iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 8 **Issue:** V **Month of publication:** May 2020

DOI: http://doi.org/10.22214/ijraset.2020.5360

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 8 Issue V May 2020- Available at www.ijraset.com

Discrete-Time Controller Design for Pitch Channel

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Abstract: This research proposes a discrete time controller design for the pitch channel for a two degree of freedom helicopter using the root locus method. The proposed lead-lag controller uses zero and pole placement to increase the stability and controllability of the system. Simulation is provided to insure the validity of the proposed controller.
Keywords: Two-degree-of-freedom, Helicopter, discrete time control, root locus, lead lag compensator

I. INTRODUCTION

The two degree of freedom (2 DOF) helicopter consists of a fixed base with two propellers that are driven by DC motors [1]. One propeller controls the elevation of the helicopter nose about the pitch axis and the other propeller controls the side to side motion about the yaw axis [1-2]. High resolution encoders are used to measure the pitch and yaw angles of the system. The 2-DOF helicopter recreates a behavior that is a subset of a real helicopter dynamics. The helicopter model is a Twin Rotor Multiple-inputs Multiple-outputs System. Helicopters has several non-linarites and open loop unstable dynamics as well as significant cross-coupling between their control channels which makes the control of such multi-input multi-output (MIMO) system a challenging task [3]. These non-linearities and model uncertainties make designing a controller for helicopters an open research problem [2, 3]. The interest in this research problem has increased recently due to their potential military and civil applications [4]. Various approaches for stabilization and tracking control of helicopters have been reported in several literatures. A fuzzy control technique was presented in [3], a State Dependent Riccati Equation (SDRE) methodology in [5], back-stepping based approach in [5], and linear and non linear feedback control was presented in [6] among others.

In this project the root locus method was used to design a lead lag controller using the linearized method of the system. Section 2 gives the system description and modeling, section 3 gives the design specification, section 4 gives the problem formulation and controller design, and section 4 gives the concluding remarks of the project.

II. SYSTEM DYNAMICS & PROBLEM STATEMENT

The two degrees of freedom (2DOF) helicopter system is a popular modeling tool due to its highly non-linear nature. The modeling and control tools of this system can be used in multiple areas such as aerospace [7, 8]. The system used in the model is a twin rotor single input single output system. The twin rotors are the yaw rotor and the pitch rotor which control the yaw and pitch of the system respectively. The system can be seen in figure 1.

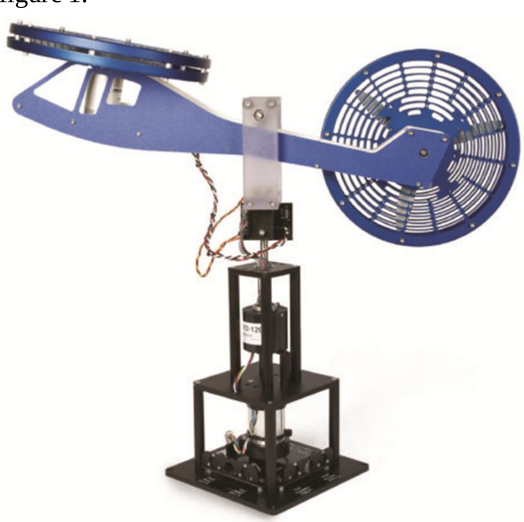

Figure 1: 2DOF helicopter system.

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
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The free body diagram of the 2-DOF helicopter is illustrated in figure 2. The diagram illustrates the degrees of freedom for the helicopter using the two rotors. In this system the two degrees of freedom are around the yaw axis and pitch axis [9, 10]. The pitch angle increases positively, $\theta(\dot{t})> 0$, when the nose is moved upwards, and the body rotates in the counter-clockwise (CCW) direction. The yaw angle increases positively, $\psi(\dot{t}) > 0$ when the body rotates in the clockwise (CW) direction. When the pitch thrust force is positive the pitch increases, and when the yaw thrust force is positive the yaw increases [11].

