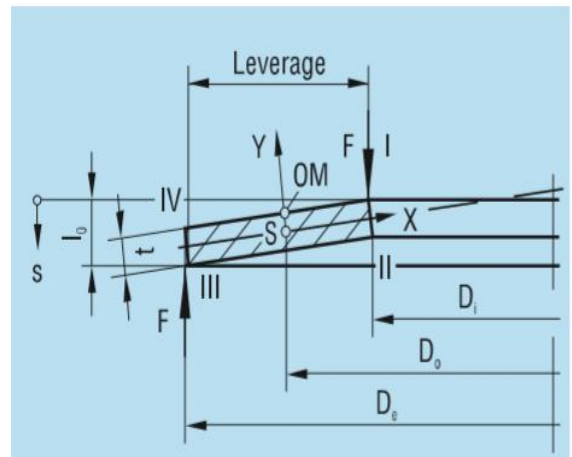
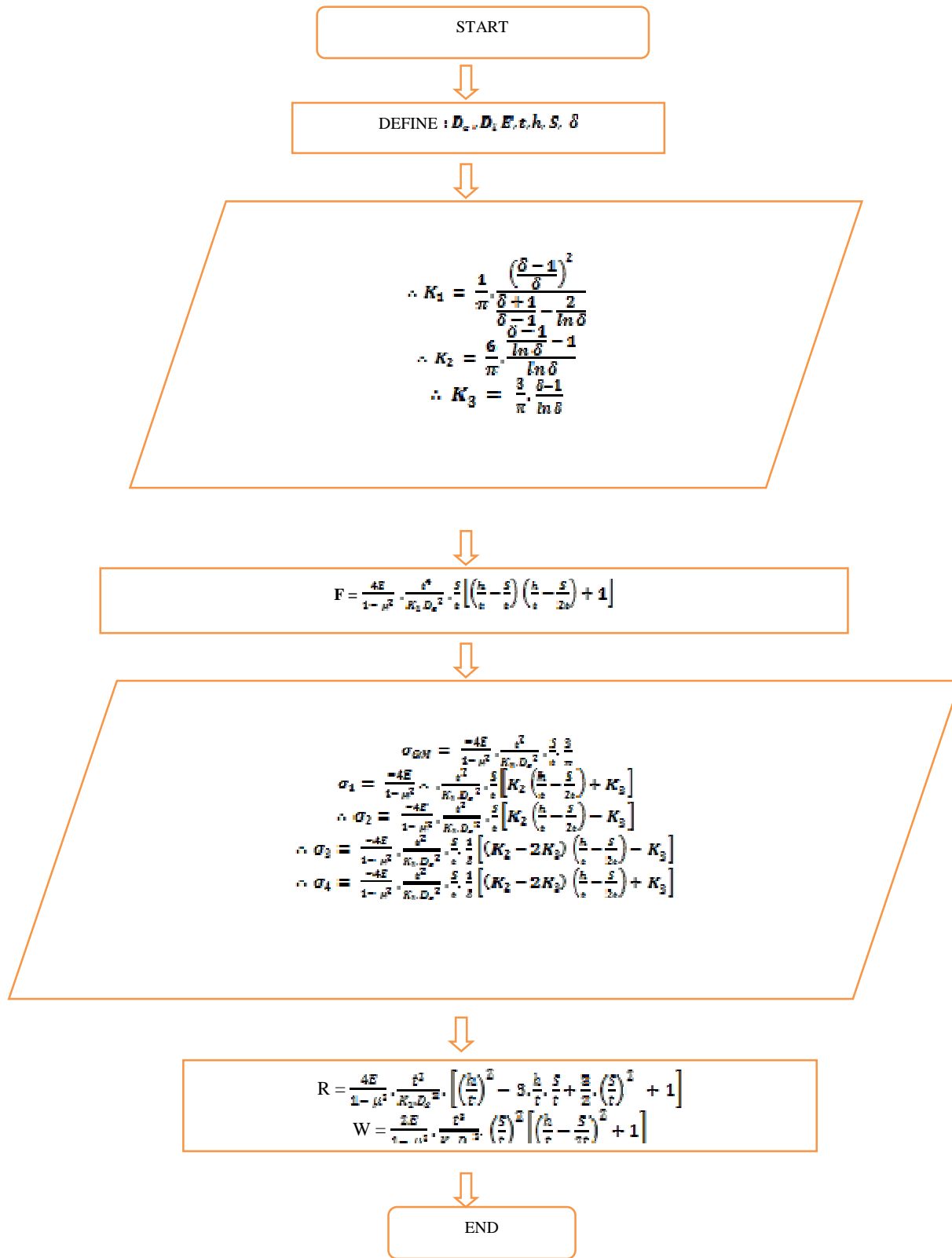


