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A Study of Behavior of Prime Number Sets with an Orderly Distribution in a Coordinate Arrangement

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Abstract: Prime Numbers distribution has been fascination and poplar amongst mathematicians for a long time. Several attempts have been made to observe patterns, provide generalized equations, and produce quantitative results to understand the distribution of primes. In this paper two ways of graphical arrangement of prime numbers been investigated to check if the different size of prime sets shows predictability will make the detection of position of unknown primes by using the known prime sets of smaller size faster and easy. The application of primality test to numbers can be focused using this study on hat position where there is most probability of the existence of prime number.
Keywords: Mathematics, Prime number, Prime number distribution, Graphical prediction of prime numbers,

I. INTRODUCTION

Prime numbers are the building blocks of numbers. These numbers appear randomly on the number line. There have been many attempts to generalize the distribution of primes. Primes are the essence of modern day communication encryption, secured bank transaction, secured passwords etc. Due to their unique properties, decoding encryption using supercomputers takes years. Recent research show prime numbers play an important role in physics and biology, so their detailed distribution has also come under increased scrutiny [6, 7].
Polish Mathematician Stanislaw Ulam’s (1909-1986) in 1963 created a spiral while doodling during a boring talk [2]. He suggested that it is the property of the visual brain which allows one to discover lines and pattern in the Ulam’s spiral [1]. He also comments of his spiral that it appears to exhibit a strongly non-random appearance (Stein et al. 1964) [3].
Euler famously quoted on the topic of distribution of primes “Mathematicians have tried in vain to this day to discover some order in the sequence of prime numbers, and it is a mystery into which the mind will never penetrate “(Ivan’s Peterson in Science News 5/4/2002) [3]. The motivation for investigation of primes in this paper has been the above quote.
In the RSA mechanism, used to protect the data secrecy has its secret key equivalent to the problem of prime number factoring. The need is to provide prime numbers in order to make it one-way hash function. To check if the numbers are prime Miller-Rabin, AKS etc. methods are used. [4]. These methods are calculation intensive and take time. To avoid wasting time on numbers that may not be primes this paper presents a graphical solution to tackle the problem.
“It is evident that the primes are randomly distributed but, unfortunately, we don't know what 'random' means." R.C. Vaughan (1990). The author in this paper [8] suggests that, defining whether a sequence is random is not an easy task.

II. ORIGIN OF IDEA

The Prime numbers are almost random based on their position on the number line. But it is difficult to sense the variations of primes on a number line. One of the reasons for this irregularity in distribution of primes is that no simple formula exists for producing all the primes [5]. The primes on the Ulam’s spiral tend to confirm that the primes become less and less likely as the number get larger and larger. The dots are the primes. These primes form diagonals. The numbers that lie on these diagonals can be represented by a quadratic equation like $x^2 - x + 41$. [9].

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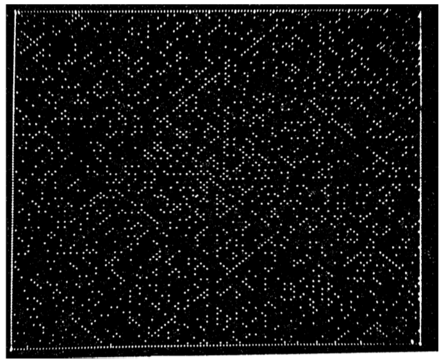

Fig 1: Ulam's Spiral [1]

The Ulam's spiral if plotted till the infinity will show the complete behavior of the primes. The plane in which it is being plotted also has to be infinite. Currently, it can be plotted finitely. Many analytical methods are used such as equations for the diagonals.
Instead of considering the infinite primes being plotted in any arrangement like Ulam's spiral, this problem if converted in such a way that finite primes plotted in a specific geometric arrangement would give the same behavior as infinite primes. Analogous to chemistry, the properties of water do not change if it is taken in small or large quantity. This solution can approximate the position of the prime numbers on the number line and also save time in primality testing.

