iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 3 **Issue:** VI **Month of publication:** June 2015

DOI:

Volume 3 Issue VI, June 2015
ISSN: 2321-9653
International Journal for Research in Applied Science & Engineering Technology (IJRASET)

An Energy Relationship in Magnetohydrodynamic Triply Diffusive Convection Problem

Jyoti Prakash[1], Renu Bala[2], Virender Singh[3]
Department of Mathematics and Statistics, Himachal Pradesh University, Shimla – 171005 (India)

An attempt has been made to establish the relationship between various energies in Magnetohydrodynamic triply diffusive convection problem, the configuration being considered analogous to thermohaline convection of Stern (Stern, M.E., Tellus. 12 [9], 172-175) type. The established relationship shows that the total kinetic energy associated with a disturbance exceeds the sum of its total magnetic and thermal energies in the parameter regime $\frac{Q\sigma_1}{\pi^2}+\frac{|R|\sigma}{\pi^4}\leq 1$, where Q, σ, σ_1 and R represent the Chandrasekhar number, the thermal Prandtl number, the magnetic Prandtl number and the Rayleigh number respectively. Further, this result is valid for the quite general nature of the bounding surfaces.
Keywords: Triply diffusive convection; Chandrasekhar number; Lewis number; Prandtl number; Rayleigh number.

I. INTRODUCTION

Convective phenomena which are driven by the differential diffusion of two properties such as heat and salt is known as thermohaline convection or more generally double diffusive convection. Double diffusive convection has matured into a subject possessing fundamental departure from its counterpart, namely single diffusive convection and its studies have importance in the fields of oceanography, limnology, chemical engineering, geophysics and astrophysics etc. For reviews of this subject one may be referred to Turner [10], Turner [11], Huppert and Turner [6], Brandt and Fernando [3]. Two fundamental configurations have been studied in the context of thermohaline instability problem, one of Veronis [12], wherein the temperature gradient is destabilizing and the concentration gradient is stabilizing and another by Stern [9], wherein the temperature gradient is stabilizing and concentration gradient is destabilizing. The main results derived by Veronis and Stern for their respective configurations are that both allow the occurrence of a stationary convection or an oscillatory convection of growing amplitude, provided the destabilizing temperature gradient or concentration gradient is sufficiently large. More interesting double diffusive phenomenon appears if the destabilizing thermal or concentration gradient is opposed by the effect of vertical magnetic field.

Chandrasekhar [4] in his investigation of the hydromagnetic Rayleigh - Benard convection problem, sought unsuccessfully the regime in terms of the parameters of the system alone in which total kinetic energy associated with a disturbance exceeds the total magnetic energy associated with it, since these considerations are of decisive significance in deciding the validity of the 'principle of the exchange of stabilities' (Banerjee et al. [1]). Banerjee and Gupta [2] showed that in the parameter regime $\frac{Q\sigma_1}{\pi^2}\leq 1$, the total kinetic energy associated with a disturbance is greater than the total magnetic energy associated with it. Banerjee et al. [1] also extended these energy considerations to a more general problem, namely, magnetothermohaline convection problem of Stern type and proved that in the parameter regime $\frac{Q\sigma_1}{\pi^2}+\frac{|R|\sigma}{\pi^4}\leq 1$, the total kinetic energy associated with a disturbance exceeds the sum of its total magnetic and thermal energy. The present analysis extends these energy considerations to another more complex problem, namely, magnetohydrodynamic triply diffusive convection problem (analogous to magnetothermohaline convection of the Stern type) wherein one stabilizing heat component and two destabilizing concentration components have been considered. We establish that in the parameter regime $\frac{Q\sigma_1}{\pi^2}+\frac{|R|\sigma}{\pi^4}\leq 1$, the total kinetic energy associated with a disturbance exceeds the sum of its total magnetic and thermal energies. Further, this result is valid for quite general nature of the bounding surfaces. Furthermore result of Banerjee et al. [1] follows as a consequence.

II. MATHEMATICAL FORMULATION AND ANALYSIS

A viscous finitely heat conducting Boussinesq fluid layer of infinite horizontal extension is statically confined between two horizontal boundaries $z = 0$ and $z = d$ which are respectively maintained at uniform temperatures T_0 and $T_1 (< T_0)$ and uniform concentrations S_{10}, S_{20} and $S_{11} (< S_{10}), S_{21} (< S_{20})$ under the simultaneous presence of a uniform vertical magnetic field, $\vec{H}$ (see Fig 1).

