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Developed Ratio Estimator for Estimating Population Mean from a Finite Population using Sample Size and Correlation

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Abstract: Estimation of the population mean is a crucial matter in our world activities today. In this paper a new improved ratio estimator for estimation of population mean from a finite population using SRSWOR by utilizing information on sample size and correlation of the auxiliary variable was proposed. The derivation of the Bias and mean square error (MSE) was done to the first order of approximation. The expression for the optimum Bias and minimum mean square error was evaluated and presented for the minimum value of a constant τ. A mathematical efficiency comparison was performed and shows that our proposed estimator is more efficient than the other estimators. The empirical study performed shows that our estimator has a smaller bias and MSE compared to the other estimators. The PRE of our proposed estimator over other estimators compared to the usual estimator is higher than all the other modified estimators. Hence our proposed estimator is better than the other estimators and should be given preference over the modified estimators when estimating a population means.
Keywords: SRSWOR, Auxiliary variable, study variable, Bias, MSE

I. INTRODUCTION

There is no argument that the significance attached to the vital role played by sampling theory in statistics cannot be overemphasized considering its vast number of applications in different spheres of life. The application of sampling theory is not only limited to the sample selection, but also to the information about the population of its parameters. The main objective of sampling theory is to estimate the parameter of interest using some statistical properties as a bedrock. In many situations, it happens that information regarding some variables apart from the variables of interest (study variables) can easily be collected. This information is called auxiliary information which is formed by auxiliary variable(s). In survey sampling, if the auxiliary variable(s) are highly correlated and suitably used, the precision of estimators of the mean of the study variable is increased by reducing the variance of the estimators of the population parameters [4]. In estimating population parameters one can propose a different number of estimators. One can have used a sample mean or sample median or any other sample statistic [1]. To estimate a population mean, the most suitable estimator is the sample mean since the best estimator of every population parameter is its corresponding statistic. But some sample statistic tends to yield estimate with a large sampling variance when estimating its corresponding parameter. Hence one has to look for a better estimator that will perform better than the other estimator.

Different techniques for estimation of population mean to exist in the literature, but the most widely used ones are ratio, product, and regression estimators. When Regression line of study variable and auxiliary variable(s) pass through the origin and their correlation is positively high then Ratio estimators are applied. Likewise, Product estimators can be applicable when the regression line of the two variables doesn't pass through the origin and their correlation is negatively low. Situations may arise when there exists a correlation between the study variable and auxiliary variable(s) but nothing can be said about the direction of the correlation (either positive or negative) and the regression line is linear in this case regression estimator is used.

Let us consider a variable under study as Y which is correlated with its auxiliary variable X. We obtain the observations x_i and y_i corresponding to X and Y respectively for each of the sampling unit using simple random sampling without replacement (SRSWOR). The usual estimator for Y is its corresponding sample mean $\bar{y}_r=\frac{1}{n}\sum_{i=1}^{N}\bar{y}_i$ which is unbiased but has a large sampling variance, with a variance $V(\bar{y}_r)=\theta\bar{Y}^2C_y^2$. [7] Cochran made his first contribution by coming up with a ratio estimator of the form $\bar{Y}_r = \bar{y}_r\frac{\bar{X}}{\bar{x}}$ in order to reduce the effect of having a large sampling variance. The estimator given by Cochran is biased and its bias and mean square error (MSE) are:

$$\text{Bias}\left(\hat{\bar{Y}}_r\right) = \theta\bar{Y}\left(C_x^2 - \rho C_x C_y\right) --------(1)$$

$$MSE\left(\hat{\bar{Y}}_r\right) = \theta\bar{Y}^2\left(C_y^2 + C_x^2 - 2\rho C_x C_y\right) ---(2)$$

A lot of researchers have contributed to the literature by proposing an estimator which will be more efficient in estimating the population means thereby modifying the estimator given by Cochran using some parameters like the coefficient of variation, skewness, kurtosis, median, correlation coefficient, number of sample size, minimum, maximum, and their combinations [13] [14]

[10] [8] [12] [2] [1] [9] [11]. To the best of our knowledge, no previous research used the information from sample size and correlation coefficient together. Table (1) below gives a summary of some estimators with their constants, bias, and MSE which we called modified estimators throughout this paper unless otherwise stated. The modified estimators in the table (1), were used in making a comparison with our proposed estimator.

