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Classical Solution of the Fingero-Imbibition Phenomenon in Displacement Processes through Homogeneous Porous Media

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Abstract: An important phenomenon of fingero-imbibition in double-phase flow through homogeneous porous media is discussed in this paper. The mathematical formulation of the phenomenon yields a non-linear partial differential equation. This partial differential equation is transformed into ordinary differential equation by using infinitesimal transformations group technique of similarity analysis. The classical solution represents the saturation of the wetting phase whenever the saturation coefficient is positive constant. The solution obtained is physically consistent with the results of earlier literature.
Keywords: Similarity solution, Infinitesimal Transformations, Porous media, Fingero-imbibition phenomenon.

I. INTRODUCTION

This paper discusses an important phenomenon of fingero-imbibition in double-phase flow through homogeneous porous media. The flow phases are assumed as two immiscible fluids of small viscosity difference. The mathematical formulation of the phenomenon yields a non-linear partial differential equation. This partial differential equation is transformed into ordinary differential equation by using infinitesimal transformations group technique of similarity analysis. A classical solution in terms of confluent hypergeometric functions, of the later has been obtained. This classical solution represents the saturation of the wetting phase whenever the saturation coefficient is positive constant.

Scheidegger and Johnson [1961] have discussed the problem of the simultaneous occurrence of fingering and imbibition in artificial replenishment of groundwater through cracked porous medium and Verma [1970] is the first researcher who has designated this phenomenon “fingero-imbibition” and displacement problems through porous media have gained considerable interest. Mehta and Verma [1978] have obtained the saturation of the wetting phase, which represents the average cross sectional area occupied by the fingers. Patel *et al* [2011] have obtained the power series solution of the fingero-imbibition phenomenon of double phase flow through homogenous porous media. Meher [2013] has derived the series solution for porous medium equation arising in fingero- imbibition phenomenon during oil recovery process. Parikh *et al* [2013] have discussed the mathematical modeling and analysis of fingero-imbibition phenomenon in vertical downward cylindrical homogenous porous media All these equations are of parabolic type and a parabolic equation related to heat conduction and quantum mechanics is solved by Essawy [1995] in terms of confluent hypergeometric functions.

II. STATEMENT OF THE PROBLEM

We consider here that a cylindrical piece of homogeneous porous matrix of length L filled with native liquid n, is completely surrounded by an impermeable surface except for one end of the cylinder which is designated as the imbibition face ($x=0$) and this end is exposed to an adjacent formulation of the injected liquid i. If the injected liquid i is less viscous and in preferentially wetting phase, the phenomenon of fingering will occur simultaneously with imbibition and then this arrangement describes a one-dimensional fingero-imbibition phenomenon in which the injection is started by imbibition and the resulting displacement produces instabilities.

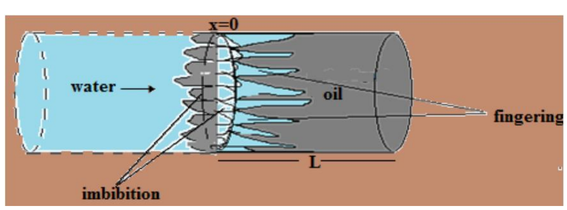

Figure: Fingero-imbibition in horizontal cylindrical homogeneous porous matrix

Our particular interest in the present paper is to derive a classical solution in terms of confluent hypergeometric function for the fingero-imbibition phenomenon in double-phase flow through homogenous porous medium with capillary pressure.

III. MATHEMATICAL FORMULATION

Assuming the validity of Darcy' law in the flow of two immiscible liquids, the seepage velocities of liquids *i* and *n* are respectively expressed as

$$v_i = -\frac{k_i}{\delta_i} K \frac{\partial p_i}{\partial x} \quad (1) \qquad v_n = -\frac{k_n}{\delta_n} K \frac{\partial p_n}{\partial x} \quad (2)$$

where k_i and k_n are relative permeabilities, δ_i and δ_n are the viscosities and p_i and p_n are the pressures of the injected liquid and native liquid respectively; *K* is the permeability of the homogeneous medium and *x* is the X co-ordinate.

