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Family of Estimators for Population Variance using Two Auxiliary Information

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***Abstract:** A family of log-type estimators using information on two auxiliary variables has been proposed for estimating the population variance of the study variable. It has been shown that these families of log-type estimators have lesser mean squared error under the optimum values of the characterizing scalars as compared to some of the commonly used estimators available in the literature. Further, an extension of the proposed classes using multiple auxiliary information have also initiated in this paper. A numerical study is included as an illustration using two auxiliary variables.*
***Keywords:** Ratio method of estimation, bias, mean squared error, efficiency.*

I. INTRODUCTION

In Sample survey, it is always advantageous to use the auxiliary variable which is highly correlated with the study variable. The use of auxiliary information enhances the precision of the estimators used for estimating the unknown population parameters. Several authors have used auxiliary information on auxiliary variable in the estimation of population parameters like Srivastava and Jhajj (1981), Bahl and Tuteja (1991), Singh and Vishwakarma (2007), Sahai and Ray (1980), Srivastava and Jhajj (1983), Srivastava (1971), Swain (1970) and Perri (2007). In this paper, we have tried to incorporate the use of auxiliary information in the class of log-type estimators. Several authors like Haq and Shabbir (2013), Shabbir and Gupta (2006), Kadilar and Cingi (2003) have proposed estimators using information on a single auxiliary variable. It is seen that many a times instead of using information on a single auxiliary variable, we have information on two auxiliary variables like Tailor et al. (2012), Koyuncu and Kadilar (2009), Bhushan and Kumari (2018), Kumari and Thakur (2020). Here, the problem of estimation of population variance using information on two auxiliary variables has been discussed.

Consider a finite population $U = U_1, U_2, \ldots, U_N$ of size N from which a sample of size n is drawn according to simple random sampling without replacement (SRSWOR). Let y_i, x_{i_1} and x_{i_2} denotes the value of the study and two auxiliary variable for the ith unit $i = 1, 2, \ldots, N$ of the population. Further, let $\bar{y}$, $\bar{x}_1$ and $\bar{x}_2$ be the sample means of study variable and two auxiliary variables. Also, $s_y^2 = N^{-1}\sum_{i=1}^{N}\left(y_i - \bar{y}\right)^2$, $s_{x_1}^2 = n^{-1}\sum_{i=1}^{n}\left(x_i - \bar{x}_1\right)^2$ and $s_{x_2}^2 = n^{-1}\sum_{i=1}^{n}\left(x_i - \bar{x}_2\right)^2$ be the sample variance of the study and two auxiliary variables respectively.

II. THE SUGGESTED GENERALIZED CLASS OF LOG-TYPE ESTIMATORS

We propose the following new classes of log type estimators for the population variance S_y^2 as

$$T_1 = w_1 s_y^2 \left[1 + \log\left(\frac{S_{x_1}^2}{s_{x_1}^2}\right)\right]^{a_1} \left[1 + \log\left(\frac{S_{x_2}^2}{s_{x_2}^2}\right)\right]^{a_2} \tag{2.1}$$

$$T_2 = w_2 s_y^2 \left[1 + b_1 \log\left(\frac{S_{x_1}^2}{s_{x_1}^2}\right)\right]\left[1 + b_2 \log\left(\frac{S_{x_2}^2}{s_{x_2}^2}\right)\right] \tag{2.2}$$

$$T_3 = w_3 s_y^2 \left[1 + \log\left(\frac{S_{x_1}^{2*}}{s_{x_1}^{2*}}\right)\right]^{c_1} \left[1 + \log\left(\frac{S_{x_2}^{2*}}{s_{x_2}^{2*}}\right)\right]^{c_2} \tag{2.3}$$

$$T_4 = w_4 s_y^2 \left[1 + d_1 \log\left(\frac{S_{x_1}^{2*}}{s_{x_1}^{2*}}\right)\right]\left[1 + d_2 \log\left(\frac{S_{x_2}^{2*}}{s_{x_2}^{2*}}\right)\right] \tag{2.4}$$

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where $S_{x_i}^{2*} = a_i S_{x_i}^2 + b_i$ and $s_{x_i}^{2*} = a_i s_{x_i}^2 + b_i$ for i = 1, 2

such that a_i, b_i, c_i and d_i are optimizing scalars or functions of the known parameters of the auxiliary variable x_i's such as the standard deviations S_{x_i}, coefficient of variation C_{x_i}, coefficient of kurtosis b_{2x_i}, coefficient of skewness b_{1x_i} and correlation coefficient $r_{x_i x_j}$ of the population $(i \neq j = 0)$.