III. ASSUMPTION

Let X={x: x is all the prime numbers} and P= {p: p are prime numbers p<n}. Now, X is plotted on an infinite plane in a specific arrangement and same done for P on a finite plane in the same arrangement as X then, the behavior shown at the particular section of the X plot will have approximately same behavior at the equivalent section of the P plot. For the sake of simplicity the arrangement is uniform.
If P= {p: p are prime numbers p<n} and Q = {q: q are prime numbers q<m} .If X plot ≡ P plot and X plot ≡ Q plots according to the assumption, P plot ≡ Q plot, provided that the Q plot is having of the same arrangement.
This property will open the possibility of predicting the positions of the Q plot primes in the P plots if the P plot is known or vice versa. If it is possible to approximately predict the position of the prime then less numbers need to be checked for primality and indeed save time.

IV. METHOD USED

The above assumption has been tested using two types of plots.

A. Radical Plots

The sets P and Q are used for P plot and Q plot respectively. The lines are drawn from the center of the circle to the circumference. These lines are the radius of the circle. Each line corresponds to individual numbers form the sets. Starting from 2 to N is plotted (N is general variable). According to the assumption, the plot is kept equidistant and evenly spread. Therefore, the angle between each line is equal. If the total angle is 360 degree then the angle between the lines will be

$$\theta = \frac{360}{N}\ , N\ is\ total\ number\ of\ Primes$$

The plot will be having the shape of a spiral

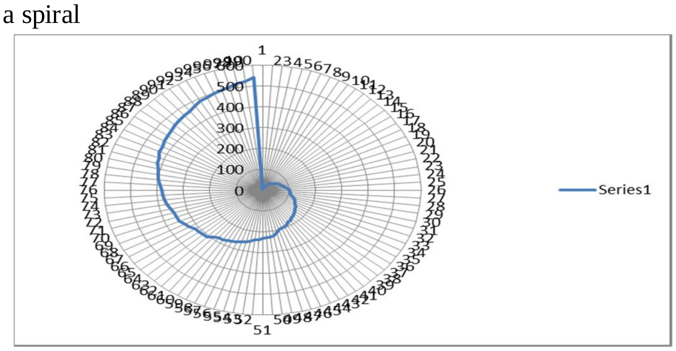

Fig 2: Plot of first 100 primes

Fig 1 shows example plot. In this the primes are points but are joined continuously to visualize the distance. This distance is used as comparison. Notice, there is a line that is joining the 100th prime and the 1st prime. This line is not a part of the calculation.

In this way both the plots of the P and Q will be produced. As the number of primes is different, the finite plane occupied by both of them will be different. In order to verify the assumption both the plots need to be made comparable. The behavior of the equivalent sections (areas) in this case is measured by the distance between the points and the radial distance. Here, the angle θ in P plot is made proportional to the θ in the Q plot.

By using the property of similar triangles the distance between on the Q plot is calculated using the data from the P plot. The actual distance on the Q plot is compared to the calculated distance. The same operation is done for all the distances of the Q plot. The average error will show the result of our assumption.

1) Calculation for Radial Plots

Let, swept angle be $\phi 1$ for P-plot and $\phi 2$ for Q plot. P plot > Q plot (Size).

The angle of sweep or ϕ will never be equal to 360° except in infinite prime number plot angle $\phi \rightarrow 360°$.

$$\phi 1 = 360 - \frac{360}{N} = 360(1 - \frac{1}{N})$$

$$\phi 2 = 360 - \frac{360}{M} = 360(1 - \frac{1}{M})$$

All the prime numbers are on the radial lines. These radial lines are at fixed angles on the plot. These angles will be used as position indicator on the plots with respect to ϕ or space.