Fig.3 Cross section of Belleville spring

B. Flowchart For Design Of Belleville Spring



C. Programming in MATLAB

```
%Design Of Belleville Spring
OD = 125;
ID = 62.5;
E = 200*(10^3);
MUE = 0.285;
t = 2;
h = 4.5;
S = 0.2422;
disp ('Diameter ratio')
DR = OD/ID
disp ('Spring cooefficient K1')
K1 = (1/3.141592654)*(((DR - 1)^2)/(DR^2))/(((DR + 1)/(DR - 1)) - (2/log(DR)))
disp ('Spring cooefficient K2')
K2 = (6/3.141592654)*(((DR - 1)/log(DR)) - 1)/(log(DR))
disp ('Spring cooefficient K3')
K3 = (3/3.141592654)*(DR - 1)/(log(DR))
disp ('Stress at OM')
SigmaOM = ((-4*E)/(1-(MUE^2)))*((t^2)/(K1*(OD^2)))*(S/t)*(3/3.141592654)
disp ('Stress at 1')
sigma1 = ((-4*E)/(1-(MUE^2)))*((t^2)/(K1*(OD^2)))*(S/t)*((K2)*((h/t) - (S/(2*t)))) + (K3))
disp ('Stress at 2')
sigma2 = (-4*E)/(1-(MUE^2))*((t^2)/(K1*(OD^2)))*(S/t)*((K2)*((h/t) - (S/(2*t)))) - (K3))
disp ('Stress at 3')
sigma3 = (-4*E)/(1-(MUE^2))*((t^2)/(K1*(OD^2)))*(S/t)*(1/DR)*(((K2) - ((2)*(K3)))*((h/t) - (S/(2*t)))) - (K3))
disp ('Stress at 4')
sigma4 = (-4*E)/(1-(MUE^2))*((t^2)/(K1*(OD^2)))*(S/t)*(1/DR)*(((K2) - 2*(K3))*((h/t) - (S/(2*t)))) + (K3))
disp ('stiffness')
R = (4*E)/(1-(MUE^2))*((t^3)/(K1*(OD^2)))*((h^2/t^2) - (3*(h/t)*(S/t)) + ((1.5)*((S^2)/(t^2)))) + 1)
disp ('spring work')
W = (2*E)/(1-(MUE^2))*((t^5)/(K1*(OD^2)))*((S^2)/(t^2))*((1/(t^2))*((h^2) - (h*S) + (S^2)) + 1)
```

IV. RESULTS AND DISCUSSION

In the recent design by taking range of forces from 1000 N to 3000 N, deflection is calculated and stresses are found at different points on belleville spring. Also spring rate and work done is determined accordingly. Here materials are changed to find effect of spring for its working capabilities. From different cross sections at OM, 1, 2 has negative value of stresses and point 3 and 4, has the stresses which are positive. In order to find safer design and associated deformation within limit, focus is made in higher positive values which is the most critical point and able to create the failure.