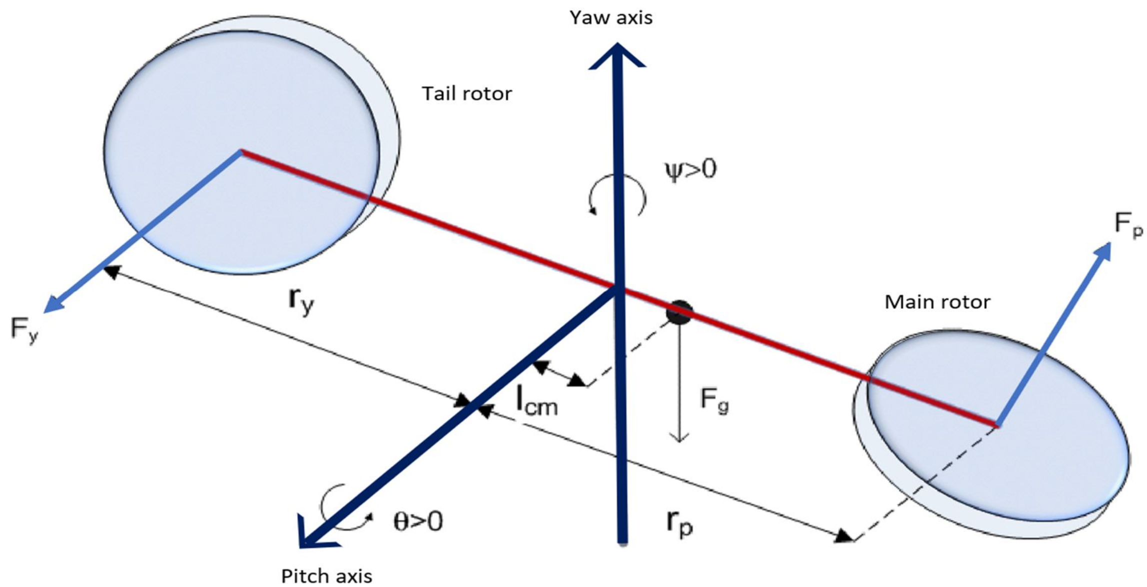

Figure 2: Simple free-body diagram of 2-DOF Helicopter.

The thrust forces acting on the pitch and yaw axes from the front and back motors are then defined [12, 13]. The non-linear equations of motion for the system are derived. Linearization can be used to simplify the non-linear dynamics of the system about a set of preselected equilibrium conditions and presented in the form:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

The (linearized) state–space equations describing the system are:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2.7451 & -0.2829 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -0.2701 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 37.2021 & 3.5306 \\ 0 & 0 \\ 2.3892 & 7.461 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(t)$$

Where:

$$x = \begin{bmatrix} \theta \\ \dot{\theta} \\ \psi \\ \dot{\psi} \end{bmatrix}, \quad u = \begin{bmatrix} v_p \\ v_y \end{bmatrix}, \quad y = \begin{bmatrix} \theta \\ \psi \end{bmatrix}$$

The closed loop system presentation for the pitch channel is shown in figure 3.

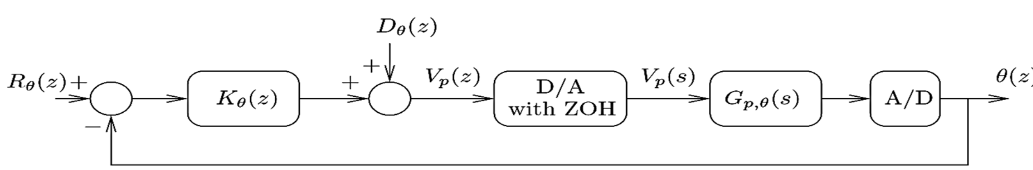

Figure 3: Closed loop system (pitch channel).

The transfer function of the system for the pitch channel is as follows.

$$G_{p,\theta}(s) = \frac{\theta(s)}{V_p(s)} = \frac{37.2021}{s^2 + 0.2830s + 2.7452}$$

The closed loop system presentation for the yaw channel is shown in figure 4.

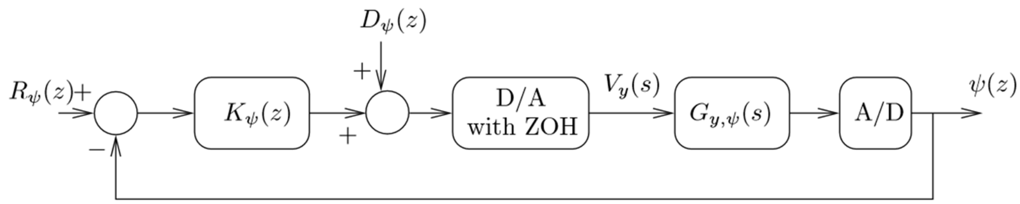

Figure 4: Closed loop system (Yaw channel).

The system for the yaw angle can be expressed in the following transfer function

$$G_{p,\psi}(s) = \frac{\psi(s)}{V_y(s)} = \frac{7.461}{s(s + 0.2701)}$$

Both part of the system needs to be controlled as per the desired specifications.

III. DESIGN SPECIFICATIONS

The desired controller for the pitch channel must have an overshoot of less then 20 percent, a setteling time of les the 16 seconds, and a rise time of less the 2 seconds [14-15]. The desired specifications are presented as follows:

$M_p \leq 20\%$,

$t_s \leq 16\ sec$,

$t_r \leq 2\ sec$,

$Steady\ state\ error\ (step\ input) = 0$,

$Steady\ state\ error\ (step\ disturbance) = 0$,

The specifications for the yaw controller are presented as follows:

$M_p \leq 20\%$,

$t_s \leq 16\ sec$,

$t_r \leq 2\ sec$,

$Steady\ state\ error\ (step\ input) = 0$,

$Steady\ state\ error\ (step\ disturbance) = 0$,

The response (θ(t)) to step disturbance must settle within 16 s. For this item, we define the settling time as follows. Let θmax = max|θ(t)| (t ≥ 0) [17]. The settling time is the time after which |θ(t)| < 0.02θmax.

IV. CONTROLLER DESIGN

1) Part 1: Design of pitch channel controller.

The continous system is digitized in order to create the discrete time controller. The discrete transfer function of the plant with a sampling time of 0.2 since the rise time must be less or equal two seconds [18, 19].

The discrete time plant transfer function using zero order hold is as follows:

$$G_{p,\theta}(z) = \frac{0.1838z + 0.1821}{z^2 - 1.945\,z + 0.972}$$

The discrete feedback control system can be observed in figure4.