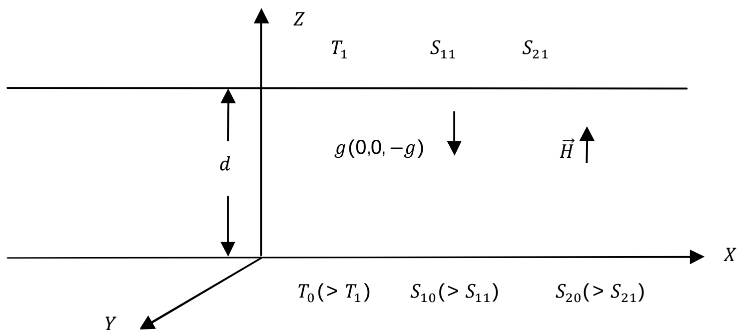


Fig.1 Physical configuration

The governing equations and boundary conditions for Magnetohydrodynamic triply diffusive convection problem, when a uniform vertical magnetic field opposite to gravity is imposed upon the system, in their non-dimensional forms are given by (Griffith [5] and Prakash et al. [7])

$$(D^2-a^2)\left(D^2-a^2-\frac{p}{\sigma}\right)w = Ra^2\theta - R_1a^2\phi_1 - R_2a^2\phi_2 - QD(D^2-a^2)h_z, \quad (1)$$

$$(D^2-a^2-p)\theta = -w, \quad (2)$$

$$\left(D^2-a^2-\frac{p}{\tau_1}\right)\phi_1 = -\frac{w}{\tau_1}, \quad (3)$$

$$\left(D^2-a^2-\frac{p}{\tau_2}\right)\phi_2 = -\frac{w}{\tau_2}, \quad (4)$$

$$\left(D^2-a^2-\frac{p\sigma_1}{\sigma}\right)h_z = -Dw, \quad (5)$$

with

$$\left.\begin{array}{l} w=0=\theta=\phi_1=\phi_2 \text{ (on both the boundaries)}, \\ D^2w=0 \quad \text{(On a dynamically free boundary)}, \\ Dw=0 \quad \text{(On a rigid boundary)} \\ h_z=0 \quad \text{(On both the boundaries if the regions outside the fluid are perfectly conducting)} \\ Dh_z=\mp ah_z \quad \text{(On both the boundaries if the regions outside the fluid are insulating).} \end{array}\right\} \quad (6)$$

where z is the real independent variable such that $0 \le z \le 1$. $D=\frac{d}{dz}$ is the differentiation with respect to z, $a^2>0$ is a constant, $\sigma>0$ is a constant, $\sigma_1>0$ is a constant, $\tau_1>0$ is a constant, $\tau_2>0$ is a constant, $R>0$, $R_1>0, R_2>0$ are constants, $Q>0$ is a constant, $p=p_r+ip_i$ is a complex constant such that p_r and p_i are real constants and as a consequence the dependent variables $w(z)=w_r(z)+iw_i(z)$, $\theta(z)=\theta_r(z)+i\theta_i(z)$, $\phi_1(z)=\phi_{1r}(z)+i\phi_{1i}(z), \phi_2(z)=\phi_{2r}(z)+i\phi_{2i}(z)$, $h_z(z)=h_{zr}(z)+ih_{zi}(z)$ are complex valued functions of the real variable z such that $w_r(z)$, $w_i(z)$, $\theta_r(z)$, $\theta_i(z)$, $\phi_{1r}(z)$, $\phi_{1i}(z)$, $\phi_{2r}(z)$, $\phi_{2i}(z)$, h_{zr} and h_{zi} are real valued functions of the real variable z. The meaning of the symbols from the physical point of view are as follows: z is the vertical coordinate, $a^2>0$ is square of the wave number, $\sigma=\frac{\nu}{\kappa}$ is the Prandtl number, $\tau_1=\frac{\kappa_1}{\kappa}$ and $\tau_2=\frac{\kappa_2}{\kappa}$ are the Lewis numbers for the two concentration components with mass diffusivities κ_1 and κ_2 respectively and κ is thermal diffusivity, R is the Rayleigh number, R_1 and R_2 are concentration Rayleigh numbers for the two concentration components, Q is the chandershekhar number, p is the complex growth rate, w is the vertical velocity, θ is the temperature, ϕ_1, ϕ_2 are the two concentrations and h_z is the magnetic field. It may further be noted that Eqs. (1) – (6) describe an eigenvalue problem for p and govern magnetohydrodynamic triply diffusive convection.