The calculated parameters are given as below:

1) *Material:* Copper alloys (Berrelium copper)

Table – 1 Calculated design parameters

F (N)	S (mm)	$\sigma_{OM}$ (N/mm <sup>2</sup> )	$\sigma_1$ (N/mm <sup>2</sup> )	$\sigma_2$ (N/mm <sup>2</sup> )	$\sigma_3$ (N/mm <sup>2</sup> )	$\sigma_4$ (N/mm <sup>2</sup> )	R (N/mm)	W (J)
1000	0.4530	-44.8684	-187.1939	-57.7311	109.4492	44.7178	1912.9	0.236
1500	0.7412	-73.4137	-299.5307	-87.7034	174.8282	68.9146	1562.9	0.599
2000	1.1108	-110.0216	-435.9064	-118.4512	253.8328	95.1052	1151.8	1.2516
2500	1.6898	-167.3699	-632.1748	-149.2424	366.6633	125.1996	593.376	2.5707
3000	2.4094	-238.6442	-846.5467	-157.9651	488.2882	143.9974	44.6951	4.4735

The maximum stresses are induced at section 3 as compared to other specified sections. The values are in increasing order along with the applied load. The same thing is happened for deflection. The graphs show the nature of increasing order. In case of copper alloy, in particular range of load, maximum deflection is 2.4094mm and the stress at point 3 is 488.28 N/mm<sup>2</sup>

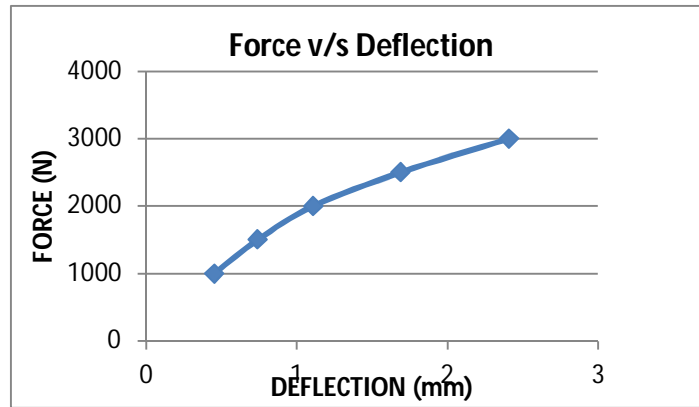


Fig.4 Force vs deflection (Copper alloys)

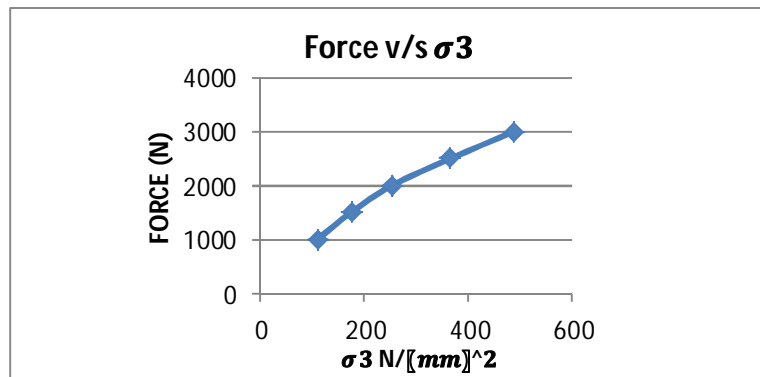


Fig.5 Force vs sigma3 (Copper alloys)

2) *Material:* Stainless steel

Table – 2: Calculated design parameters

F (N)	S (mm)	$\sigma_{OM}$ (N/mm <sup>2</sup> )	$\sigma_1$ (N/mm <sup>2</sup> )	$\sigma_2$ (N/mm <sup>2</sup> )	$\sigma_3$ (N/mm <sup>2</sup> )	$\sigma_4$ (N/mm <sup>2</sup> )	R (N/mm)	W (J)
1000	0.2817	-42.6690	-180.380	-57.2354	105.5534	43.9950	3262.2	0.144
1500	0.4430	-67.1010	-280.1644	-86.5517	163.8171	67.0108	2944.6	0.3471
2000	0.6234	-94.4262	-388.8142	-116.3579	227.1033	90.8751	2604	0.6639
2500	0.8304	-125.7804	-509.6055	-146.6801	297.2794	115.8167	2232.4	1.1313
3000	1.0778	-163.2540	-648.5339	-177.4826	377.7291	142.2034	1894.9	1.8141