Table (1) Modified estimators with their corresponding Constants, Bias, and MSE

ESTIMATORS	CONSTANTS	BIAS	MSE
$\hat{\hat{Y}}_1 = \bar{y}(\frac{\bar{X}+C_x}{\bar{x}+C_x})$	$\gamma_1 = \frac{\bar{X}}{\bar{X}+C_x}$	$\theta\bar{Y}(\gamma_1^2C_x^2 - \gamma_1\rho C_xC_y)$	$\theta\bar{Y}^2(C_y^2 + \gamma_1^2C_x^2 - 2\gamma_1\rho C_xC_y)$
$\hat{\hat{Y}}_2 = \bar{y}(\frac{\bar{X}C_x+\beta_2}{\bar{x}C_x+\beta_2})$	$\gamma_2 = \frac{\bar{X}C_x}{\bar{X}C_x+\beta_2}$	$\theta\bar{Y}(\gamma_2^2C_x^2 - \gamma_2\rho C_xC_y)$	$\theta\bar{Y}^2(C_y^2 + \gamma_2^2C_x^2 - 2\gamma_2\rho C_xC_y)$
$\hat{\hat{Y}}_3 = \bar{y}(\frac{\bar{X}+\rho}{\bar{x}+\rho})$	$\gamma_3 = \frac{\bar{X}}{\bar{X}+\rho}$	$\theta\bar{Y}(\gamma_3^2C_x^2 - \gamma_3\rho C_xC_y)$	$\theta\bar{Y}^2(C_y^2 + \gamma_3^2C_x^2 - 2\gamma_3\rho C_xC_y)$
$\hat{\hat{Y}}_4 = \bar{y}(\frac{\bar{X}+\beta_2}{\bar{x}+\beta_2})$	$\gamma_4 = \frac{\bar{X}}{\bar{X}+\beta_3}$	$\theta\bar{Y}(\gamma_4^2C_x^2 - \gamma_4\rho C_xC_y)$	$\theta\bar{Y}^2(C_y^2 + \gamma_4^2C_x^2 - 2\gamma_4\rho C_xC_y)$
$\hat{\hat{Y}}_5 = \bar{y}(\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1})$	$\gamma_5 = \frac{\bar{X}}{\bar{X}+\beta_1}$	$\theta\bar{Y}(\gamma_5^2C_x^2 - \gamma_5\rho C_xC_y)$	$\theta\bar{Y}^2(C_y^2 + \gamma_5^2C_x^2 - 2\gamma_5\rho C_xC_y)$
$\hat{\hat{Y}}_6 = \bar{y}(\frac{\bar{X}+M_d}{\bar{x}+M_d})$	$\gamma_6 = \frac{\bar{X}}{\bar{X}+M_d}$	$\theta\bar{Y}(\gamma_6^2C_x^2 - \gamma_6\rho C_xC_y)$	$\theta\bar{Y}^2(C_y^2 + \gamma_6^2C_x^2 - 2\gamma_6\rho C_xC_y)$
$\hat{\hat{Y}}_7 = \bar{y}(\frac{\bar{X}+n}{\bar{x}+n})$	$\gamma_7 = \frac{\bar{X}}{\bar{X}+n}$	$\theta\bar{Y}(\gamma_7^2C_x^2 - \gamma_7\rho C_xC_y)$	$\theta\bar{Y}^2(C_y^2 + \gamma_7^2C_x^2 - 2\gamma_7\rho C_xC_y)$
$\hat{\hat{Y}}_8 = \bar{y}\{\alpha + (1-\alpha)\left(\frac{\bar{X}+n}{\bar{x}+n}\right)\}$	$\gamma_8 = \frac{\bar{X}}{\bar{X}+n}$	$\theta\bar{Y}(\gamma_8^2C_x^2 - \gamma_8\rho C_xC_y + \alpha\gamma_8^2C_x^2)$	$\theta\bar{Y}^2(C_y^2 + \gamma_8^2C_x^2 - 2\gamma_8\rho C_xC_y + \alpha^2\gamma_8^2C_x^2 + 2\alpha\gamma_8\rho C_xC_y - 2\alpha\gamma_8\rho\gamma_8^2C_x^2)$

Where: $\theta = \frac{1-f}{n}$ and $f = \frac{n}{N}$

II. METHODOLOGY OF OUR PROPOSED ESTIMATOR

Motivated by [1] [2] and [13] We proposed the following estimator:

$$\hat{\hat{Y}}_p = \bar{y}\left\{\alpha + (1-\alpha)\left(\frac{\bar{X}\rho + n}{\bar{x}\rho + n}\right)\right\} ------(3)$$

Define; $\varepsilon_0 = \frac{\bar{y}}{\bar{Y}} - 1$, $\varepsilon_1 = \frac{\bar{x}}{\bar{X}} - 1$, $\varepsilon_2 = \frac{s_y^2}{S_y^2} - 1$, $\varepsilon_3 = \frac{s_x^2}{S_y^2} - 1$, $\varepsilon_4 = \frac{s_{xy}}{S_{xy}} - 1$

Such that;

$$E[\varepsilon_0] = [\varepsilon_1] = E[\varepsilon_2] = E[\varepsilon_3] = E[\varepsilon_4] = 0$$

And

$$E[\varepsilon_0^2] = \theta C_y^2; E[\varepsilon_1^2] = \theta C_x^2; E[\varepsilon_0\varepsilon_1] = \theta\rho C_xC_y$$

Where $\theta = \frac{1-f}{n}$

It implies that;

$$\bar{y} = \bar{Y}(1+\varepsilon_0) and\ \bar{x} = \bar{X}(1+\varepsilon_1)$$

Putting these values in the propose estimator in (1) above gives;

$$\hat{\hat{Y}}_p = \bar{Y}(1-\varepsilon_0)\left\{\alpha + (1-\alpha)\left(\frac{\bar{X}\rho + n}{\bar{X}(1-\varepsilon_1)\rho + n}\right)\right\}$$