The condition of linear countercurrent imbibition is,

$$v_i = -v_n \quad (3)$$

The equation of the continuity for the injected liquid *n* can be written as

$$\phi \frac{\partial s_i}{\partial t} + \frac{\partial v_i}{\partial x} = 0 \quad (4)$$

where s_i is the saturation of the injected liquid *i*, ϕ is the porosity of the medium and *t* is the time.

The capillary pressure p_c is defined as the pressure discontinuity between the flowing phases, is a function of saturation of injected fluid and may be written as

$$p_c = p_n - p_i \quad \text{[Scheidegger (1960)]} \quad (5)$$

The fluid can flow through inter connected pores which constitute capillaries with irregular shape, size and walls. So the capillary pressure depends on saturation of injected fluid. We have linear relationship between capillary pressure (P_c) and saturation of injected fluid (s_i) as

$$p_c = -\beta g(s_i) \quad \text{[Mehta (1977)]} \quad (6)$$

where β is the positive constant capillary pressure coefficient and negative sign indicates the direction of p_c.

Substituting the values of v_i and v_n from equation (1) and (2) into the equation (3), we get

$$\frac{k_i}{\delta_i} K \frac{\partial p_i}{\partial x} + \frac{k_n}{\delta_n} K \frac{\partial p_n}{\partial x} = 0 \quad (7)$$

Eliminating p_n between equations (5) and (7), we get

$$\left(\frac{k_i}{\delta_i} + \frac{k_n}{\delta_n}\right) K \frac{\partial p_i}{\partial x} + \frac{k_n}{\delta_n} K \frac{\partial p_c}{\partial x} = 0 \quad (8)$$

Eliminating $\frac{\partial p_i}{\partial x}$ from (1) and (8), we obtain $v_i = \frac{k_i k_n}{k_i \delta_n + k_n \delta_i} K \frac{\partial p_c}{\partial x}$ (9)

Combining equations (4) and (9), we obtain $\phi \frac{\partial s_i}{\partial t} + \frac{\partial}{\partial x}\left(\frac{k_i k_n}{k_i \delta_n + k_n \delta_i} K \frac{\partial p_c}{\partial x}\right) = 0$ (10)

Now substituting the value P_c from equation (6) into the equation (10), we get $\phi \frac{\partial s_i}{\partial t} - \beta K \frac{\partial}{\partial x}\left\{D(s_i) . \frac{\partial s_i}{\partial x}\right\} = 0$ (11)

where $D(s_i) = \frac{k_i k_n}{k_i \delta_n + k_n \delta_i} g'(s_i)$ (12)

The appropriate set of boundary conditions related to the phenomenon is $s_i(0,t) = s_{io}; (t > 0)$ (13)

$$s_i(L,t) = s_{iL}; (t > 0) \quad (14)$$

where the boundary condition (13) defines the saturation s_{io} at the imbibition face $x=0$ and the boundary condition (14) gives the saturation at $x=L$, of the porous matrix.
Using the dimensionless variables

$$X=\frac{x}{L} \tag{15}$$

$$T=\frac{\beta K t}{\phi L} \tag{16}$$

where $0 \le X \le 1$ and $0 < T \le 1$, in equation (11), we obtain

$$\frac{\partial s_i}{\partial T}=\frac{\partial}{\partial X}\left\{D(s_i).\frac{\partial s_i}{\partial X}\right\} \tag{17}$$

This is a non-linear partial differential equation governing the fingero-imbibition phenomenon in the double-phase flow through the unsaturated porous medium.
The boundary conditions (13) and (14) are transformed into

$$s_i(0,T)=s_{io} \quad (18) \qquad s_i(1,T)=s_{iL} \quad (19)$$

The actual flow of liquid in the porous medium is rotational, but due to homogeneity of the medium, the flow is considered to be one-dimensional and it concerns with the physical meaning of Laplace, which occurs in the equation of motion for saturation. Now $D(s_i)$ is a function of saturation s_i and $0 \le s_i \le 1$. Moreover, in the one-dimensional flow analysis, if this type of non-uniformity of the actual flow is not too large, valuable results are obtained. Hence, at the stage, for definiteness, we assume that the average value of $D(s_i)=D=1$ as constant.