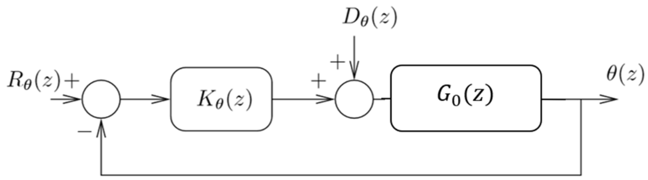

Figure 5: discrete system block diagram

A. *Percentage overshoot Calculations*

Table 1: Zeta vs. Overshoot table.

ζ	$\leq M_p$
0.7	5 %
0.6	10%
0.5	15%
0.46	20%

Since the design specification for the overshoot is $M_p \leq 20\%$, then is selected from table 1 the dampening ratio can be select as $\zeta = 0.7$ in order to ensure that the system is dynamic and operates within the desired specification then the dampening ratio is chosen from the information gathered from figures 5 and 6.

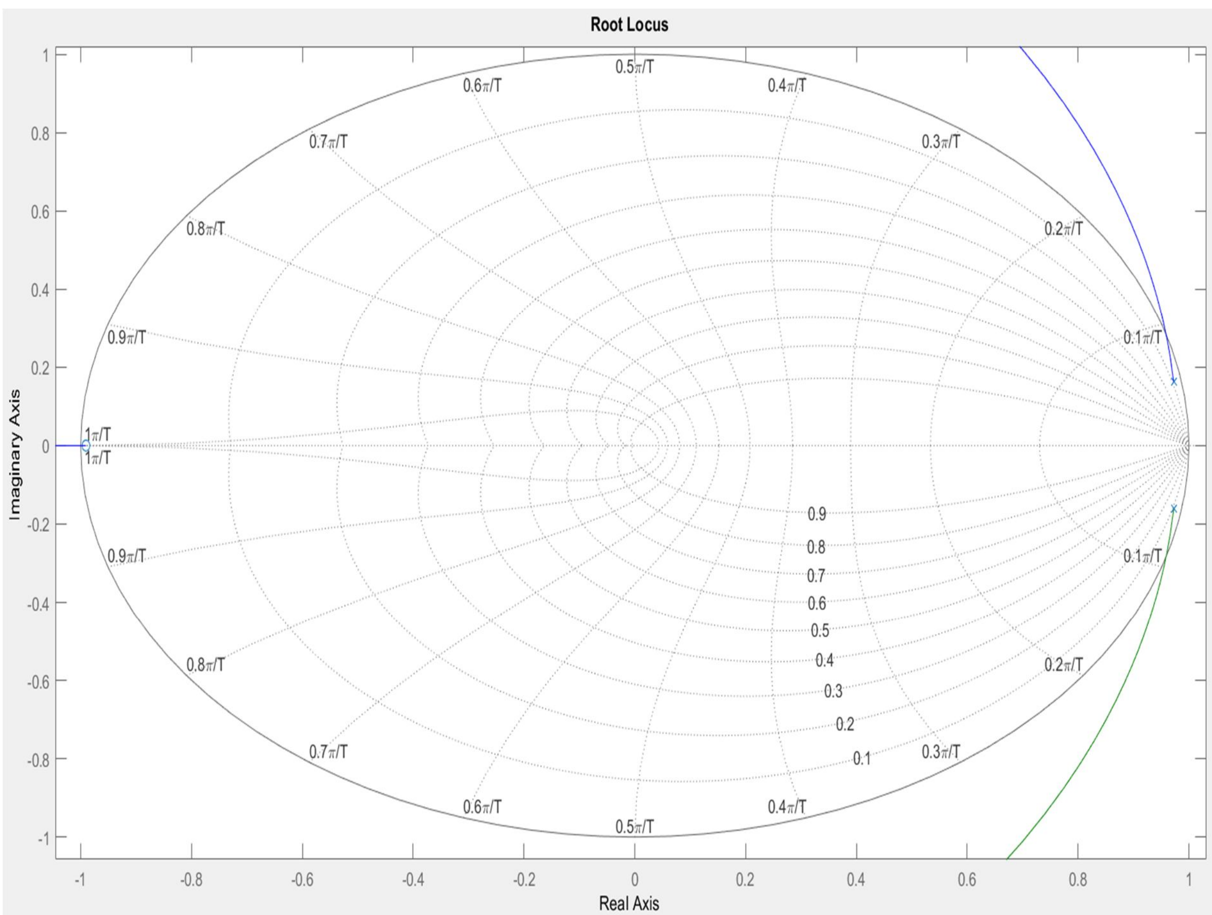

Figure 6: the root locus with the poles and zeros of the open loop system.

B. Rise time calculations.

Since the design specification for the rise time is $t_r \leq 2\,sec$, w_n can be calculated using the rise time as follows:
The rise time of the system as per the desired specifications is 2 sec.
The equation for the rise time calculations is:

$$T_r = \frac{1.8}{w_n}$$

With $T_r = 2$ as per the desired specification w_n can be calculated as follows

$$w_n \geq \frac{1.8}{2} \rightarrow w_n \geq 0.9$$

C. Settling time calculations

The settling time of the system can be estimated using the below equation

$$T_s = \frac{4.6}{w_n * \zeta}$$

With $\zeta = 0.7$ and $w_n = 0.9$

$$T_s = \frac{4.6}{0.7 * 0.9} = 7.3\,sec$$

This means that the settling time for our chosen parameters is estimated to be 7.3 sec which is less than the desired settling time of less than 16 sec.