$\hat{\hat{Y}}_p = \bar{Y}(1-\varepsilon_0)\{\alpha + (1-\alpha)(1+\gamma\varepsilon_1)^{-1}\}$, Where $\gamma = \frac{\bar{X}\rho}{\bar{x}\rho + n}$

Using the fact that; $(1+\gamma\varepsilon_1)^{-1} = 1 - x + x^2 - x^3 + ---$

$$\hat{\hat{Y}}_p = \bar{Y}(1-\varepsilon_0)\{\alpha + (1-\alpha)(1 - \gamma\varepsilon_1 + \gamma^2\varepsilon_1^2 - \gamma^3\varepsilon_1^3 + ---)\}$$

Taking the expansion up to the first order of approximation and expanding through gives;

$$\hat{\hat{Y}}_p = \bar{Y}\{(1 - \gamma\varepsilon_1 + \alpha\gamma\varepsilon_1 + \gamma^2\varepsilon_1^2 - \alpha\gamma^2\varepsilon_1^2 + \varepsilon_0 - \gamma\varepsilon_0\varepsilon_1 + \alpha\gamma\varepsilon_0\varepsilon_1)\}$$

Taking the expectation gives;

$$E[\hat{\hat{Y}}_p] = E[\bar{Y}\{(1 - \gamma\varepsilon_1 + \alpha\gamma\varepsilon_1 + \gamma^2\varepsilon_1^2 - \alpha\gamma^2\varepsilon_1^2 + \varepsilon_0 - \gamma\varepsilon_0\varepsilon_1 + \alpha\gamma\varepsilon_0\varepsilon_1)\}]$$

$$E[\hat{\hat{Y}}_p] = \bar{Y}\{(1 - \gamma\theta\rho C_xC_y + \gamma^2\theta C_x^2 + \alpha\gamma\theta\rho C_xC_y - \alpha\gamma^2\theta C_x^2)\}$$

$\text{Bias}\left(\hat{\hat{Y}}_p\right) = E\left[\hat{\hat{Y}}_p\right] - \bar{Y}$

$\text{Bias}\left(\widehat{\bar{Y}_p}\right) = \theta\bar{Y}\{(\gamma^2C_x^2 - \gamma\rho C_xC_y + \alpha\gamma\rho C_xC_y - \alpha\gamma^2C_x^2)\} - - - - - - - - - - - - -(4)$

$\text{MSE}\left(\widehat{\bar{Y}_p}\right) = E[\widehat{\bar{Y}_p} - \bar{Y}]^2$

$= E[\bar{Y}(\varepsilon_0 - \gamma\varepsilon_1 - \gamma\varepsilon_0\varepsilon_1 + \gamma^2\varepsilon_1^2 + \alpha\gamma\varepsilon_1 + \alpha\gamma\varepsilon_0\varepsilon_{1-}\alpha\gamma^2\varepsilon_1^2)]^2$

Expanding and taking the expansion up to the first order approximation yield; $MSE\left(\widehat{\bar{Y}_p}\right) =$
$\theta\bar{Y}^2(C_y^2 + \gamma^2C_x^2 - 2\gamma\rho C_xC_y + \alpha^2\gamma^2C_x^2 + 2\alpha\gamma\rho C_xC_y - 2\alpha\gamma^2C_x^2) - - - - - -(5)$

The Min. $MSE\left(\widehat{\bar{Y}_p}\right)$ is found by finding the optimum value of alpha (α), which is found by differentiating equation (5) w.r.t. α and equating to zero. Which yield;

$\frac{\partial MSE(\widehat{\bar{Y}_p})}{\partial\alpha} = 0$

$$\alpha\gamma^2C_x^2 = \gamma^2C_x^2 - \gamma\rho C_xC_y$$

Which is minimum for;

$$\alpha = \frac{\gamma^2C_x^2 - \gamma\rho C_xC_y}{\gamma^2C_x^2}$$

To find the Min. $MSE\left(\widehat{\bar{Y}_p}\right)$ we substitute the value of α into equation (5) to yield;

$$\text{Min.}\,MSE\left(\widehat{\bar{Y}_p}\right) = \theta\bar{Y}^2(C_y^2 + \gamma^2C_x^2 - 2\gamma\rho C_xC_y - \tau) - - - - - - - - - - - - - - -(6)$$

Where $\tau = (\frac{\gamma^2C_x^2 - \gamma\rho C_xC_y}{\gamma C_x})^2 \geq 0$

To find the optimum Bias, we substitute the value of α into equation (4) to yield;

$$Bias_{opt}\left(\widehat{\bar{Y}_p}\right) = \theta\bar{Y}(\gamma^2C_x^2 - \gamma\rho C_xC_y - \tau) - -(7)$$

Where $\tau = (\frac{\gamma^2C_x^2 - \gamma\rho C_xC_y}{\gamma C_x})^2 \geq 0$

III. MATHEMATICAL EFFICIENCY COMPARISON BETWEEN OUR PROPOSED ESTIMATOR AND OTHER ESTIMATORS

The efficiency of our proposed estimator in (1) above is compared with that of existing and modified estimators