The maximum stresses are induced at section 3 as compared to other specified sections. The values are in increasing order along with the applied load. The same thing is happened for deflection. The graphs show the nature of increasing order. In case of stainless steel, in particular range of load maximum deflection is 1.0778mm and stress at point 3 is 377.72 N/mm<sup>2</sup>

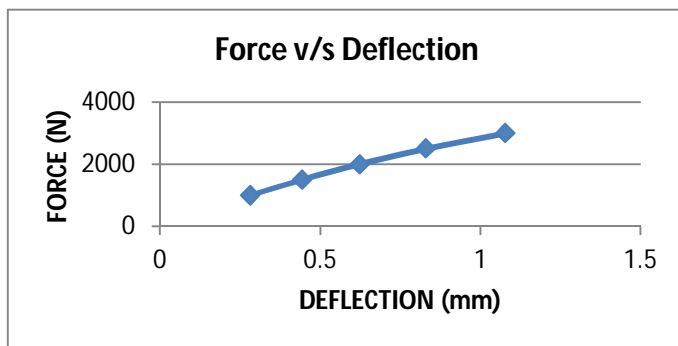


Fig.6 Force vs deflection (Stainless steel)

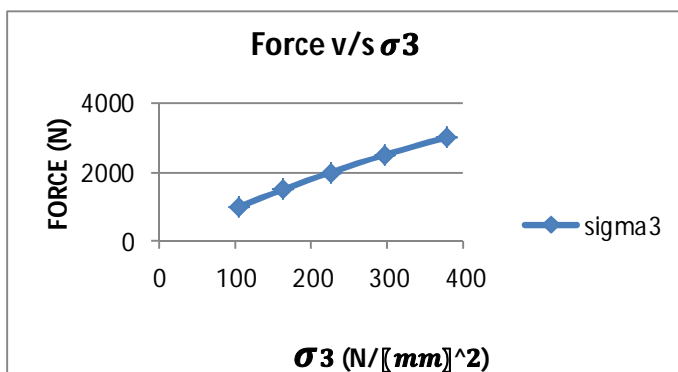


Fig.7 Force vs sigma3 (Stainless steel)

3) Material: Aluminium

Table – 3: Calculated design parameters

F (N)	S (mm)	$\sigma_{OM}$ (N/mm <sup>2</sup> )	$\sigma_1$ (N/mm <sup>2</sup> )	$\sigma_2$ (N/mm <sup>2</sup> )	$\sigma_3$ (N/mm <sup>2</sup> )	$\sigma_4$ (N/mm <sup>2</sup> )	R (N/mm)	W (J)
1000	0.9221	-51.7967	-208.3403	-58.8865	121.4658	46.7389	769.247	0.507
1500	2.006	-112.6822	-414.2952	-89.1032	239.6951	77.1291	188.5334	1.9208
2000	2.2915	-128.7195	-461.455	-90.0492	266.4224	80.7195	70.0714	2.3554
3000	9.5402	-535.8976	-680.691	865.582	388.370	-444.7578	1881.2	10.903

Here at section 3, the maximum stresses are induced. The graphs show the nature of increasing order. In case of aluminium in particular range of load, maximum deflection is 9.5402mm and stress at that point is 388.37 N/mm<sup>2</sup>

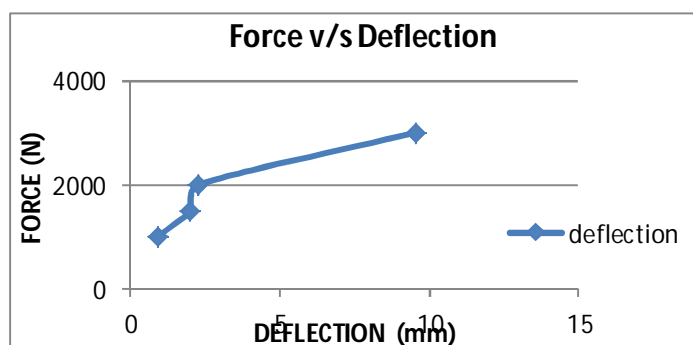


Fig. 8 Force vs deflection (Aluminium)