D. Sampling time calculations

The sampling frequency is calculated using the bandwidth frequency w_{bw} which can be derived as follows.

$$w_{bw} = (-1.196 \times \zeta + 1.85)w_n$$
$$w_{bw} = (-1.196 \times 0.7 + 1.85) * 0.9$$
$$w_{bw} = 0.9115\,rad/sec$$

From the bandwidth frequency w_s can be calculated as follows:

$$w_s = 30 \times w_{bw} = 30 \times 0.9115 = 27.3450$$
$$w_s = 27.3450 = 2\pi f_s \quad \rightarrow f_s = 4.3521$$
$$T_s = \frac{1}{f_s} = 0.229\text{ sec}$$

For this project the rise time will be chosen as 0.15 sec to give the system more dynamic freedom in our calculation.

E. Desired pole calculation

Desired pole calculation:

$$s_{1,2} = -\zeta w_n \pm j w_n \sqrt{(1-\zeta^2)}$$
$$w_n = 0.9\ ,\ \zeta = 0.7\ ,\ T = 0.15$$
$$s_{1,2} = -(0.7)(0.9) \pm j(0.9)\sqrt{(1-(0.7)^2)} = -0.7200 \pm j\,0.5400$$

In z domain

$$z = e^{sT}\ ,$$
$$z_1 = e^{(-0.7200 + j\,0.5400)\times 0.15} = e^{-0.7200 \times 0.15} e^{j\,0.54 \times 0.15} = e^{-0.108}[\cos(0.081) + j\sin(0.081)]$$

The desired poles of the system are:

$$z_1 = 0.9056 + j\ 0.0876$$
$$z_2 = 0.9056 - j\ 0.0876$$

The poles of the open loop system are:

$$G_{p,\theta}(z) = \frac{(z + 0.9859)}{(z - 0.9491 + j\,0.2400)(z - 0.9491 - j\,0.2400)}$$

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$P_1 = 0.9491 + j\,0.2400$
$P_2 = 0.9491 - j\,0.2400$
The zero of the open loop system is:
$z_1 = -0.9859$
These poles and zeros can be seen in the root locus of the system:

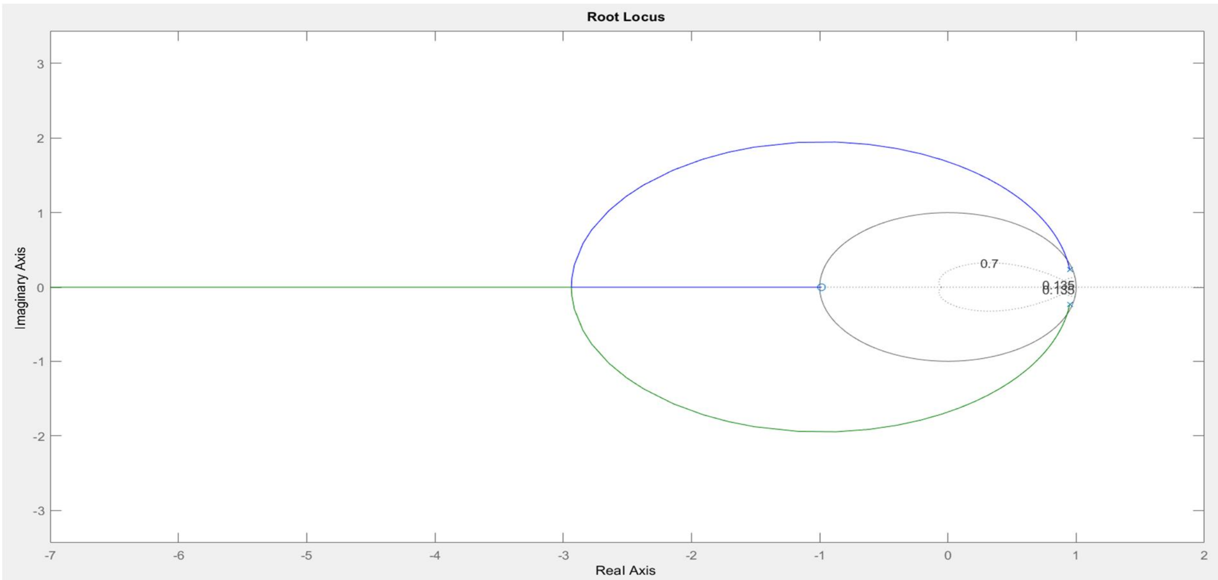

Figure 7: root locus of the open loop system.

In order to stabilize the system and meet the steady state error requirements a pole will be placed near the systems zero in order to decrease and counteract its effect on the system ($P = -0.9854$) And the complex poles will be counteracted using zeros in the controller [20, 21, 22]. In order to use two zeros a lead lag compensator was used.

$$K_\theta(z) = K\,\frac{(z-a_1)(z-a_2)}{(z-b_1)(z-b_2)}$$

After adding the selected poles or zeros

$$K_\theta(z) = K\,\frac{(z-0.9090 + j\,0.2396)(z-0.9090 + j\,0.2396)}{(z+0.9854)(z-b_2)}$$

To find the pole of the controller the desired poles were used to approximate the position of the controller.