1) Comparison with the usual sample mean($\bar{y}_r$)

Our proposed estimator is better than the sample mean ($\bar{y}_r$) if :

$MSE\left(\widehat{\bar{Y}_p}\right) \leq V(\bar{y}_r)$

That is: $V(\bar{y}_r)$- $MSE\left(\widehat{\bar{Y}_p}\right) \geq 0$

$$\theta\bar{Y}^2C_y^2 - \theta\bar{Y}^2(C_y^2 + \gamma^2C_x^2 - 2\gamma\rho C_xC_y - \tau)$$
$$-\theta\bar{Y}^2(\gamma^2C_x^2 - (2\gamma\rho C_xC_y + \tau) > 0$$

If $\gamma^2C_x^2 < 2\gamma\rho C_xC_y + \tau$

2) Comparison with Cochran estimator($\widehat{\bar{Y}_r}$)

Our proposed estimator is better than the estimator given by Cochran if:

$\text{MSE}(\widehat{\bar{Y}_r}) - \text{MSE}(\widehat{\bar{Y}_p}) = \theta\bar{Y}^2(C_y^2 + C_x^2 - 2\rho C_xC_y) - \theta\bar{Y}^2(C_y^2 + \gamma^2C_x^2 - 2\gamma\rho C_xC_y - \tau)$

$= \theta\bar{Y}^2((C_x - \rho C_y)^2 - (\gamma C_x - \rho C_y)^2 + \tau) > 0$

If $C_x - \rho C_y)^2 > (\gamma C_x - \rho C_y)^2$

3) Comparison with other modified estimators $(\widehat{\bar{Y}_j})\ \forall\ j = 1, 2, 3, 4, 5, 6, and\ 7$

Our proposed estimator performs better than the modified estimators if:

$\text{MSE}(\widehat{\bar{Y}_j}) - \text{Min.MSE}(\widehat{\bar{Y}_p}) = \theta\bar{Y}^2(C_y^2 + \gamma_j^2C_x^2 - 2\gamma_j\rho C_xC_y) - \theta\bar{Y}^2(C_y^2 + \gamma_p^2C_x^2 - 2\gamma_p\rho C_xC_y - \tau)$

$= \theta\bar{Y}^2[C_x^2(\gamma_j^2 - \gamma_p^2) - 2\rho C_xC_y\,(\gamma_j - \gamma_p) + \tau] > 0$

If $C_x^2(\gamma_j^2 - \gamma_p^2) - 2\rho C_xC_y\,(\gamma_j - \gamma_p) > 0$

If: $\rho \leq \frac{\gamma_j + \gamma_p}{2} \bullet \frac{C_x}{C_y}$

4) *Comparison with $\widehat{\widehat{Y}}_8$ estimator our proposed estimator is better than $\widehat{\widehat{Y}}_8$ if*

$$MSE\left(\widehat{\widehat{Y}}_8\right) - MSE\left(\widehat{\widehat{Y}}_p\right)$$

$$\theta\bar{Y}^2\left(C_y^2 + \gamma_8^2 C_x^2 - 2\gamma_8\rho C_x C_y - \tau\right) - \theta\bar{Y}^2\left(C_y^2 + \gamma_p^2 C_x^2 - 2\gamma_p\rho C_x C_y - \tau\right)$$

$$\theta\bar{Y}^2\left[C_x^2\left(\gamma_8^2 - \gamma_p^2\right) - 2\gamma\rho C_x C_y\left(\gamma_8 - \gamma_p\right) - \left(\tau_8 - \tau_p\right)\right] > 0$$

If:

$$C_x^2\left(\gamma_8^2 - \gamma_p^2\right) - 2\gamma\rho C_x C_y\left(\gamma_8 - \gamma_p\right) > \left(\tau_8 - \tau_p\right)$$

IV. RESULT AND DISCUSSION

To buttress our point that our proposed estimator performs better than the modified estimators, in addition to the mathematical efficiency comparison an empirical study was performed. We used data from [5] [15] and [16]. The summary of the data is given in table (2). Using the data in the table (2), the value of the constants, bias, and MSE were obtained and presented in table (3&4). Table (5) gives the summary of the efficiency of our proposed estimator relative to the modified estimators for population 1, 2, 3 &4. The Summary of Percentage relative efficiency of our proposed estimator over modified estimators compared to the usual estimator $\bar{y}_r$ for population 1,2,3, and 4 is presented in table (6). For clear vision, we use figures 1, 2, 3 & 4 to show the representation of the numerical values obtained in table 3 while fig 5, 6 7 & 8 was used to represent the numerical values obtained in the table (6).