For example a 100 prime plot, $\theta = \frac{360}{100} = 3.6°$. So, the primes will be having the positions as $2 \rightarrow 0°, 3 \rightarrow 3.6°, 5 \rightarrow 7.2°, 7 \rightarrow 10.8°$ … ….. and $\phi = 356.4°$

The region $(18° \rightarrow 21.6°)$ in the 100 plot will be equal to region $(18.1727273° \rightarrow 21.8072727°)$ on the 2000 number plot.

$$18.1727273° = \frac{18°}{356.4°} * 359.82°$$

$$21.8072727° = \frac{21°}{356.4°} * 359.82°$$

359.82° is the ϕ for 2000 prime number plot. According to the assumption the behavior between $(18° \rightarrow 21.6°)$ in 100 prime plot will be equal to $(18.1727273° \rightarrow 21.8072727°)$ region in the 2000 prime plot. The above calculation is done to find equivalent region.

The generalized formula is:

$$Pos1 = \frac{pos2}{\phi 2} * \phi 1 - - - (1)$$

Pos1 and $\phi 1$ are the position and swept angle in P plot respectively and pos2 and $\phi 2$ are the position and swept angle in Q plot respectively.

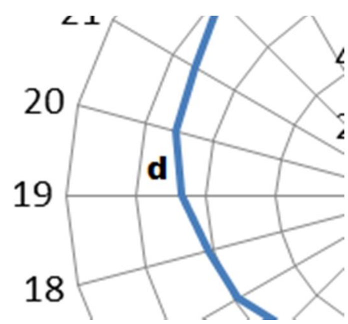

Fig 3: Single distance in the Q plot marked 'd'.

The distance between the prime numbers is the blue line. It is calculated using the cosine rule,

$$d = \sqrt{[p1^2 + p2^2 - 2\,p1\,p2\,cos\theta]}$$

For this calculation, the Microsoft Excel software has been used. This distance is found out for both the plots P and Q. This data is necessary for verification.

Since we have made the plots comparable the region OBC and OAD are considered as similar triangles. The distance BC is the sum of all the distances in the equivalent range calculated from (1-equation) in P plot. OB is the distance from the center to the point where it intersects the distance (blue) line on the P plot. Both are represented in same triangle for simplicity.
Similarly, AO is the distance from the center to the point where it intersects the line on Q plot.
Using the similar triangles formula:

$$\frac{x}{AO} = \frac{BC}{BO}$$

$$x = \frac{BC}{BO} * AO$$

Here 'x' is the calculated distance between the two prime numbers in the Q plot. The error in the calculated data will tell whether the assumption is consistent.

$$Error = |d - x|$$

2) *Analysis for Radial Plots:* The P plot is 1-2000 prime number and Q plot size 100 primes. The error vs position chart has maximum scatter near zero.The average error in the above case is 0.716472426. The average percentage error is 1.810549 (excluding the end values.) The results for plot (1-200) also shows the same behaviour. That all the error is located near line 0. The average error is -0.060381986. Average % error is -4.28795. These results show that the assumption considered is demonstrable.

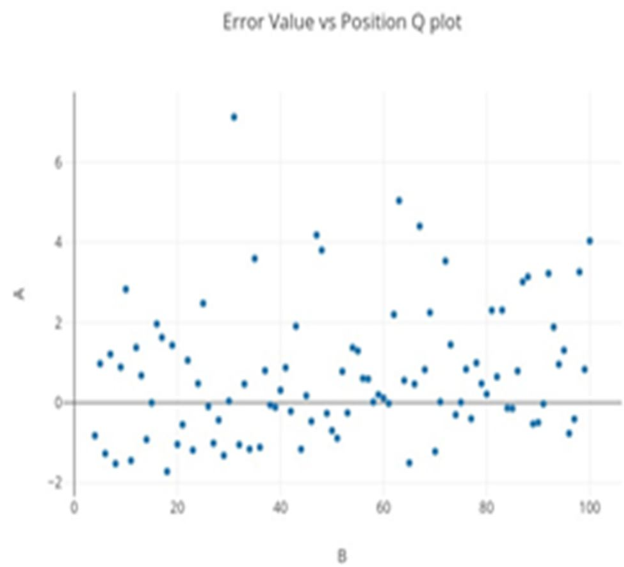

Fig 4

B. Cartesian Plots

The Cartesian plots are produced in the same (XY) plane. The P plot will be brought to the scale of the Q plot vice versa. The plot is shown in Figure 5. This particular arrangement was studied because it is most widely used. The Q plot is plotted to scale but the P plot being different in size is brought to the scale of the Q plot. The both the scatter plots should coincide. This is done to verify the assumption.