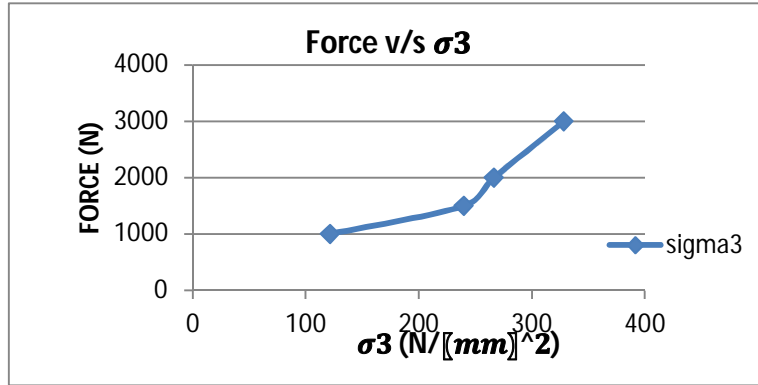


Fig.9 Force vs sigma3 (Aluminium)

4) Material: Titanium

Table – 4: Calculated design parameters

F (N)	S (mm)	$\sigma_{OM}$ (N/mm <sup>2</sup> )	$\sigma_1$ (N/mm <sup>2</sup> )	$\sigma_2$ (N/mm <sup>2</sup> )	$\sigma_3$ (N/mm <sup>2</sup> )	$\sigma_4$ (N/mm <sup>2</sup> )	R (N/mm)	W (J)
1000	0.5278	-45.8785	-190.3123	-57.935	111.2234	45.0347	1596.8	0.2782
1500	0.8826	-70.7191	-309.5525	-88.1879	180.5191	180.5191	1229.2	0.725
2000	1.3871	-120.5723	-467.069	-119.1719	271.4782	271.4782	765.553	1.61
2500	2.4463	-212.6422	-751.8035	-138.2479	433.5084	433.5084	18.34	4.01
3000	2.3145	-201.1856	-719.76	-139.2671	415.4821	415.4821	94.63	3.69

Here at section 3 & 4, the maximum stresses are induced at a load of 2500 N. The graphs show the nature of increasing order after that little bit decrease. In case of aluminium in particular range of load, maximum deflection is 2.4463mm and stress at point 3 & 4 is 433.5084 N/mm<sup>2</sup>. Here load for spring material, load resistance capacity is decreased after some maximum load and hence stress induced is also decrease along with work done.

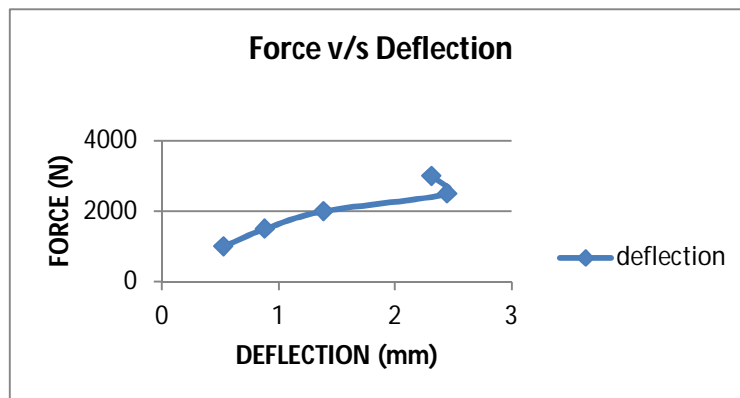


Fig.10 Force vs deflection (Titanium)