Table (2): summary of the data used in assessing our comparison

POPULATION	MURTHY		MUKHOPADHYAY		COCHRAN	
PARAMETERS	P_1	P_2	P_3	P_4	P_5	P_6
N	80	80	40	40	49	10
n	20	20	8	8	5	4
F	0.25	0.25	0.2	0.2	0.102	0.102
$\bar{Y}$	51.8264	51.8264	50.7858	50.7858	127.796	101.1
$\bar{X}$	11.2646	2.8513	2.3033	9.4543	103.143	58.8
ρ	0.9413	0.9150	0.8006	0.8349	0.982	0.6515
S_y	18.3566	18.3566	16.7352	16.7352	123.121	14.6523
S_x	8.4561	2.7043	1.9360	6.3869	104.405	7.5339
C_y	0.3542	0.3542	0.3295	0.3295	0.963	0.1449
C_x	0.7507	0.9485	0.8406	0.6756	1.012	0.1281
C_y^2	0.1255	0.1255	0.1086	0.1086	0.927	0.0210
C_x^2	0.5635	0.8996	0.7065	0.4564	1.024	0.0164
β_1	1.0500	1.3006	0.9740	0.8799	4.777	0.5764
β_2	-0.0634	0.6977	-0.5344	-0.4622	7.511	0.3814
M_d	7.5750	1.4800	1.250	7.0700	64	58

Table (3): Constants of the modified Estimators

$\widehat{\widehat{Y}}_j$	P_1	P_2	P_3	P_4	P_5	P_6	Constants
$\widehat{\widehat{Y}}_1 = \bar{y}(\frac{\bar{X} + C_x}{\bar{x} + C_x})$	0.9375	0.7504	0.7326	0.9333	0.9903	0.9978	$\gamma_1 = \frac{\bar{X}}{\bar{X} + C_x}$
$\widehat{\widehat{Y}}_2 = \bar{y}(\frac{\bar{X}C_x + \beta_2}{\bar{x}C_x + \beta_2})$	1.0075	0.7949	1.3813	1.0780	0.9329	0.9518	$\gamma_2 = \frac{\bar{X}C_x}{\bar{X}C_x + \beta_2}$
$\widehat{\widehat{Y}}_3 = \bar{y}(\frac{\bar{X} + \rho}{\bar{x} + \rho})$	0.9229	0.7571	0.7421	0.9189	0.9906	0.9890	$\gamma_3 = \frac{\bar{X}}{\bar{X} + \rho}$
$\widehat{\widehat{Y}}_4 = \bar{y}(\frac{\bar{X} + \beta_2}{\bar{x} + \beta_2})$	1.0057	0.8034	1.3021	1.0514	0.9321	0.9936	$\gamma_4 = \frac{\bar{X}}{\bar{X} + \beta_3}$
$\widehat{\widehat{Y}}_5 = \bar{y}(\frac{\bar{X} + \beta_1}{\bar{x} + \beta_1})$	0.9147	0.6868	0.7028	0.9149	0.9557	0.9903	$\gamma_5 = \frac{\bar{X}}{\bar{X} + \beta_1}$
$\widehat{\widehat{Y}}_6 = \bar{y}(\frac{\bar{X} + M_d}{\bar{x} + M_d})$	0.5979	0.6583	0.6482	0.5722	0.6171	0.5034	$\gamma_6 = \frac{\bar{X}}{\bar{X} + M_d}$
$\widehat{\widehat{Y}}_7 = \bar{y}(\frac{\bar{X} + n}{\bar{x} + n})$	0.3606	0.1248	0.2235	0.5417	0.9538	0.9363	$\gamma_7 = \frac{\bar{X}}{\bar{X} + n}$
$\widehat{\widehat{Y}}_8 = \bar{y}\{\alpha + (1-\alpha)\left(\frac{\bar{X} + n}{\bar{x} + n}\right)\}$	0.3603	0.1248	0.2235	0.5417	0.9538	0.9363	$\gamma_8 = \frac{\bar{X}}{\bar{X} + n}$
$\widehat{\widehat{Y}}_p = \bar{y}\{\alpha + (1-\alpha)\left(\frac{\bar{X}\rho + n}{\bar{x}\rho + n}\right)\}$	0.3465	0.1955	0.1813	0.4966	0.9530	0.9055	$\gamma_p = \frac{\bar{X}\rho}{\bar{X}\rho + n}$

Table (4): Summary of the Bias and MSE of the proposed and usual estimators

Estimator	Population 1		Population 2		Population 3		Population 4	
	BIAS	MSE	BIAS	MSE	BIAS	MSE	BIAS	MSE
$\bar{y}$	0	12.6409	0	12.6409	0	28.0101	0	28.0101
$\widehat{Y_r}$	0.6087	18.9764	1.1509	41.3171	2.4621	95.8438	1.3740	49.8521
$\widehat{Y_1}$	0.5065	15.2496	0.5362	17.1936	1.1010	42.0409	1.1376	41.0305
$\widehat{Y_2}$	0.6215	19.4498	0.6299	20.6686	5.2909	217.7358	1.6759	61.4365
$\widehat{Y_3}$	0.4837	14.4438	0.5500	17.7073	1.1407	43.5111	1.0899	39.3070
$\widehat{Y_4}$	0.6184	19.3390	0.6485	21.3737	4.6175	188.0493	1.5693	57.2840
$\widehat{Y_5}$	0.4715	14.0208	0.4144	12.5502	0.9807	37.6305	1.0762	38.7911
$\widehat{Y_6}$	0.1007	2.7800	0.3644	11.1401	0.7780	30.4603	0.2184	11.6580
$\widehat{Y_7}$	-0.0332	1.8332	-0.0474	6.3154	-0.0721	11.5806	0.1686	10.6005
$\widehat{Y_8}$	-0.0410	1.4303	-0.1300	2.0377	-0.1011	10.1074	0.1270	8.4858
$\widehat{Y_p}$	-0.0472	1.4001	-0.1353	1.6619	-0.1422	9.8792	-0.2031	8.3824