1) *Calculation for Cartesian Plots:* Let, $P \to the\ last\ prime\ from\ P\ plot$ and $p \to the\ last\ prime\ from\ Q\ plot$. Let $P > p$. In order to plot to the scale of the Q plot, the following is done,

$$P_{Y-axis} = \frac{p}{P} * P\ (prime\ numbers)$$

Let, $x \to position\ of\ last\ prime\ on\ Q\ plot$ and $X \to position\ of\ last\ prime\ on\ P\ plot$. The position of P prime in Q plot scale is given by,

$$P_{X-axis} = \frac{x}{X} * P\ (position\ of\ prime\ no.)$$

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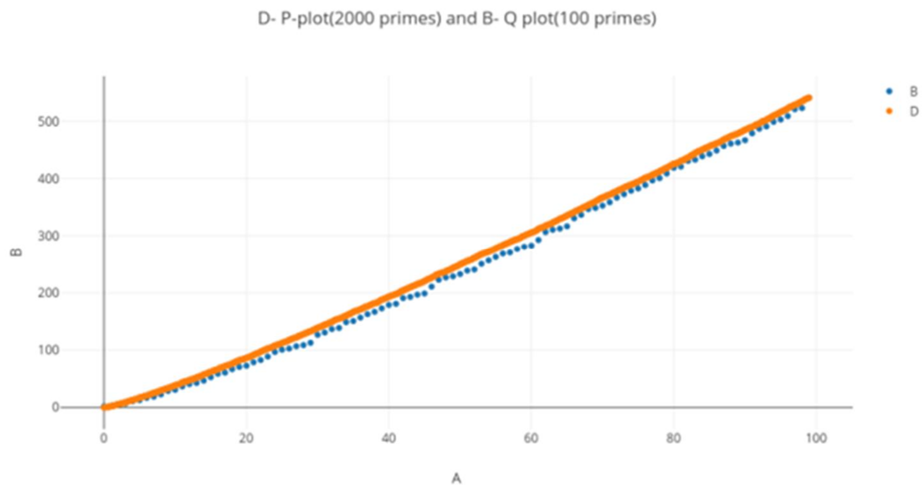

Fig 5

For Q plot of 100 primes and P plots of 2000 primes, the prime 9803 with position 1209 in the 'P' will be represented as 304.9872333 at the position 59.82591296 in 'Q' using the above relations. This type of plot is easier to calculate and analyze.

2) *Analysis for Cartesian Plots:* The following analysis is made using the Fig 5. The example chosen is the P plot (2000 primes) and Q plots (100 primes). The values that were predicted were close to the actual values. These values can be used to start the search of prime numbers. This approach will lead to the prime numbers faster. The following are some results.

Q Plot Prime numbers	The Values by Scaling P Plot to Q Plot Scale.
5	5.631204
431	437.1494
97	107

V. CONCLUSION

The process of large prime generation requires a lot of time and computer resources is suggested by the authors of [4]. In his paper, it has been demonstrated that the prime numbers when taken in groups of first 'n' primes and arranged in a particular arrangement and made comparable by mathematical scaling show the behavior of prime in the equivalent positions.

The Radial plots and Cartesian plots were taken for the investigation of the assumption. The test results of particular examples have been used to justify the merit of the assumption. Different combinations on investigation also gave the similar result indeed justifying the assumption. The above investigation was aimed at the detection of the probable position of the larger prime numbers by arrangement of small set of known prime numbers. The ideas suggested in [9] were interpreted in a different way and were investigated. The further correction methods in the prediction of the values using the ideas suggested will make detection of prime numbers faster.