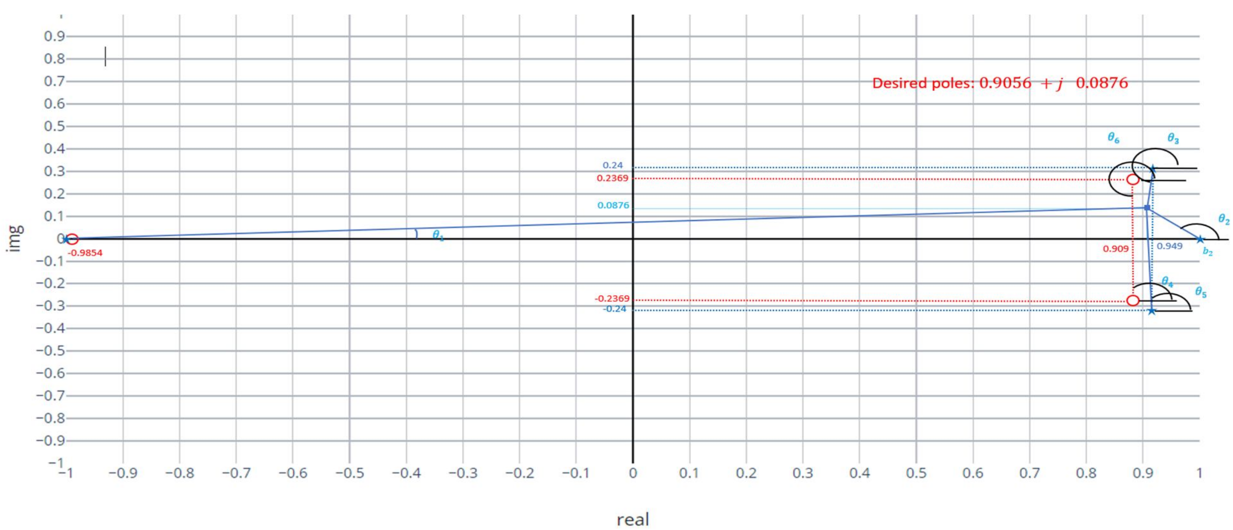

Figure 8: Angle and magnitude criteria.

$$\theta_1 = tan^{-1} (\frac{0.0876}{0.9859 + 0.9056}) = 2.65°(zero\ system)$$

$$\theta_2 = tan^{-1} (\frac{0.0876}{0.9854 + 0.9056}) = 2.65°(pole\ controller)$$

$$\theta_3 = 360 - tan^{-1} \left(\frac{0.24 - 0.0876}{0.909 - 0.9056}\right) = 271.3°(pole\ system)$$

$$\theta_4 = tan^{-1} \left(\frac{0.0876 + 0.2369}{0.909 - 0.9056}\right) = 89.33°(controller\ zero)$$

$$\theta_5 = 180 - tan^{-1} \left(\frac{0.24 + 0.0876}{0.949 - 0.909}\right) = 90.6°(system\ pole)$$

$$\theta_6 = 360 - tan^{-1} \left(\frac{0.2369 - 0.0876}{0.949 - 0.909}\right) = 271.27°(zero\ controller)$$

$$\sphericalangle Zeros - \sphericalangle Poles = -180$$

$$(\theta_1 + \theta_4 + \theta_6) - (\theta_3 + \theta_2 + \theta_5) = -180$$

$$2.65° + 89.33° + 271.25° - 90.6° - 271.3° - 2.65° - \theta_2 = -180$$

$$\theta_2 = 180 - 133.93 = 46.065$$

$$\tan(46.065) = \frac{0.0876}{0.9056 - b_2}$$

$$b_2 = 0.99$$

After some trial and error the pole was selected as 0.99 as to not be placed on the unit circle but be in a position to stabilize the system.

The final lead lag controller is as follows:

$$K_\theta(z) = K \frac{(z - 0.9090 + j\,0.2396)(z - 0.9090 + j\,0.2396)}{(z + 0.9854)(z - 0.99)}$$

The closed loop system with the controller is

$$G_{cl}(z) = \frac{K_\theta(z)G_0(z)}{1 + K_\theta(z)G_0(z)}$$

The characteristic equation (C.E) = $1 + K_\theta(z)G_0(z) = 0.0$

$$|K_\theta(z)||G_0(z)| = |-1|$$

$$\frac{K\,|(z - 0.9090 + j\,0.2396)(z_1 - 0.9090 - j\,0.2396)|}{|(z_1 + 0.9854)(z_1 - 0.99)|} \cdot \frac{|0.1838z + 0.1821|}{|z^2 - 1.945\,z + 0.972|} = 1$$

By solving the characteristic equation, the gain is found to be 0.34. After some trial and error the gain was chosen to be 2.52 (K = 2.52) in order to reach the systems specifications [23-24].