Table (5): Summary of Relative efficiency of the proposed estimator compared to another estimator

Estimators($\widehat{Y_j}$)	$Efficiency\ e\ (\widehat{Y_j}/\widehat{Y_p}) = MSE(\widehat{Y_p})/MSE(\widehat{Y_j})$			
	Population 1	Population 2	Population 3	Population 4
$\widehat{Y_r}$	0.0738	0.0402	0.1031	0.1681
$\widehat{Y_1}$	0.0918	0.0967	0.2350	0.20430
$\widehat{Y_2}$	0.0720	0.0804	0.0454	0.1364
$\widehat{Y_3}$	0.0969	0.0939	0.2271	0.2126
$\widehat{Y_4}$	0.0724	0.0778	0.0525	0.1463
$\widehat{Y_5}$	0.0999	0.1324	0.2625	0.2161
$\widehat{Y_6}$	0.5036	0.1492	0.3243	0.7190
$\widehat{Y_7}$	0.7637	0.2632	0.8531	0.7865
$\widehat{Y_8}$	0.9789	0.8156	0.9774	0.9878

Table (6): Summary of Percentage relative efficiency of proposed estimators over modified estimators compared to the estimator $\bar{y}$ for population 1,2,3, and 4

Estimators($\widehat{Y_j}$)	$PRE = V(\bar{y})/MSE(\widehat{Y_j}) \times 100$			
	Population 1	Population 2	Population 3	Population 4
$\widehat{Y_r}$	66.6138	30.5874	29.2247	56.1864
$\widehat{Y_1}$	82.9129	73.5210	66.6258	68.2590
$\widehat{Y_2}$	64.9924	61.1599	12.8643	45.5920
$\widehat{Y_3}$	87.5178	71.3881	64.3746	71.2598
$\widehat{Y_4}$	65.1999	59.1423	14.8951	48.8969
$\widehat{Y_5}$	90.1582	100.7227	74.4346	72.2075
$\widehat{Y_6}$	454.7086	113.4721	91.9561	240.2651
$\widehat{Y_7}$	689.5528	200.1599	277.1247	330.0820
$\widehat{Y_8}$	883.7936	620.3515	277.1247	330.0820
$\widehat{Y_p}$	902.8569	760.6294	283.5260	334.1537