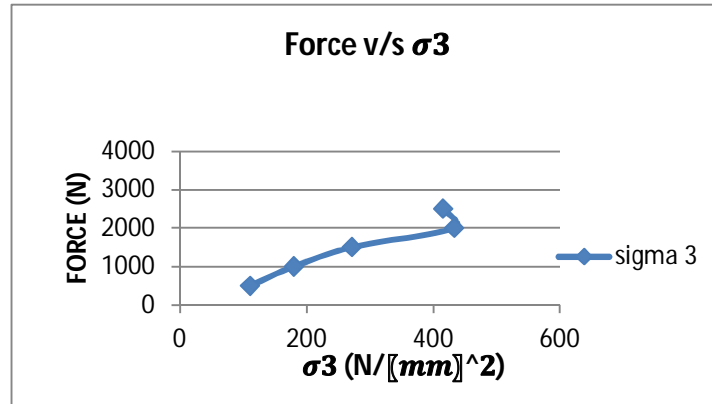


Fig.11 Force vs stress (Titanium)

5) *Material:* High carbon steel

Table - 6: Calculated design parameters

F (N)	S (mm)	$\sigma_{OM}$ (N/mm <sup>2</sup> )	$\sigma_1$ (N/mm <sup>2</sup> )	$\sigma_2$ (N/mm <sup>2</sup> )	$\sigma_3$ (N/mm <sup>2</sup> )	$\sigma_4$ (N/mm <sup>2</sup> )	R (N/mm)	W (J)
1000	0.2785	-42.61	-178.40	-59.076	104.48	44.8185	3301.96	0.143
1500	0.4326	-66.95	-66.95	-86.863	163.37	66.82	2985.15	0.343
2000	0.6153	-94.146	-94.146	-116.368	226.35	90.616	2646.09	0.655
2500	0.8186	-125.25	-125.25	-176.06	319.541	133.936	2276.01	1.1151
3000	4.3820	-670.48	-670.48	-22.047	1105.18	134.535	977.89	14.38

Here at section 3, the maximum stresses are induced. The graphs show the nature of increasing order. In case of HCS in particular range of load, maximum deflection is 4.382mm and stress at that point is 1105.18 N/mm<sup>2</sup>

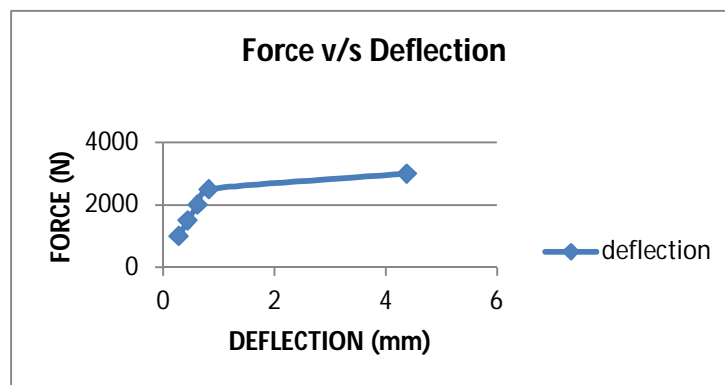


Fig.12 Force vs deflection (High carbon steel)

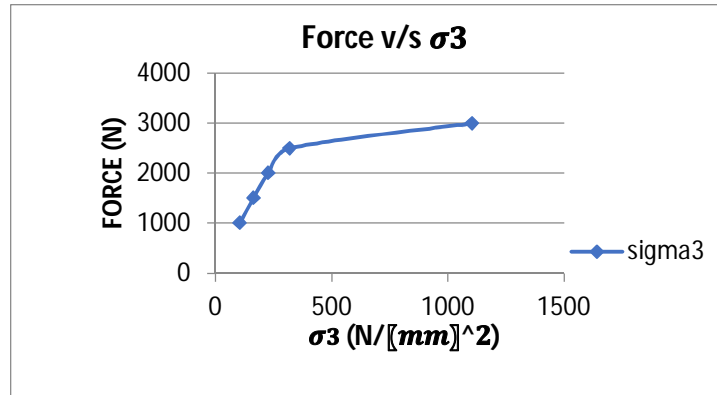


Fig.13 Force vs sigma3 (High carbon steel)

## V. CONCLUSION

Belleville spring has been analysed in this investigation with its unique features the forces and deflections are determined by using the standard formulas. Also the values of the stresses at different cross sections are generated on the spring's washer and which are calculated by using Matlab programming. We are finding the forces and stresses in this work and observed that the values which is positive are tensile stress and negative are compressive stress.  $\sigma_3, \sigma_4$  are tensile stresses and  $\sigma_{OM}, \sigma_1, \sigma_2$  are compressive stresses. For using different materials at various loads, it is essential to determine capability for various mechanical properties like stress, deflection, stiffness and work done. By grouping loading range in to low load and higher load, the best suited materials are found for particular property or application.