III. PROPERTIES OF THE SUGGESTED CLASS OF ESTIMATORS

In order to obtain the bias and mean square error (MSE), let us consider

$$\varepsilon_0 = \frac{\left(s_y^2 - S_y^2\right)}{S_y^2}, \quad \varepsilon_1 = \frac{\left(s_{x_1}^2 - S_{x_1}^2\right)}{S_{x_1}^2} \text{ and } \varepsilon_2 = \frac{\left(s_{x_2}^2 - S_{x_2}^2\right)}{S_{x_2}^2}$$

$E(\varepsilon_0) = E(\varepsilon_1) = E(\varepsilon_2) = 0$, $E(\varepsilon_0^2) = I b_{2y}^*$, $E(\varepsilon_1^2) = I b_{2x_1}^*$, $E(\varepsilon_2^2) = I b_{2x_2}^*$, $E(\varepsilon_0 \varepsilon_1) = I\, I_{22yx_1}^*$, $E(\varepsilon_0 \varepsilon_2) = I\, I_{22yx_2}^*$ and $E(\varepsilon_1 \varepsilon_2) = I\, I_{22x_1x_2}^*$ where $b_{2y}^* = b_{2y} - 1$, $b_{2x_1}^* = b_{2x_1} - 1$, $b_{2x_2}^* = b_{2x_2} - 1$ and $I_{22yx_1}^* = I_{22yx_1} - 1$, $I_{22yx_2}^* = I_{22yx_2} - 1$, $I_{22x_1x_2}^* = I_{22x_1x_2} - 1$; $I_{pq} = m_{pq} \big/ m_{20}^{p/2} m_{02}^{q/2}$, $m_{pq} = \sum_{i=1}^{N} \left(Y_i - \bar{Y}\right)^p \left(X_i - \bar{X}\right)^q \Big/ N$, $I = 1/N$, $b_{2y} = m_{40} \big/ m_{20}^2$, $b_{2x} = m_{04} \big/ m_{02}^2$ are the coefficient of kurtosis of y and x respectively.

1) *Theorem 1:* The bias and the mean squared error of the proposed estimator considered upto the terms of order n^{-1} are given by

$$Bias(T_1) = S_y^2 \left[w_1 \left\{ 1 + I \left(\frac{a_1^2}{2} b_{2x_1}^* + \frac{a_2^2}{2} b_{2x_2}^* - a_1 r_{yx_1} \sqrt{b_{2y}^* b_{2x_1}^*} - a_2 r_{yx_2} \sqrt{b_{2y}^* b_{2x_2}^*} + a_1 a_2 r_{x_1x_2} \sqrt{b_{2x_1}^* b_{2x_2}^*} \right) \right\} - 1 \right]$$

$$MSE(T_1) = S_y^4 + w_1^4 S_y^4 \left[1 + I \left\{ b_{2y}^* + 2a_1^2 b_{2x_1}^* + 2a_2^2 b_{2x_2}^* - 4a_1 r_{yx_1} \sqrt{b_{2y}^* b_{2x_1}^*} - 4a_2 r_{yx_2} \sqrt{b_{2y}^* b_{2x_2}^*} + 4a_1 a_2 r_{x_1x_2} \sqrt{b_{2x_1}^* b_{2x_2}^*} \right\} \right]$$
$$-2w_1 S_y^4 \left\{ 1 + I \left(\frac{a_1^2}{2} b_{2x_1}^* + \frac{a_2^2}{2} b_{2x_2}^* - a_1 r_{yx_1} \sqrt{b_{2y}^* b_{2x_1}^*} - a_2 r_{yx_2} \sqrt{b_{2y}^* b_{2x_2}^*} + a_1 a_2 r_{x_1x_2} \sqrt{b_{2x_1}^* b_{2x_2}^*} \right) \right\}$$

where $r_{yx_1} = \dfrac{I_{22yx_1}^*}{\sqrt{b_{2y}^* b_{2x_1}^*}}$, $r_{yx_2} = \dfrac{I_{22yx_2}^*}{\sqrt{b_{2y}^* b_{2x_2}^*}}$ and $r_{x_1x_2} = \dfrac{I_{22x_1x_2}^*}{\sqrt{b_{2x_1}^* b_{2x_2}^*}}$