Fig1

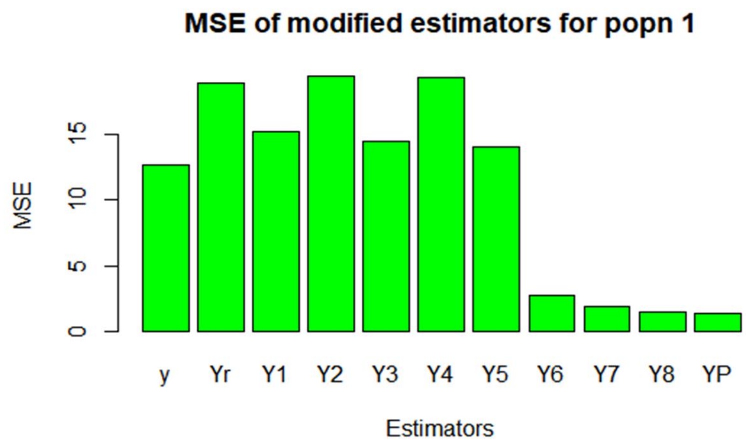


Fig2

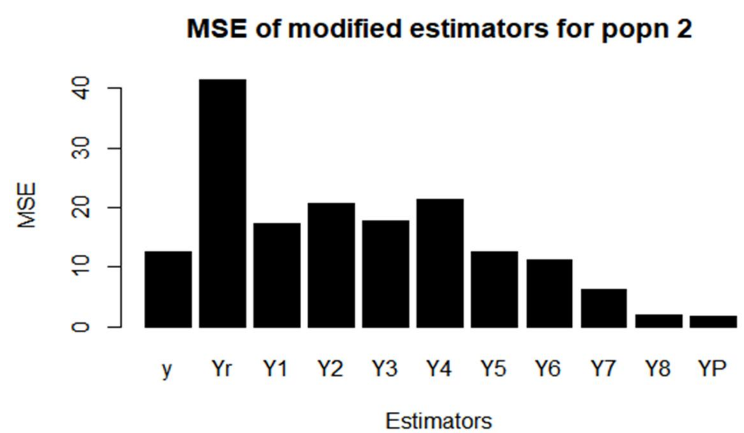
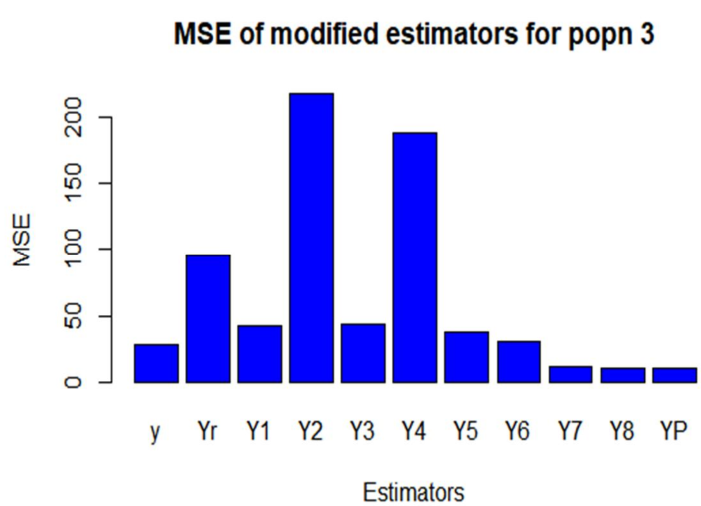


Fig.3



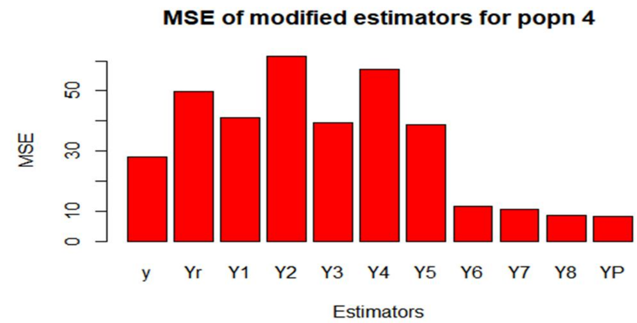


Fig 4: MSE of modified and existed estimators corresponding to population 1

Fig5

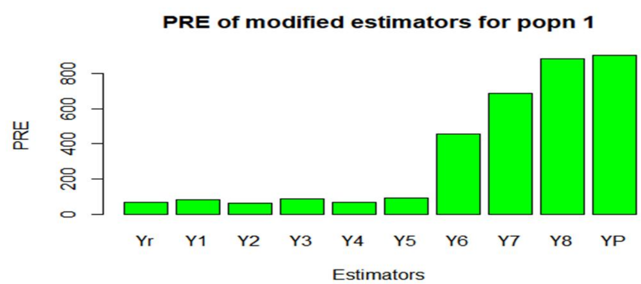


Fig6

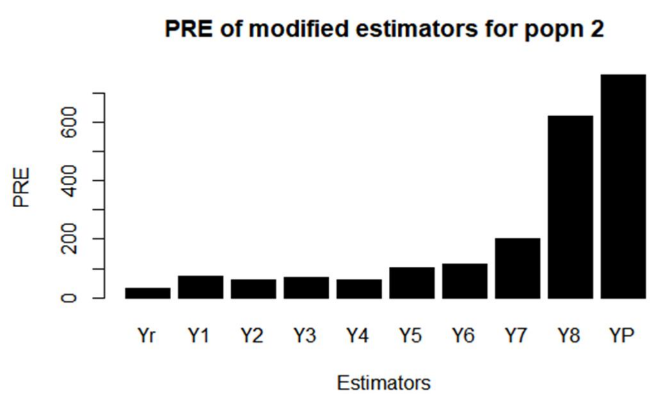


Fig7

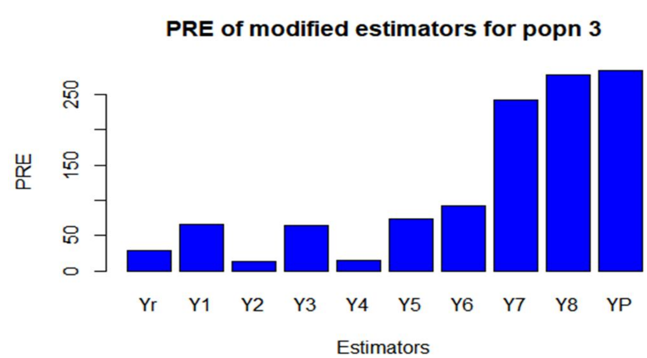


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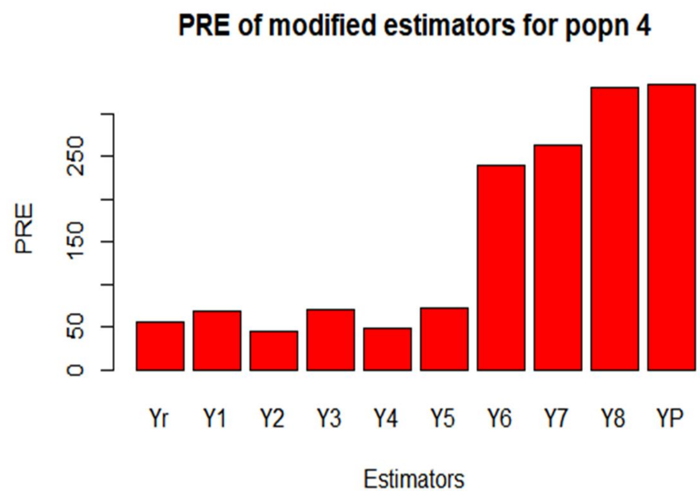
Fig 8