we prove the following Theorem:

Theorem: If $(w, \theta, \phi_1, \phi_2, h_z, p)$, $p_r \ge 0$ with $R>0, R_1>0, R_2>0, Q>0$, is a solution of Eqs. (1) - (5) together with boundary conditions (6) and $\frac{Q\sigma_1}{\pi^2} + \frac{|R|\sigma}{\pi^4} \leq 1$, then

$$\int_0^1 (|Dw|^2 + a^2|w|^2)\,dz > Q\sigma_1 \int_0^1 (|Dh_z|^2 + a^2|h_z|^2)dz + |R|\,a^2\sigma \int_0^1 |\theta|^2\,dz. \quad (7)$$

Proof: Multiplying Eq. (5) by h_z^*(the superscript $*$ denotes the complex conjugation) throughout, integrating the resulting equation, by parts, over the vertical range of z, and making use of boundary conditions (6), we get

$$a\{(|h_z|^2)_0 + (|h_z|^2)_1\} + \int_0^1 (|Dh_z|^2 + a^2|h_z|^2)dz + \frac{p\sigma_1}{\sigma}\int_0^1 |h_z|^2 dz = -\int_0^1 wDh_z^*dz. \quad (8)$$

Equating the real parts of Eq. (8), we obtain

$$a\{(|h_z|^2)_0 + (|h_z|^2)_1\} + \int_0^1 (|Dh_z|^2 + a^2|h_z|^2)dz + \frac{p_r\sigma_1}{\sigma}\int_0^1 |h_z|^2 dz = \text{Real part of } \left(-\int_0^1 wDh_z^*\,dz\right)$$

$$\leq \left|\int_0^1 wDh_z^*dz\right| \leq \int_0^1 |w||Dh_z|\,dz \leq \left(\int_0^1 |w|^2\,dz\right)^{1/2}\left(\int_0^1 |Dh_z|^2\,dz\right)^{1/2}. \quad (9) \quad \text{(using Schwartz inequality)}$$

Since $p_r \geq 0$, therefore we have from inequality (9), that

$$\int_0^1 |Dh_z|^2\,dz < \left(\int_0^1 |w|^2\,dz\right)^{1/2}\left(\int_0^1 |Dh_z|^2\,dz\right)^{1/2}$$

or $\left(\int_0^1 |Dh_z|^2\,dz\right)^{1/2} < \left(\int_0^1 |w|^2\,dz\right)^{1/2}$. (10)

Utilizing inequality (10) in inequality (9), we have

$$\int_0^1 (|Dh_z|^2 + a^2|h_z|^2)dz < \int_0^1 |w|^2\,dz\,. \quad (11)$$

Since $w(0) = 0 = w(1)$, therefore using Rayleigh Ritz inequality (Schultz [8]), we obtain

$$\int_0^1 |Dw|^2 dz \geq \pi^2 \int_0^1 |w|^2 dz\,. \quad (12)$$

It follows from inequality (11) and (12) that

$$\int_0^1 (|Dh_z|^2 + a^2|h_z|^2)dz < \frac{1}{\pi^2}\int_0^1 |Dw|^2\,dz < \frac{1}{\pi^2}\int_0^1 (|Dw|^2 + a^2|w|^2)dz. \quad (13)$$

Now multiplying Eq. (2) by θ^* and integrating the resulting equation by parts for a suitable number of times and making use of the boundary condition (6) and equating the real parts of resulting equation, we have

$$\int_0^1 (|D\theta|^2 + a^2|\theta|^2)dz + p_r\int_0^1 |\theta|^2 dz = \text{Real part of } \left(-\int_0^1 \theta^* w dz\right)$$

$$\leq \left|\int_0^1 \theta^* w dz\right| \leq \int_0^1 |\theta||w|\,dz$$

$$\leq \left(\int_0^1 |\theta|^2\,dz\right)^{1/2}\left(\int_0^1 |w|^2\,dz\right)^{1/2} \quad \text{(using Schwartz inequality) (14)}$$

Since $p_r \geq 0$, therefore we have from inequality (14), we obtain

$$\int_0^1 |D\theta|^2\,dz < \left(\int_0^1 |\theta|^2\,dz\right)^{1/2}\left(\int_0^1 |w|^2\,dz\right)^{1/2}. \quad (15)$$