The discrete time controller can be written as follows:

$$K_\theta(z) = 2.52 * \frac{(z - 0.9090 + j\,0.2396)(z - 0.9090 + j\,0.2396)}{(z + 0.9854)(z - 0.99)}$$

$$K_\theta(z) = 2.52 * \frac{z^2 - 1.817z + 0.88366}{z^2 - 0.0046z + 0.9755}$$

The closed loop transfer function is

$$\frac{\theta(z)}{R_\theta(z)} = \frac{1.035\,z^3 - 0.8598\,z^2 - 0.9392z + 0.9014}{z^4 - 0.8683\,z^3 - 0.8682\,z^2 + 0.9083z - 0.03365}$$

The closed loop root locus is shown in the following figure

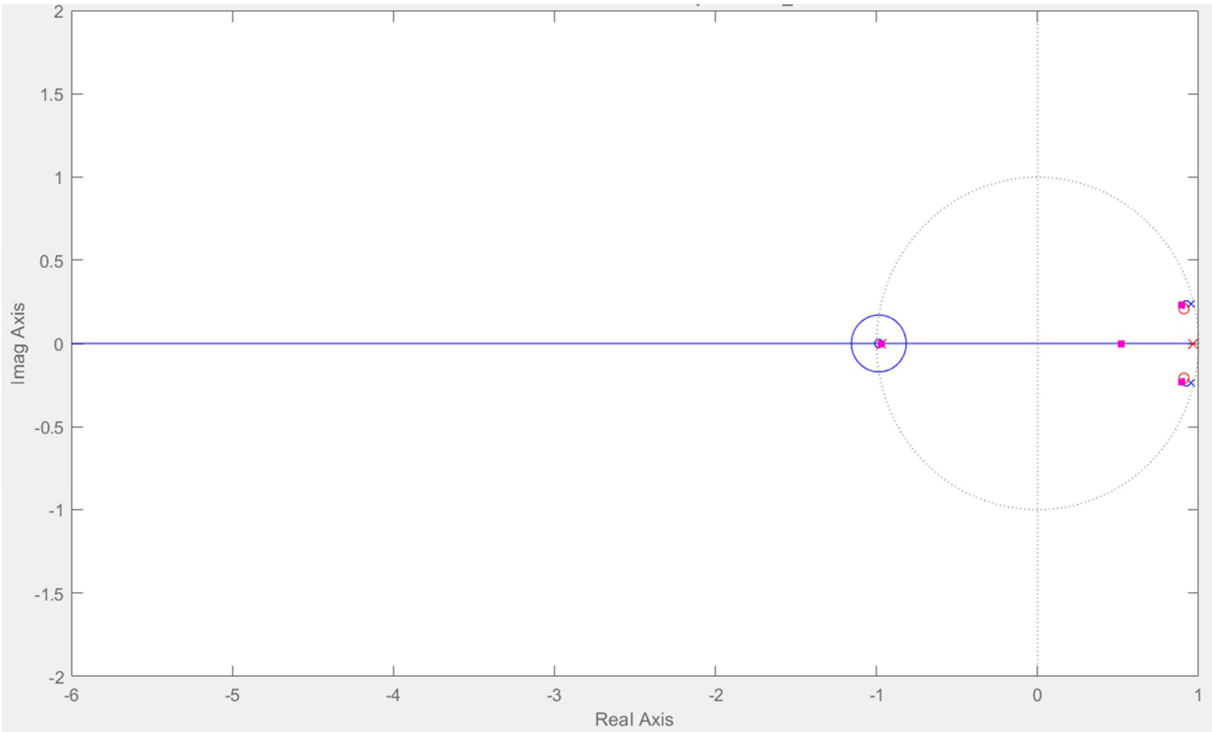

Figure 9: the root locus for the closed loop system

F. The response of the closed–loop system (θ[n]) to unit step reference input.

In order to check if the system meets the required criteria the step response is simulated in MATLAB.