V. CONCLUSION

In this paper, we proposed a new modified ratio class of estimators for the estimation of population mean under simple random sampling without replacement (SRSWOR). The derivation of the Bias and mean square error (MSE) was done to the first order of approximation. The expression for the optimum Bias and minimum mean square error was evaluated and presented for the minimum value of a constant τ. The mathematical efficiency comparison shows that our proposed estimator is more efficient than the other estimators. The empirical study performed shows that our estimator has a smaller bias and MSE compared to the other estimators. The PRE of our proposed estimator over other estimators compared to the usual estimator is higher than all the other modified estimators. Hence our proposed estimator is better than the other estimators and should be given preference over the modified estimators when estimating the population mean.

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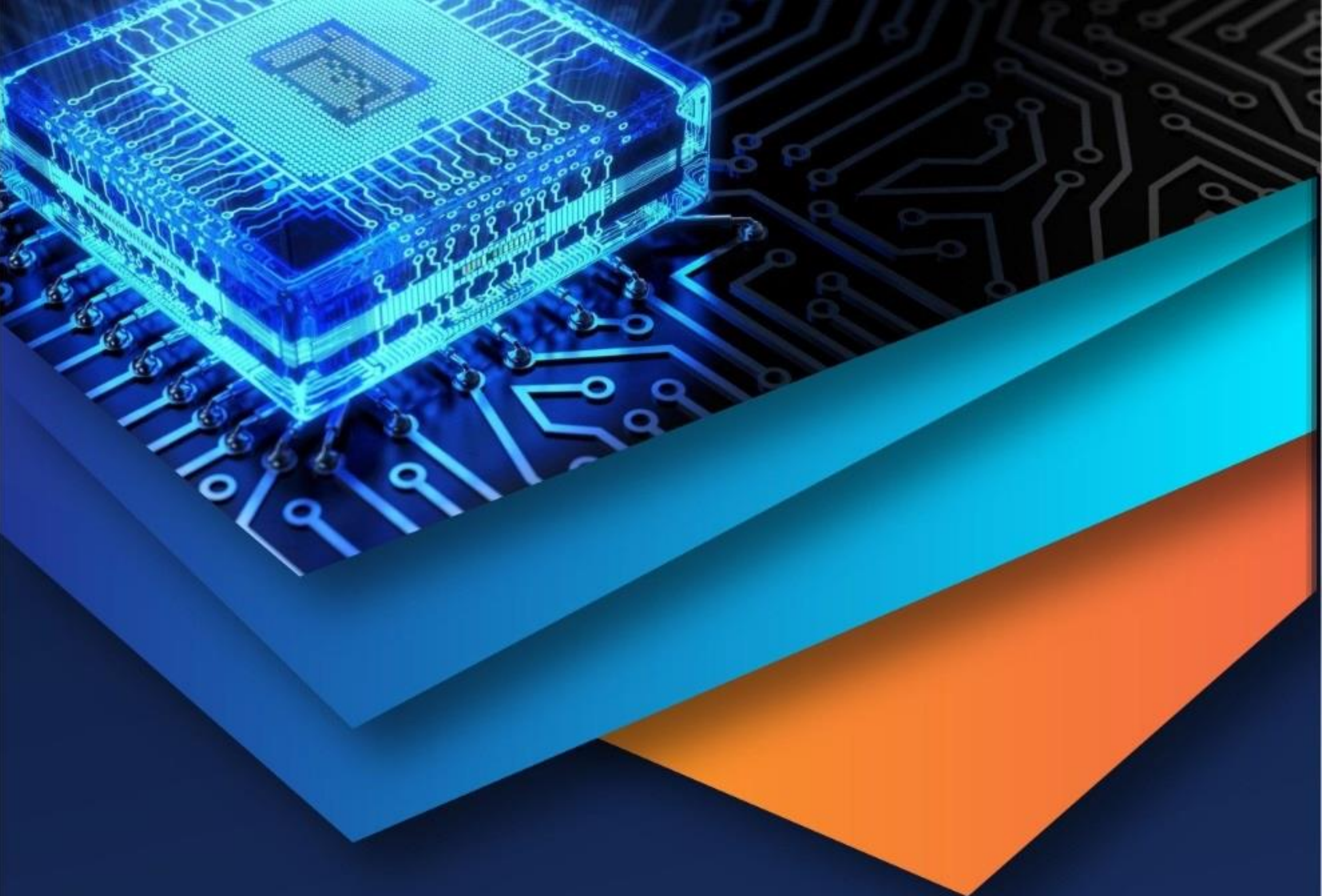

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