Since $\theta(0) = 0 = \theta(1)$, therefore using Rayleigh Ritz inequality (Schultz [8]), we obtain

$$\int_0^1 |D\theta|^2 dz \geq \pi^2 \int_0^1 |\theta|^2 dz. \quad (16)$$

Combining inequalities (15) and (16), we have

$$\int_0^1 |\theta|^2 dz < \frac{1}{\pi^4}\int_0^1 |w|^2\, dz. \quad (17)$$

From inequalities (14) and (17), we get

$$a^2\int_0^1 |\theta|^2 dz < \frac{1}{\pi^2}\int_0^1 |w|^2\, dz. \quad (18)$$

Combining inequalities (12) and (18), we have

$$a^2\int_0^1 |\theta|^2 dz < \frac{1}{\pi^4}\int_0^1 |Dw|^2\, dz < \frac{1}{\pi^4}\int_0^1 (|Dw|^2 + a^2|w|^2) dz. \quad (19)$$

Finally from inequalities (13) and (19), we obtain

$$Q\sigma_1\int_0^1 (|Dh_z|^2 + a^2|h_z|^2) dz + |R|a^2\sigma\int_0^1 |\theta|^2 dz < \left(\frac{Q\sigma_1}{\pi^2} + \frac{|R|\sigma}{\pi^4}\right)\int_0^1 (|Dw|^2 + a^2|w|^2) dz. \quad (20)$$

Thus, if $\frac{Q\sigma_1}{\pi^2} + \frac{|R|\sigma}{\pi^4} \leq 1$, then inequality (20) yields

$$\int_0^1 (|Dw|^2 + a^2|w|^2) dz > Q\sigma_1\int_0^1 (|Dh_z|^2 + a^2|h_z|^2) dz + |R|a^2\sigma\int_0^1 |\theta|^2 dz \quad (21)$$

which completes the proof the theorem.

It is clear from inequality (21), that left hand side represents total kinetic energy associated with a disturbance while the right hand side represents the sum of its total magnetic and thermal energies and thus theorem 1 may be stated in equivalent form as: 'At the neutral or unstable state in the magnetohydrodynamic triply diffusive convection problem of the Stern type, the total kinetic energy associated with a disturbance is greater than the sum of its total magnetic and thermal energies in the parameter regime $\frac{Q\sigma_1}{\pi^2} + \frac{|R|\sigma}{\pi^4} \leq 1$, and this result is uniformly valid for any combination of a dynamically free or a rigid boundary that are either perfectly conducting or insulating'.