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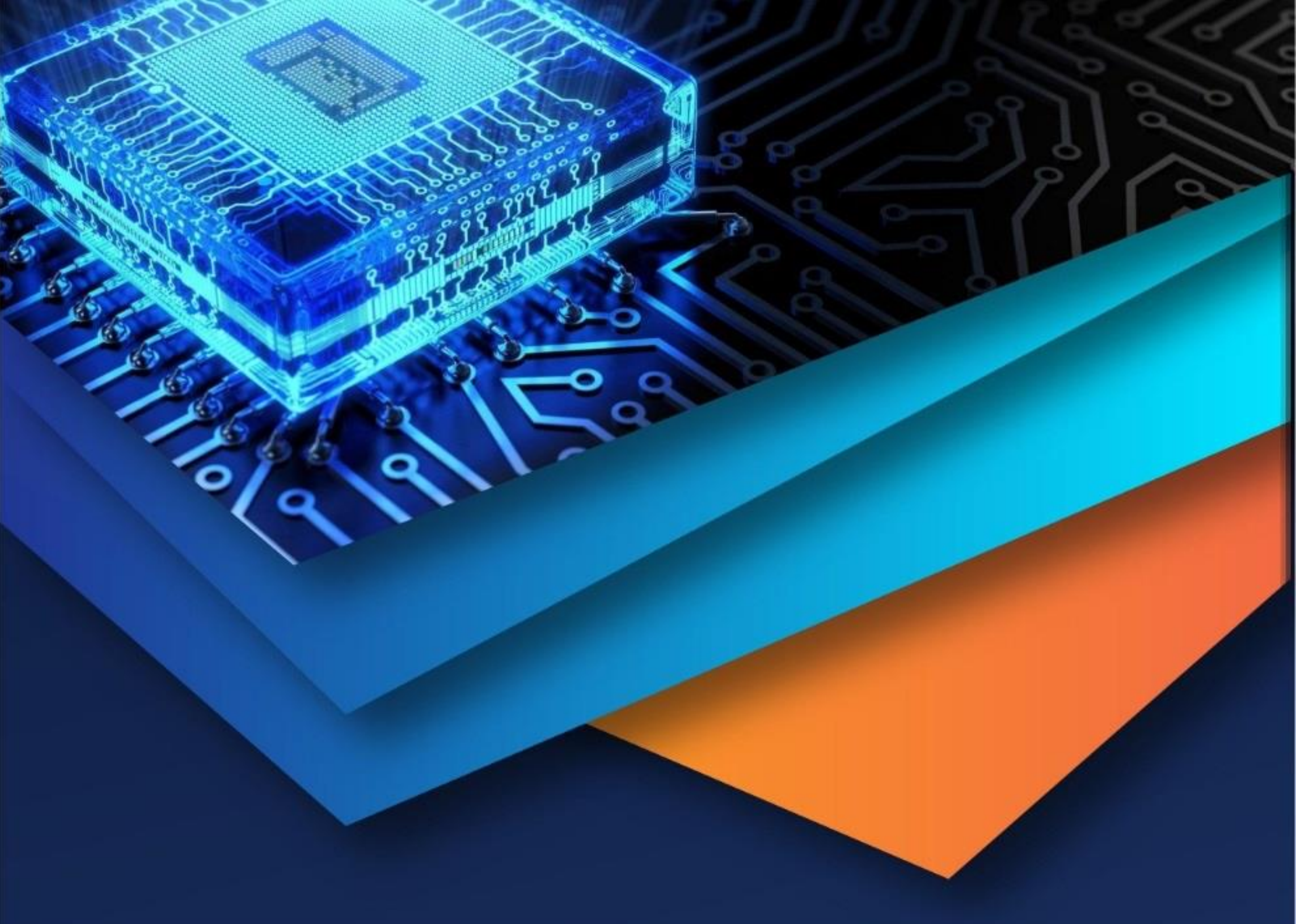

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