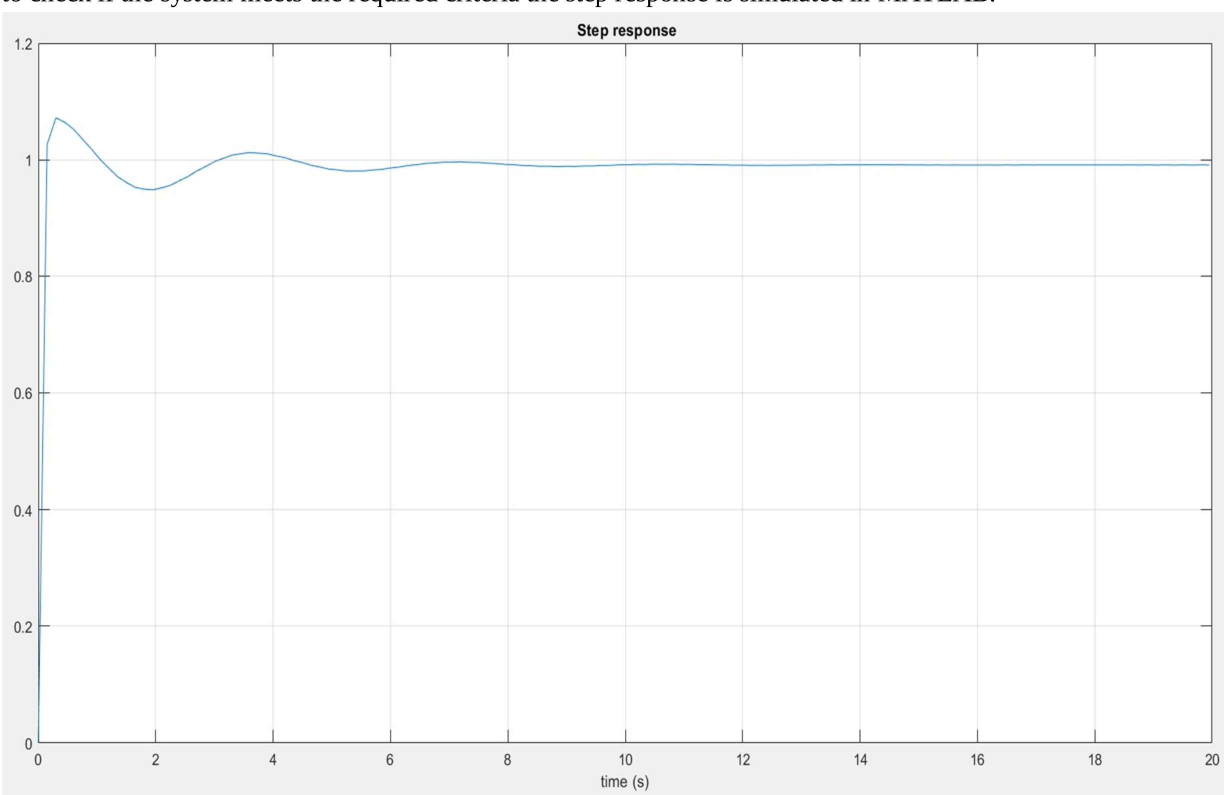

Figure 10: The step response of the closed loop system.

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The result of the closed loop unit step response can be observed in table 2.

Table 2: closed loop system specifications

Overshoot	8.2441
Rise Time	0
Settling Time	3.9000
Steady state error	0

The steady state error of the system is 0.0 since a pole was placed on the unit circle.

$$e_{ss} = \frac{1}{1 + k_p}$$

$$K_p = \lim_{z \to 1} G(z)K(z)$$

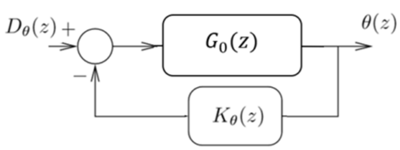
$$K_\theta(z) = 2.52 * \frac{z^2 - 1.817z + 0.88366}{z^2 - 0.0046z + 0.9755}$$

$$G(z) = \frac{0.4106\, z + 0.4048}{z^2 - 1.898\, z + 0.9584}$$

$$K_p = \lim_{z \to 1} \frac{2.52 * (z^2 - 1.817z + 0.88366)(0.4106\, z + 0.4048)}{(z^2 - 0.0046z + 0.9755)(z^2 - 1.898\, z + 0.9584)} = \infty$$

So $e_{ss} = 0$.

G. The response of the closed–loop system (θ[n]) to unit step disturbance



$$\frac{\theta(z)}{D(Z)} = \frac{G_\theta(z)}{1 + K(z)G_\theta(z)}$$

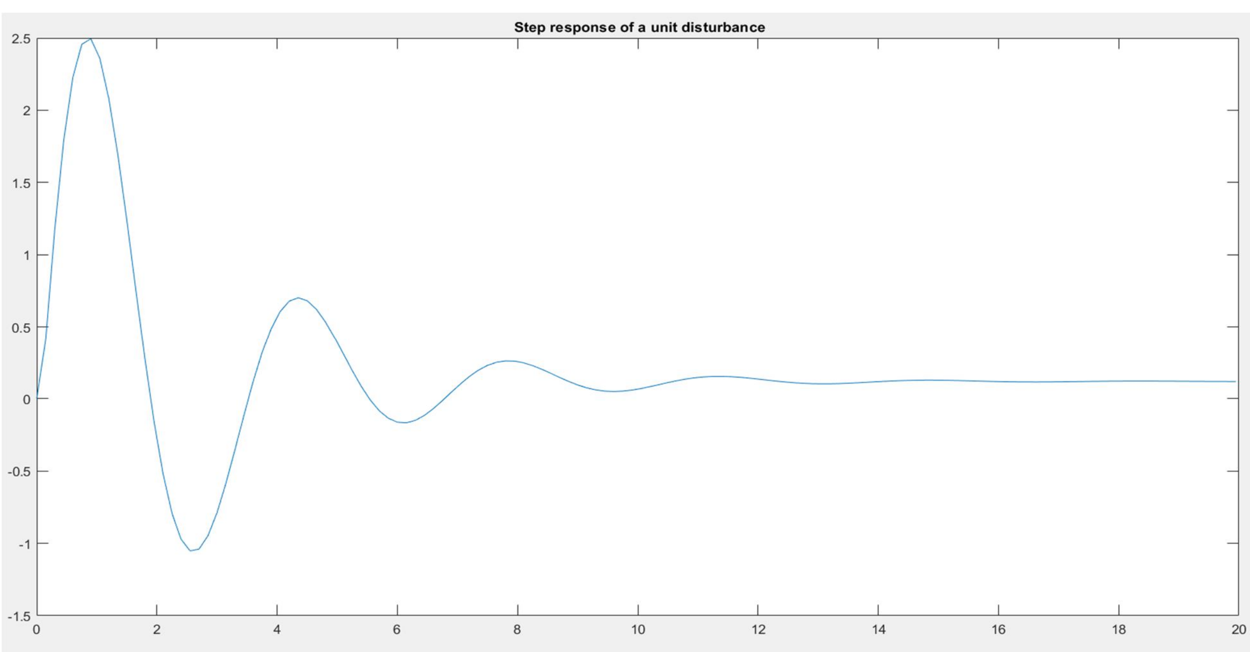


Figure 11: the response of the system to a unit step disturbance.