- 1) For induced stress in material at various cross section, the noticeable value is found at point 3, here High carbon steel shows lower value i.e. 104.48 N/mm<sup>2</sup> at load of 1000 N, while Stainless steel has lower value i.e 377.729 N/mm<sup>2</sup> at load of 3000 N
- 2) For deflection of spring, High carbon steel shows its lesser value at low load. i.e 0.2783 mm at 1000 N and Stainless steel shows lesser value at higher load. i.e 1.0778 mm at 3000 N.
- 3) For stiffness of spring, High carbon steel has much better value at low load. i.e 3301.98 N/mm at 1000N and Stainless steel shows more value at higher load. i.e 1894.9 N/mm at 3000 N.
- 4) Similarly, for work done within different materials, Aluminium shows satisfactory value at low load i.e 0.507 J at 1000 N and High carbon steel has higher value at heavy load. i.e 14.38 J at 3000 N.

Ultimately, for low load condition, high carbon steel is much better than other materials and for high load application Stainless steel is more suitable to manufacture such kind of spring.

## REFERENCES

- [1] Higley, Joseph Edward; Mischke, Charles R.; Brown, Thomas H. (2004), Standard handbook of Machine design (3rd ed.), McGraw-Hill Professional, ISBN: 978-0-07-144164-3.
- [2] Jump up to: Smith, Carroll (1990), Carroll Smith's Nuts, Bolts, Fasteners, and Plumbing Handbook, Motor Books/MBI Publishing Company, p. 116, ISBN 0-87938-406-9.
- [3] Bhandari, V. B. (2010), Design of Machine Elements (3rd ed.), Tata McGraw-Hill, New Delhi, ISBN: 978-0-07-068179-8.
- [4] Bhandari, V.B.(2014), Machine Design Data Book, McGraw Hill, New Delhi, ISBN: 978-93-5134-284-7
- [5] Khurmi R.S. (2013), Machine Design, Eurasia Publishing House, Ram Nagar, New Delhi, ISBN: 81-219-2537-1
- [6] Bansal, R.K (2001), Strength of materials (Reprint), Laxmi Publications (P) Ltd, New Delhi, ISBN:81-7008-147-5

#### AUTHORS' BIOGRAPHY



Prof. Nitinchandra R. Patel is an Assistant Professor in Mechanical Engineering Department of G. H. Patel College of Engineering & Technology, Vallabh Vidyanagar, Gujarat, India. He did Master degree in Machine Design in 2004 from Sardar Patel University, Vallabh Vidyanagar and Bachelor degree in Mechanical Engineering in 1997 from B.V.M Engineering College, Sardar Patel University. He has more than 22 years' experience including teaching and industries. He has presented 2 technical research papers in International conferences & published 1 technical research paper in National journal and 23 research papers in International journals. He reviewed a book published by Tata McGraw Hill. He is a Member of Institute of Engineers (I) and Life member of ISTE. He is a reviewer / member in Editorial board of various Peer-reviewed journals. He is also recognized as a Chartered Engineer by Institute of Engineers (I) in Mechanical Engineering Division in 2012.



Vatsal K Dabhi is a final year undergraduate student at Department of Mechanical Engineering, G H Patel College of Engineering & Technology, Vallabh Vidhyanagar, Gujarat, India



Adnan H Fancy is a final year undergraduate student at Department of Mechanical Engineering, G H Patel College of Engineering & Technology, Vallabh Vidhyanagar, Gujarat, India



10.22214/IJRASET



45.98



IMPACT FACTOR:  
7.129



IMPACT FACTOR:  
7.429



# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089  (24\*7 Support on Whatsapp)