Proof. Consider the estimator

$$T_1 = w_1 s_y^2 \left[1 + \log \left(\frac{S_{x_1}^2}{s_{x_1}^2} \right) \right]^{a_1} \left[1 + \log \left(\frac{S_{x_2}^2}{s_{x_2}^2} \right) \right]^{a_2}$$

$$= w_1 S_y^2 (1 + \varepsilon_0) \left[1 + \log (1 + \varepsilon_1)^{-1} \right]^{a_1} \left[1 + \log (1 + \varepsilon_2)^{-1} \right]^{a_2}$$

$$T_1 - S_y^2 = (w_1 - 1) S_y^2 + w_1 S_y^2 \left[\frac{a_1^2 \varepsilon_1^2}{2} + \frac{a_2^2 \varepsilon_2^2}{2} + a_1 a_2 \varepsilon_1 \varepsilon_2 - a_1 \varepsilon_0 \varepsilon_1 - a_2 \varepsilon_0 \varepsilon_2 + \varepsilon_0 - a_1 \varepsilon_1 - a_2 \varepsilon_2 - a_1 \varepsilon_1^2 - a_2 \varepsilon_2^2 \right] \quad (3.1)$$

Taking expectation on both the sides, we get

$$Bias(T_1) = S_y^2 \left[w_1 \left\{ 1 + I \left(\frac{a_1^2}{2} b_{2x_1}^* + \frac{a_2^2}{2} b_{2x_2}^* - a_1 r_{yx_1} \sqrt{b_{2y}^* b_{2x_1}^*} - a_2 r_{yx_2} \sqrt{b_{2y}^* b_{2x_2}^*} + a_1 a_2 r_{x_1x_2} \sqrt{b_{2x_1}^* b_{2x_2}^*} \right) \right\} - 1 \right]$$

Squaring and by considering expectation on both the sides of equation (3.1), we get

$$MSE(T_1) = S_y^4 + w_1^4 S_y^4 \left[1 + I \left\{ b_{2y}^* + 2a_1^2 b_{2x_1}^* + 2a_2^2 b_{2x_2}^* - 4a_1 r_{yx_1} \sqrt{b_{2y}^* b_{2x_1}^*} - 4a_2 r_{yx_2} \sqrt{b_{2y}^* b_{2x_2}^*} + 4a_1 a_2 r_{x_1x_2} \sqrt{b_{2x_1}^* b_{2x_2}^*} \right\} \right]$$
$$-2w_1 S_y^4 \left\{ 1 + I \left(\frac{a_1^2}{2} b_{2x_1}^* + \frac{a_2^2}{2} b_{2x_2}^* - a_1 r_{yx_1} \sqrt{b_{2y}^* b_{2x_1}^*} - a_2 r_{yx_2} \sqrt{b_{2y}^* b_{2x_2}^*} + a_1 a_2 r_{x_1x_2} \sqrt{b_{2x_1}^* b_{2x_2}^*} \right) \right\}$$

2) *Corollary 1:* The optimum values of constant are obtained as

$$w_{1opt} = \frac{B}{A}$$

where $A = \left[1 + I\left\{b^*_{2y} + 2a_1^2 b^*_{2x_1} + 2a_2^2 b^*_{2x_2} - 4a_1 r_{yx_1}\sqrt{b^*_{2y}b^*_{2x_1}} - 4a_2 r_{yx_2}\sqrt{b^*_{2y}b^*_{2x_2}} + 4a_1 a_2 r_{x_1x_2}\sqrt{b^*_{2x_1}b^*_{2x_2}}\right\}\right]$

$$B = \left\{1 + I\left(\frac{a_1^2}{2}b^*_{2x_1} + \frac{a_2^2}{2}b^*_{2x_2} - a_1 r_{yx_1}\sqrt{b^*_{2y}b^*_{2x_1}} - a_2 r_{yx_2}\sqrt{b^*_{2y}b^*_{2x_2}} + a_1 a_2 r_{x_1x_2}\sqrt{b^*_{2x_1}b^*_{2x_2}}\right)\right\}$$

The optimum mean squared error is given by

$$M\left(T_1\right)_{opt} = S_y^4\left(1 - \frac{B^2}{A}\right) \tag{3.4}$$

IV. MULTIVARIATE EXTENSION OF PROPOSED CLASS OF ESTIMATORS

Let there are k auxiliary variables then we can use the variables by taking a linear combination of these k estimators of the form given in section 2, calculated for every auxiliary variable separately, for estimating the population variance. Then the estimators for population variance will be defined as

$$T_1^* = w_1 s_y^2 \, \pi_{i=1}^{k}\left[1 + \log\left(\frac{S^2_{x_i}}{s^2_{x_i}}\right)\right]^{a_i}$$