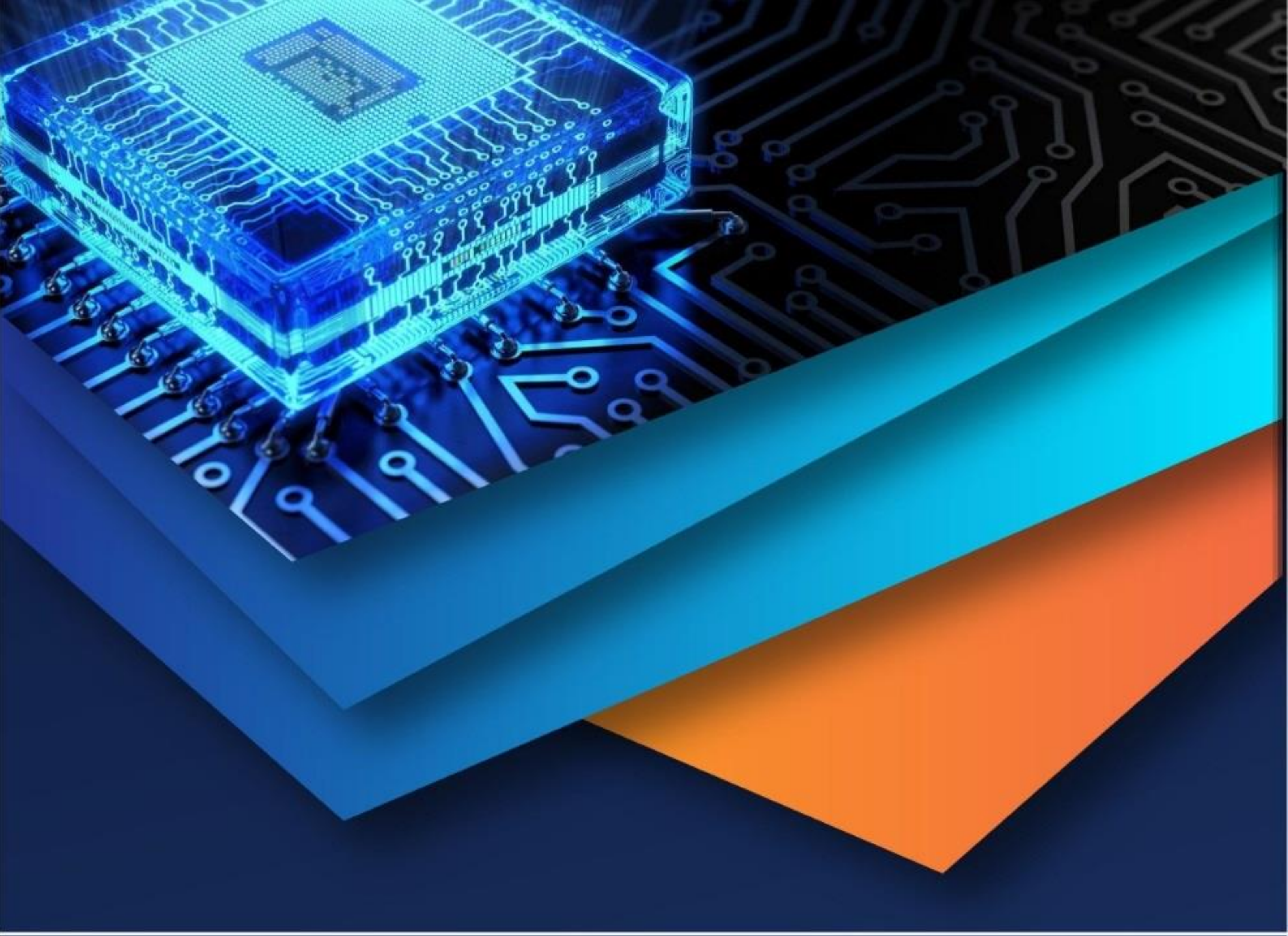
Note: If we put $R_2 = 0$, we obtain the result of Banerjee et al [1].

III.CONCLUSION

Linear stability of triply diffusive configuration has been analyzed in the presence of a uniform vertical magnetic field. An energy relationship has been established for this configuration which proves that the total kinetic energy associated with the disturbance exceeds the sum of its total magnetic and thermal energies in the parameter regime $\frac{Q\sigma_1}{\pi^2} + \frac{|R|\sigma}{\pi^4} \leq 1$. The result derived herein is uniformly valid for the quite general nature of the bounding surfaces.

REFERENCES

[1] M. B. Banerjee, J. R. Gupta and S. P. Katyal, "An Energy Relationship In Magnetothermohaline Convection Problem of the Stern Type", Ind. J. Pure Appl. Math. 18(9) 856-860, (1987).
[2] M. B. Banerjee and J. R. Gupta, "Studies in Hydrodynamic and Hydromagnetic Stability", Silver Line Publications, Shimla, (1991).
[3] A. Brandt and H. J. S. Fernando, "Double Diffusive Convection", Am. Geophys. Union Washington, Dc, (1996).
[4] S. Chandrasekhar, "On the inhibition of convection by a Magnetic Field", Philos. Mag. 43, (1952), 501-532.
[5] R. W. Griffiths, "The influence of a third diffusing component upon the onset of convection", J Fluid Mech, 92, (1979), 659-670.
[6] H. E. Huppert and J. S. Turner, "Double-Diffusive Convection', J. Fluid Mechanics, 106, (1981), 299-329.
[7] J. Prakash, K. Vaid and R. Bala, "Upper limits to the complex growth rates in triply diffusive convection", Proc. Ind. Nat. Sc. Acad., 2014, 80 (1), 115–122.
[8] M. H. Schultz, "Spline Analysis", Prentice- Hall Inc. Englewood Cliffs Nj, (1973).
[9] M. E. Stern, "The salt fountain and thermohaline convection", Tellus, 12, (1960), 172-175.
[10] J. S. Turner, "Double Diffusive Phenomena", Ann. Rev. Fluid Mech., 6, (1974), 37-54.
[11] J. S. Turner, "Multicomponent Convection", Ann. Rev. Fluid Mech., 17, (1985), 11-44.
[12] G. Veronis, "On finite amplitude instability in thermohaline convection", J Mar Res, 23, (1965), 1-17.



10.22214/IJRASET

INDEX COPERNICUS 45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429