The steady state error of the system is 0.0 to a step input disturbance.

$$\theta_{ss} = \lim_{z\to 1} \frac{G(z)}{1 + K(z)G(z)}$$

$$\theta_{ss} = \lim_{z\to 1}(1 + K(z)G(z)) = \infty$$

$$\theta_{ss} = 0$$

Table 3: closed loop system specifications (with step response to the disturbance)

Overshoot	2.0032e+03
Rise Time	0
Settling Time	10.0500
Steady state error	0

From the table it can be seen that the settling time of systems reaction to a step disturbance is less than 16 seconds which means that the system specifications has been met.

H. Obtain motor voltage vp[n] in response to step reference input in the pitch channel.

From the step response the peak voltage can be derived [25, 26]. From this the maximum size of step input that does not result in motor saturation is calculated as follows.

$$\frac{V_{max}}{V_p} = \frac{8}{1.0720} = 7.46$$

V. CONCLUSION

In this project a controller for a 2-DOF helicopter was designed using the root locus method. The poles and zeros of the lead lag controller were strategically placed to allow the maximum controllability and stability of the system. Simulation results were presented to show the result of the proposed controller under various conditions.

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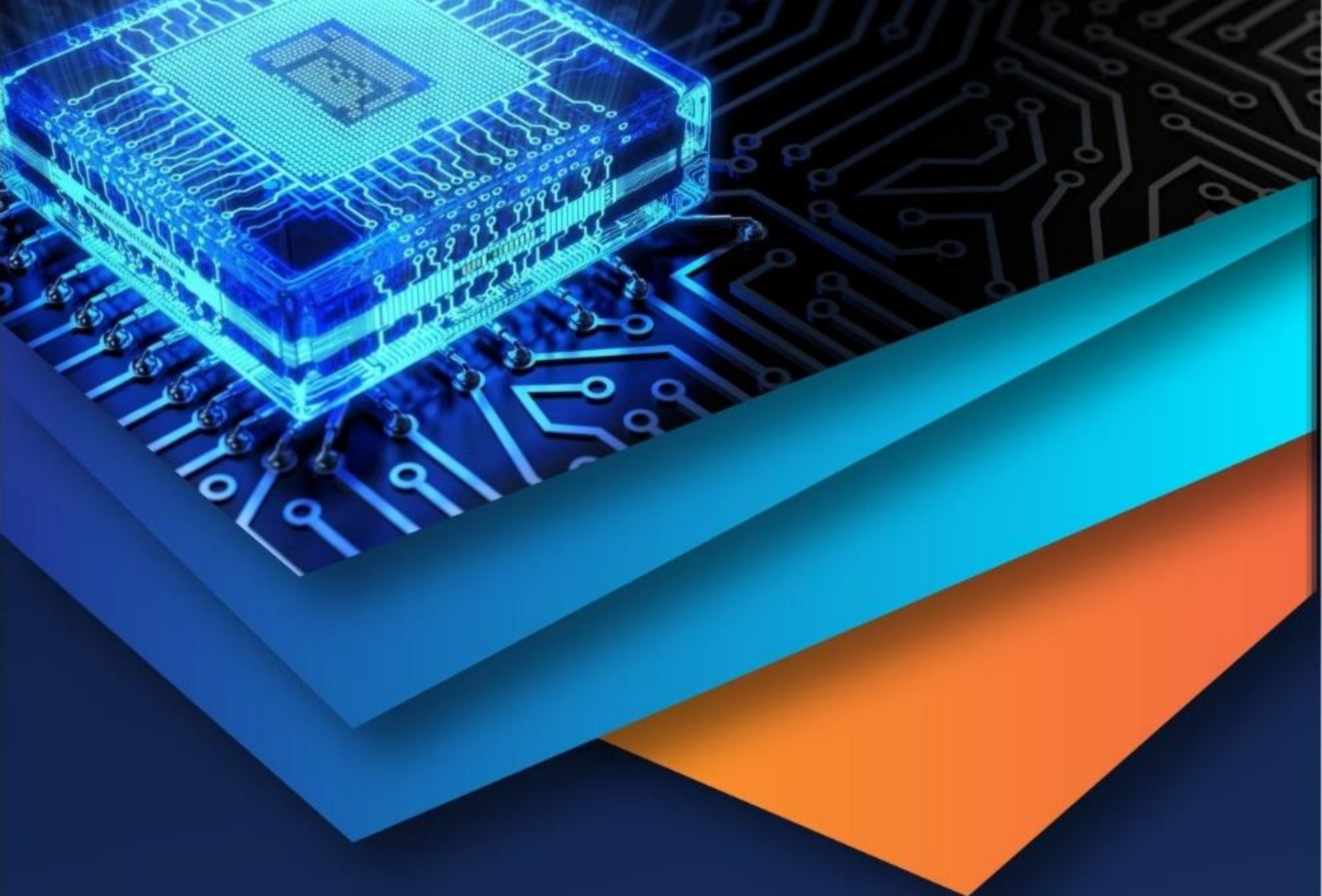

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