$$T_2^* = w_2 s_y^2 \, \pi_{i=1}^{k}\left[1 + b_i \log\left(\frac{S^2_{x_i}}{s^2_{x_i}}\right)\right]$$

$$T_3^* = w_3 s_y^2 \, \pi_{i=1}^{k}\left[1 + \log\left(\frac{S^{2*}_{x_i}}{s^{2*}_{x_i}}\right)\right]^{c_i}$$

$$T_4^* = w_4 s_y^2 \, \pi_{i=1}^{k}\left[1 + d_i \log\left(\frac{S^{2*}_{x_i}}{s^{2*}_{x_i}}\right)\right]$$

where a_i, b_i, c_i and d_i are the optimizing scalars i = 1,2,...,k.

V. PROPERTIES OF PROPOSED CLASS OF ESTIMATORS USING MULTIPLE AUXILIARY INFORMATION

Theorem 2. *The bias of the proposed estimators are given by*

$$Bias\left(T_1^*\right) = S_y^2\left[w_1\left\{1 + I\left(\sum_{i=1}^{k}\frac{a_i^2}{2}b^*_{2x_i} - \sum_{i=1}^{k}a_i r_{yx_i}\sqrt{b^*_{2y}b^*_{2x_i}} + \sum_{i\neq j=1}^{k}a_i a_j r_{x_ix_j}\sqrt{b^*_{2x_i}b^*_{2x_j}}\right)\right\} - 1\right]$$

$$MSE\left(T_1^*\right) = S_y^4 + w_1^4 S_y^4\left[1 + I\left\{b^*_{2y} + 2\sum_{i=1}^{k}a_i^2 b^*_{2x_i} - 4\sum_{i=1}^{k}a_i r_{yx_i}\sqrt{b^*_{2y}b^*_{2x_i}} + 4\sum_{i\neq j=1}^{k}a_i a_j r_{x_ix_j}\sqrt{b^*_{2x_i}b^*_{2x_j}}\right\}\right]$$

$$-2w_1 S_y^4\left\{1 + I\left(\sum_{i=1}^{k}\frac{a_i^2}{2}b^*_{2x_i} - \sum_{i=1}^{k}a_i r_{yx_i}\sqrt{b^*_{2y}b^*_{2x_i}} + \sum_{i\neq j=1}^{k}a_i a_j r_{x_ix_j}\sqrt{b^*_{2x_i}b^*_{2x_j}}\right)\right\}$$

VI. EFFICIENCY COMPARISON

In this section, we compare the proposed classes of estimators with some important estimators. The comparison will be in terms of their MSE up to the order of n^{-1}. The optimum mean squared error of proposed estimator is given by

$$M\left(T_1\right)_{opt} = S_y^4\left(1-\frac{B^2}{A}\right)$$

A. General Variance Estimator

$$\hat{S}_y^2 = s_y^2$$

It's mean squared error is given by

$$MSE(\hat{S}_y^2) = S_y^4\, I\, b_{2y}^* > MSE(T_1)_{opt}$$

B. The Usual Ratio Type Variance Estimator

$$\hat{S}_r^2 = s_y^2\left(\frac{S_{x_1}^2}{s_{x_1}^2}\right)\left(\frac{S_{x_2}^2}{s_{x_2}^2}\right)$$

It's mean squared error is given by

$$MSE(\hat{S}_r^2) = S_y^4\, I\left[b_{2y}^* + b_{2x_1}^* + b_{2x2}^* - 2I_{22yx_1}^* - 2I_{22yx_2}^* + 2I_{22x_1x_2}^*\right] > MSE(T_1)_{opt}$$

C. The Product Type Variance Estimator

$$\hat{S}_p^2 = s_y^2\left(\frac{s_{x_1}^2}{S_{x_1}^2}\right)\left(\frac{s_{x_2}^2}{S_{x_2}^2}\right)$$

Its mean squared error is given by

$$MSE(\hat{S}_r^2) = S_y^4\, I\left[b_{2y}^* + b_{2x_1}^* + b_{2x2}^* + 2I_{22yx_1}^* + 2I_{22yx_2}^* + 2I_{22x_1x_2}^*\right] > MSE(T_1)_{opt}$$

D. Isaki (1983) Variance Estimator

$$\hat{S}_I^2 = w_1\left(\frac{s_y^2}{s_{x_1}^2}\right)S_{x_1}^2 + w_2\left(\frac{s_y^2}{s_{x_2}^2}\right)S_{x_2}^2$$