Therefore, $$\frac{\partial s_i}{\partial T}=\frac{\partial^2 s_i}{\partial X^2} \tag{20}$$

together with boundary conditions (18) and (19) the equation (20) represents the boundary value problem under the investigation.
The equation (20) is the parabolic differential equation describing the saturation of the injected liquid. We derive the classical solution of (20) in terms of confluent hypergeometric functions.

IV. SIMILARITY SOLUTION

We seek the following one-parameter group G of infinitesimal transformation which takes the (x,t,s_i)- space into itself and under which equation (20) is invariant: $$G=\left\{\begin{aligned}\bar{x}&=x+\varepsilon X\\ \bar{t}&=t+\varepsilon T\\ \bar{s_i}&=s_i+\varepsilon S\end{aligned}\right\} \tag{21}$$

where the generators X, T and S are functions of x, t and s_i. Invariance of equation (20) under (21) gives

$$\frac{\partial \bar{s_i}}{\partial T}=\frac{\partial^2 \bar{s_i}}{\partial X^2} \tag{22}$$

Applying transformations (21) into the (22) we get the group of infinitesimal transformation explicitly is, $$G=\left\{\begin{aligned}X&=\frac{a_0}{2}x+a_1\\ T&=a_2 t+a_3\\ S&=a_4\end{aligned}\right\}$$

(23)

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Thus, the characteristic equations are $\dfrac{dx}{\frac{a_0}{2}x+a_1}=\dfrac{dt}{a_2t+a_3}=\dfrac{ds_i}{a_4}$ (24)

Now, for simplicity we assume that $a_1=a_3=0, a_0=a_2=2$ and $a_4=-1$ then we have

$$\frac{dx}{x}=\frac{dt}{2t}=\frac{ds_i}{-1} \qquad (25)$$

From these,

we can have the similarity variable $\eta=\dfrac{x}{\sqrt{t}}$ (26)

and also we have

$$s_i=\frac{1}{\sqrt{t}}\phi(\eta) \text{ where } \eta=\frac{x}{\sqrt{t}} \qquad (27)$$

Using (20), (26), (27) to obtain the following ordinary differential equation,

$$2\phi''(\eta)+\eta\phi'(\eta)+\phi(\eta)=0 \qquad (28)$$

V. CLASSICAL SOLUTION

We determine the classical solution of the following boundary value problem associated with the fingero-imbibition phenomenon:

$$\frac{d^2\phi}{d\eta^2}+\frac{\eta}{2}\frac{d\phi}{d\eta}+\frac{\phi}{2}=0 \text{ with; } \phi(0)=\sqrt{t}s_{i0}$$

$$\text{and } \phi\left(\frac{L}{\sqrt{t}}\right)=\sqrt{t}s_{iL} \qquad (29)$$

the solution of the (29) equation is given by

$$\phi(\eta)=c_0\,{}_1F_1\left(1;1;\frac{-1}{4}\eta^2\right)+c_1\frac{\eta}{2}\,{}_1F_1\left(1;\frac{3}{2};\frac{-1}{4}\eta^2\right)$$

where c_0 and c_1 are constants and

$${}_1F_1(a,b;\eta)=\sum_{k=0}^{\infty}C_k\eta^k$$

where $C_k=\dfrac{a(a+1)......(a+k-1)}{b(b+1)......(b+k-1)k!}$ is Kummer confluent hypergeometric function.

VI. CONCLUSION

We have obtained the classical solution in terms of confluent hypergeometric functions of the non- linear differential equation governing the fingero-imbibition phenomenon of double-phase through homogeneous porous media. The solution represents the saturation of the wetting phase whenever the saturation coefficient is positive constant. The solution is consistent with the physical phenomenon for the restricted values of the parameter. We have not included any numerical illustration as well as graphical representation due to our particular interest of the classical solution. In spite of the limitations of the present work, it is believed that the present classical solution in terms of confluent hypergeometric functions will provide useful theoretical information to at least one complicated phenomenon of the fingero-imbibition flow through unsaturated porous media.