The mean squared error is given by

$$MSE\left(\hat{S}_I^2\right)_{opt} = I\, S_y^4\left[b_{2y}^* + b_{2x_2}^* - 2I_{22x_2}^* - \frac{\left(b_{2x_2}^* - I_{22x_2}^*\right)^2}{b_{2x_1}^* + b_{2x_2}^* - 2I_{22x_1x_2}^*}\right] > MSE\left(T_1\right)_{opt}$$

E. Singh, Chauhan, Sawan and Smarandache (2011) Type Variance Estimator

$$\hat{S}_s^2 = s_y^2 \exp\left(\frac{S_{x_1}^2 - s_{x_1}^2}{S_{x_1}^2 + s_{x_1}^2}\right)\left(\frac{S_{x_2}^2 - s_{x_2}^2}{S_{x_2}^2 + s_{x_2}^2}\right)$$

It's mean squared error is given by

$$MSE(\hat{S}_s^2) = S_y^4\, I\left[b_{2y}^* + \frac{b_{2x_1}^*}{4} + \frac{b_{2x2}^*}{4} - I_{22yx_1}^* - I_{22yx_2}^* + \frac{I_{22x_1x_2}^*}{4}\right] > MSE(T_1)_{opt}$$

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F. Olufadi And Kadilar (2014) Variance Estimator

$$\hat{S}_K^2 = s_y^2 \left(\frac{S_{x_1}^2}{s_{x_1}^2}\right)^{a_1} \left(\frac{S_{x_2}^2}{s_{x_2}^2}\right)^{a_2}$$

It's mean squared error is given by

$$MSE(\hat{S}_K^2) = S_y^4\, I\left[b_{2y}^* + a_1^2 b_{2x_1}^* + a_2^2 b_{2x2}^* - 2a_1 I_{22yx_1}^* - 2a_2 I_{22yx_2}^* + 2a_1 a_2 I_{22x_1x_2}^*\right] > MSE(T_1)_{opt}$$

G. Das and Tripathi (1978) type Variance Estimator

$$\hat{S}_D^2 = s_y^2 \left(\frac{S_{x_1}^2}{S_{x_1}^2 + a_1\left(s_{x_1}^2 - S_{x_1}^2\right)}\right)\left(\frac{S_{x_2}^2}{S_{x_2}^2 + a_2\left(s_{x_2}^2 - S_{x_2}^2\right)}\right)$$

It's mean squared error is given by

$$MSE(\hat{S}_D^2) = S_y^4\, I\left[b_{2y}^* + a_1^2 b_{2x_1}^* + a_2^2 b_{2x2}^* - 2a_1 I_{22yx_1}^* - 2a_2 I_{22yx_2}^* + 2a_1 a_2 I_{22x_1x_2}^*\right] > MSE(T_1)_{opt}$$

VII. EMPIRICAL STUDY

The data on which we performed the numerical calculation is taken from some natural populations. The source of the data is given as follows.

Population 1. (Chochran, Pg. no. 155). The data concerns about weekly expenditure on food per family.

y : weekly expenditure on food

x_1 : number of persons

x_2 : the weekly family income

Population 2. (Choudhary F. S., Pg. no. 117).

y : area under wheat (in acres) in 1974

x_1 : area under wheat (in acres) in 1971

x_2 : area under wheat (in acres) in 1973

The summary and the percent relative efficiency of the following estimators are as follows:

Table 2: Parameters of the data

Parameter	Population 1	Population 2
N	33	34
n	11	10
b_{2y}^*	4.032	2.725
$b_{2x_1}^*$	1.388	12.366
$b_{2x_1}^*$	1.143	1.912
$I_{22yx_1}^*$	0.305	0.224
$I_{22yx_2}^*$	1.155	2.104
$I_{22x_1x_2}^*$	0.492	0.152

Table 3: PRE of the estimators

Estimator	Pop. 1	Pop. 2
$\hat{S}_y^2$	100	100
$\hat{S}_r^2$	87.167	21.544
$\hat{S}_p^2$	38.525	12.407
$\hat{S}_s^2$	116.860	67.423
$\hat{S}_I^2$	141.940	637.142
$\hat{S}_D^2$	142.235	666.034
$\hat{S}_K^2$	142.235	666.034
$T_{1_{opt}}$	159.192	794.969

VIII. CONCLUSION

This paper has proved that the proposed class of estimators are better than conventional estimators in terms of their percent relative efficiency (PRE) over different populations. This work provides the better use of auxiliary information in form of various auxiliary variables (two or more than two). Hence, it’s an appeal to survey practitioners that they can used such class of estimators for their practical utility.

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