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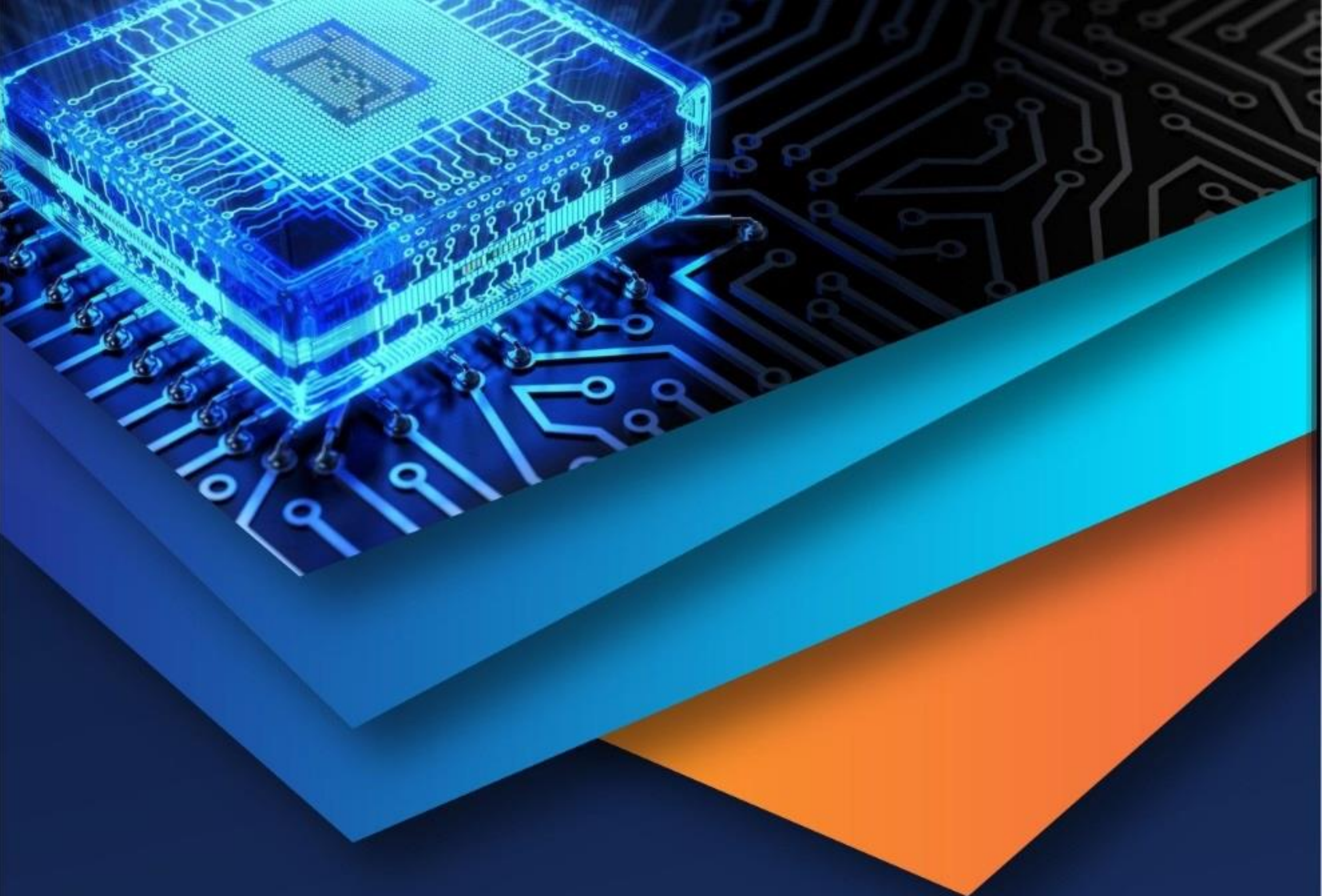

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